Resolución del examen

Teoría

$$\oiint_{S_{Load}} \vec{\tilde{S}}_{AV}(\vec{r})\, d\vec{s} = \frac{1}{2}\mathrm{Re}\Big[\underbrace{\oiint_{S_{Load}} (\vec{\tilde{E}}(\vec{r}) \wedge \vec{\tilde{H}}^*(\vec{r}))\, d\vec{s}}_{\xi}\Big]$$

$$\xi = \cancel{\int_0^{2\pi} \hat{e}_z d\varphi \int_0^a r\,dr\, (\vec{\tilde{E}}(r,\varphi,z=0) \wedge \vec{\tilde{H}}^*(r,\varphi,z=0))} \to 0 \text{ pq } \vec{S}_{AV} = S\hat{e}_r,\ d\vec{s} = \hat{e}_z ds \Big\}\text{ además tiende a cero cuand } a\to 0 \quad +$$

$$+ \cancel{\int_0^{2\pi} \hat{e}_z d\varphi \int_0^a r\,dr \big(\vec{\tilde{E}}(r,\varphi,z=\Delta) \wedge \vec{\tilde{H}}^*(r,\varphi,z=\Delta)\big)} \to 0 \text{ Mismas razones} \quad +$$

$$\int_0^{2\pi} d\varphi \int_0^{\Delta} \hat{e}_r dz \,(\vec{\tilde{E}}(r=a,\varphi,z) \wedge \vec{\tilde{H}}^*(a,\varphi,z)) = \underbrace{\int_0^{2\pi} |\vec{\tilde{H}}^*(a,\varphi,z)|\, d\varphi}_{\tilde{I}^*} \underbrace{\int_0^{\Delta} -|\vec{\tilde{E}}(a,\varphi,z)|\, dz}_{\tilde{V}}$$

Notar que $\vec{\tilde{E}}(a,\varphi,z) \neq 0$ cuando $a\to 0$. Igual $\vec{\tilde{H}}^*(a,\varphi,z)$

Además
$$\left.\begin{aligned} \vec{\tilde{H}}^*(r=a,\varphi,z) &= h(a,\varphi,z)\,\hat{e}_\varphi \\ \vec{E}(a,\varphi,z) &= e(a,\varphi,z)\,\hat{e}_z \end{aligned}\right\} \hat{e}_z \wedge \hat{e}_\varphi = -\hat{e}_r$$

$$\oiint_{S_{Load}} \vec{\tilde{S}}_{AV}(\vec{r})\, d\vec{s} = \frac{1}{2}\mathrm{Re}\left(-\int_{\Delta V} \vec{\tilde{E}}(\vec{r})\, d\vec{l}_z \oint \vec{\tilde{H}}^*(\vec{r})\, d\vec{l}_\varphi\right) = \frac{1}{2}\mathrm{Re}\left(\tilde{V}\tilde{I}\right)$$

Problema 1

Orden cero implica $\psi(x',y',z') = Ae^{-jBz'}\psi_0(x',y')$

$$\left(1+\frac{y'}{R}\right)\cancel{Ae^{-jBz'}}\left(\frac{\partial^2\psi_0}{\partial x'^2}+\frac{\partial^2\psi_0}{\partial y'^2}+\frac{1}{R}\frac{\partial\psi_0}{\partial y'}+K^2\psi_0\right)+$$

$$\cancel{\frac{\partial^2\psi_0}{\partial z'^2}}\cancel{Ae^{-jBz'}} - B^2\cancel{Ae^{-jBz'}}\psi_0 = 0$$

0

Desprecio todos los términos que van con $\frac{1}{R^\alpha}$ con $\alpha = 1,2,\dots,\infty$, Me quedo con $\left(\frac{1}{R}\right)^0 = 1$

Ese es el orden cero de la ecuación diferencial

$$\frac{\partial^2\psi_0}{\partial x'^2}+\frac{\partial^2\psi_0}{\partial y'^2}+K^2\psi_0-B^2\psi_0=0$$

En orden cero $B^2=K'^2$, en consecuencia $K'^2=\frac{4\pi}{d}-\left(\frac{\pi}{a}\right)^2$

Por definición $B=\frac{2\pi}{d} \Rightarrow \quad K'^2=K^2-\left(\frac{\pi}{a}\right)^2$

$$\frac{\partial^2\psi_0}{\partial x'^2}+\frac{\partial^2\psi_0}{\partial y'^2}+\left(\frac{\pi}{a}\right)^2\psi_0=0$$

; A partir de la solución propuesta supongo como solución particular $\psi_0(x',y')=C_1\cos(Nx')+C_2\cos(My')$; Sustituimos en la ecuación diferencial:

$$-C_1N^2\cos(Nx')-C_2M^2\cos(My')+\left(\frac{\pi}{a}\right)\left(C_1\cos(Nx')+C_2\cos(My')\right)=0$$

Se verifica sii $N=\frac{\pi}{a}$ y $M=\frac{\pi}{a}$

Determino C_1 y C_2 a partir de las condiciones de frontera.

