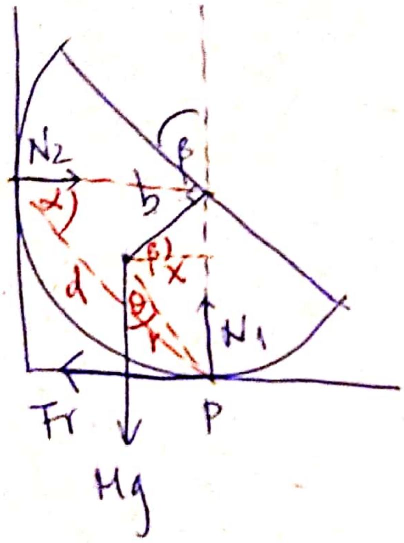
Pauta AUX # 9

P1

a)



b) $\tau = Mg \underbrace{r \sin\theta}_{x = b\cos\beta}$

$= Mgx$

$= \boxed{\frac{1}{2} \cdot \frac{4}{3} \pi R^3 \rho g\, b\cos\beta}$

c) $\sum F_x = N_2 - Fr = 0 \Rightarrow Fr = N_2$

$\sum F_y = N_1 - Mg = 0 \Rightarrow N_1 = Mg$

$\Rightarrow Fr = \mu N_1 = \mu Mg$

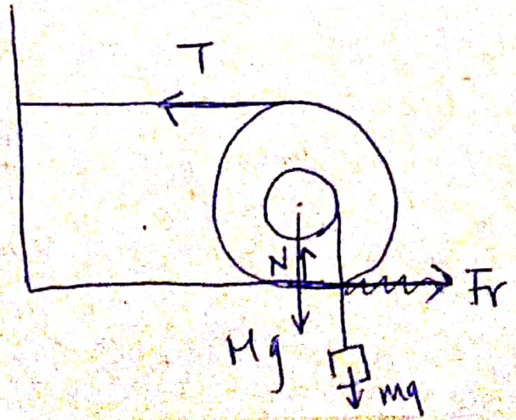
$= \frac{3}{16} \cdot \frac{4}{3} \pi R^3 \rho g = \boxed{\frac{1}{8} \pi R^3 \rho g}$

d) $\sum \tau = Mgx - \overset{= Fr = \mu Mg}{N_2} \underbrace{d \sin\alpha}_{R}$

$= \cancel{Mg} b\cos\beta - \mu \cancel{Mg} R = 0$

$\Rightarrow \beta = \cos^{-1}\left(\frac{\mu R}{b}\right) = \cos^{-1}\left(\frac{\frac{3}{16}R}{\frac{3}{8}R}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \boxed{60^\circ}$

P2



$\sum \tau = TR + FrR - mgr = 0$

$\Rightarrow m = \frac{TR + FrR}{rg}$

$$\sum F_x = Fr - T = 0 \Rightarrow T = Fr = \mu N_1$$

$$\sum F_y = N_1 - Mg - mg = 0 \Rightarrow N_1 = (M+m)g.$$

$$\Rightarrow T = Fr = \mu (M+m) g.$$

$$\Rightarrow m = \frac{2\mu(M+m)gR}{rg}$$

hay que despejarlo.

$$\Rightarrow m\cancel{g}r = 2\mu M\cancel{g}R + 2\mu m\cancel{g}R$$

$$\Rightarrow \boxed{m = \frac{2\mu MR}{r - 2\mu R}}$$

P3

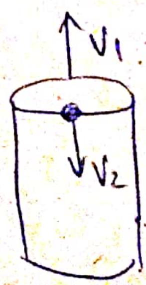
- Vel. inicial permite llegar a 2h:

$$\frac{1}{2}mv_i^2 = mg(2h) \Rightarrow v_i = \sqrt{4gh}$$

- Antes de chocar con el techo:

$$\frac{1}{2}\cancel{m}\cdot 4gh = \cancel{m}gh + \frac{1}{2}\cancel{m}v_f^2 \Rightarrow v_f = \sqrt{2gh} = V.$$

(*) Choque elástico:



- Conservación $\vec{p}$: $mV = mV_1 - mV_2$

$$\Rightarrow V = V_1 - V_2 = \sqrt{2gh} \quad (1)$$

- Conservación E: $\cancel{\frac{1}{2}m}V^2 = \cancel{\frac{1}{2}m}V_1^2 + \cancel{\frac{1}{2}m}V_2^2$.

$$\Rightarrow V^2 = V_1^2 + V_2^2 = 2gh \quad (2)$$

$(1) \Rightarrow V_1 = V_2 + V. \quad (3)$

(3) en $(2) \Rightarrow \cancel{V^2} = V_2^2 + 2V_2V + \cancel{V^2} + V_2^2$

$\Rightarrow 2V_2^2 + 2V_2V = 0.$

$\Rightarrow V_2(V_2 + V) = 0.$

$\Rightarrow V_2 = \begin{cases} 0 & \text{la pelota se queda ahí.} \\ -V & \text{se devuelve con la misma vel.} \end{cases}$

Para efectos de este problema, las dos soluciones son válidas.

① $V_2 = 0 \Rightarrow V_1 = V \Rightarrow$ la pelota cae libremente, y el cilindro sube con velocidad V.

$\therefore$ Pelota: $y = h - \frac{1}{2}gt^2.$
Base cilindro: $y = vt - \frac{1}{2}gt^2.$

Se encuentran $\Rightarrow h - \cancel{\frac{1}{2}gt^2} = vt - \cancel{\frac{1}{2}gt^2}$

$\Rightarrow t = \frac{h}{v}.$

$\Rightarrow y = h - \frac{1}{2}g\frac{h^2}{v^2}.$

$= h - \frac{1}{2}g\frac{h^2}{2gh} = \boxed{\frac{3}{4}h}$

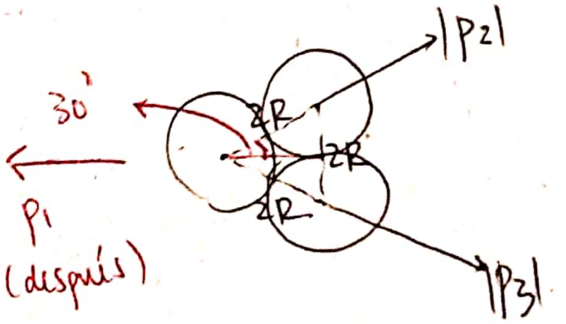
② $V_2 = -V \Rightarrow V_1 = 0 \Rightarrow$ cilindro no se mueve, y la pelota se devuelve con la misma velocidad.

$\therefore$ Ni una gracia, la pelota simplemente cae, y choca con la base en $y = 0$.

Scanned by CamScanner

P4

a) Justo antes del choque:



b) Después: $p_i = p_f$

$\hat{x}$ | $p = 2p_2 \sin 30° - p_1$ (1)

$\hat{y}$ | $p_2 \sin 30 = p_3 \sin 30$

$\Rightarrow p_2 = p_3$.

$E_i = E_f$:

$$\cancel{\frac{1}{2}} \frac{p^2}{\cancel{m}} = \cancel{\frac{1}{2}} \frac{p_2^2}{\cancel{m}} \cdot 2 + \cancel{\frac{1}{2}} \frac{p_1^2}{\cancel{m}}.$$

$\Rightarrow p^2 = 2p_2^2 + p_1^2$. (2)

(1) $\Rightarrow$ $p_1 = 2p_2 \sin 30° - p$. (3)

(3) en (2) $\Rightarrow$ $\cancel{p^2} = \cancel{4}^{2} p_2^2 + \cancel{4}^{2} p_2^2 \sin^2 30° - \cancel{4}^{2} p p_2 \sin 30° + \cancel{p^2}$.

$\Rightarrow p_2 (p_2 + p_2 \sin^2 30° - p \sin 30°) = 0$.

Tomando sol. no nula $\Rightarrow$ $p_2 + \frac{p_2}{4} - \frac{p}{2} = 0$.

$\Rightarrow p_2 = \frac{p}{2} \cdot \frac{4}{5} = \boxed{\frac{2}{5} p}$ (4)

(4) en (3) $\Rightarrow$ $p_1 = \frac{2}{5}p - p = \boxed{\frac{-3}{5} p}$ $\Rightarrow$ se devuelve.

c) Máx elongación:

$$\frac{1}{2}\frac{p_2^2}{m}\cdot 2 = \frac{1}{2}kx^2 + \frac{1}{2}\frac{p_{2x}^2}{m}\cdot 2.$$

p_{2x} constante (no hay fuerza ext. en $\hat{x}$).

$$\Rightarrow p_{2x} = p_2 \sin 30^\circ = \frac{p_2}{2}.$$

$$\therefore \frac{p_2^2}{m} = \frac{1}{2}kx^2 + \frac{p_2^2}{4m}.$$

$$\Rightarrow x^2 = \frac{\frac{3}{4}\frac{p_2^2}{m}}{\frac{1}{2}k}. \quad \Rightarrow x = \sqrt{\frac{3p_2^2}{2mk}} = \sqrt{\frac{3\cdot\frac{\cancel{4}^{2}}{25}p^2}{\cancel{2}mk}} = \boxed{\sqrt{\frac{6p^2}{25mk}}}$$