$$\left.\frac{\partial \Psi_0}{\partial x'}\right|_{x'=0}=0 \;;\; \left.\frac{\partial \Psi_0}{\partial y'}\right|_{y'=0}=0 \;;\; \frac{\partial \Psi_0}{\partial x'}+\frac{\partial \Psi_0}{\partial y'}=0 \text{ si } y'+x'=a$$

$$\Downarrow$$

$$y'=a-x'$$

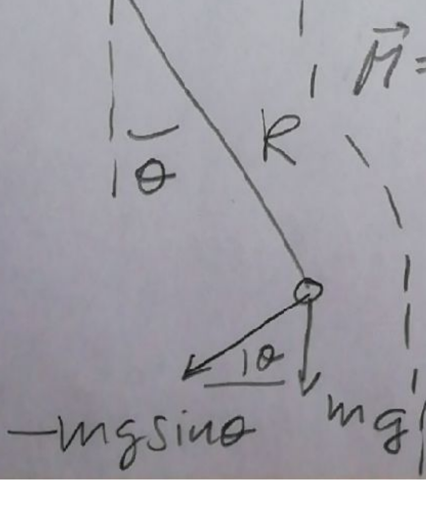
$$C_1\sin\left(\frac{\pi}{a}x'\right)+C_2\sin\left(\frac{\pi}{a}(a-x')\right)=0$$

$$\sin\left(\frac{\pi}{a}(a-x')\right)=\sin\pi\cos\left(\frac{\pi}{a}x'\right)-\cos(\pi)\sin\left(\frac{\pi}{a}x'\right)=\sin\left(\frac{\pi}{a}x'\right)$$

$$C_1+C_2=0 \Rightarrow C_1=-C_2 \Rightarrow \boxed{C_1=1 \text{ con lo cual } C_2=-1}$$

Hay infinitas posibilidades, la más simple ↑

<u>Problema 2</u>



$$\sum\vec{M}=I\ddot{\theta} \Rightarrow -mg\sin\theta R=mR^2\ddot{\theta}$$

$$\vec{M}=\vec{r}\wedge\vec{F}=R|\vec{F}|\hat{e}_\theta$$

↓ Paraxial $\sin\theta\simeq\theta$

$$\ddot{\theta}+\frac{g}{R}\theta=0 \Rightarrow \theta(t)=A\sin(wt+\phi)$$

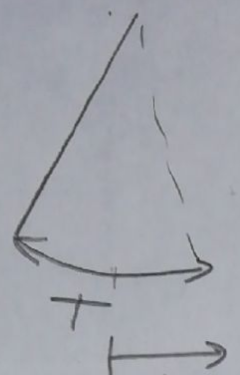
Propuesta

$$\ddot{\theta}=-Aw^2\sin(wt)$$

$$\downarrow$$

$$-Aw^2\sin(wt+\phi)+\frac{g}{R}A\sin(wt+\phi)=0 \Rightarrow w=\sqrt{\frac{g}{R}}$$

$$\theta(t) = A \sin\left(\sqrt{\frac{g}{R}}\, t + \phi\right) \quad \text{Periodo} \quad \frac{2\pi}{T} = \sqrt{\frac{g}{R}} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$



Velocidad: $\dot{\theta}(t) = A\sqrt{\frac{g}{R}} \cos\left(\sqrt{\frac{g}{R}}\, t + \phi\right)$

$d = R\sin\theta$; $\dot{\theta}(t=0) = 0 \Rightarrow A\sqrt{\frac{g}{R}} \cos(\phi) = 0$

$$\phi = \frac{\pi}{2}$$

$R \sin(\theta(t=0)) = d \Rightarrow$ Paraxial $R\theta(t=0) \simeq d$

$$R A\sqrt{\frac{g}{R}} \sin\left(\frac{\pi}{2}\right) = d \Rightarrow A = \frac{d}{\sqrt{gR}}$$

$$\boxed{\theta(t) = \frac{d}{\sqrt{gR}} \sin\left(\sqrt{\frac{g}{R}}\, t + \frac{\pi}{2}\right)} \qquad t = \frac{T}{4} \quad \sin\left(\sqrt{\frac{g}{R}}\, \frac{\pi}{2}\sqrt{\frac{R}{g}} + \frac{\pi}{2}\right) = 0 \ \text{cqd}$$

$$\text{Area} = \frac{R^2\theta(t)}{2} \Rightarrow \text{fem} = -\partial_t \phi = \frac{B_0 R}{2} \partial_t \theta(t) =$$

$$= -\frac{B_0 R^2}{2} \frac{d}{\sqrt{gR}} \sqrt{\frac{g}{R}} \cos\left(\sqrt{\frac{g}{R}}\, t + \frac{\pi}{2}\right) = -\frac{B_0}{2} dR \cos\left(\sqrt{\frac{g}{R}}\, t + \frac{\pi}{2}\right)$$

$$\left.\frac{d(f.e.m)}{dt}\right]_{t=\frac{T}{4}} = 0 \ ; \ \left.\frac{d^2(f.e.m)}{dt^2}\right]_{t=\frac{T}{4}} < 0 \ \text{es un maximo}$$

$$f.e.m.\left(t = \frac{T}{4}\right) = -\frac{B_0}{2} dR \cancelto{-1}{\cos\left(\sqrt{\frac{g}{R}}\, \frac{2\pi}{4}\sqrt{\frac{R}{g}} + \frac{\pi}{2}\right)} = \frac{B_0 R d}{2} = \underline{\underline{50\,mV}}$$

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b) $$Q = \int_{-\frac{d}{2}}^{\frac{d}{2}} dz \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} dx \; 2\frac{\mu C}{cm^3} \ln(|z|)\,\delta(x)\,\delta(y) =$$

$$= 2\int_0^{\frac{d}{2}} 2\frac{\mu C}{cm} \ln(z)\,dz = 4\frac{\mu C}{cm}\int_0^{\frac{d}{2}} \ln(z)\,dz$$

$$\int v\,du = uv - \int u\,dv \quad \left\{\begin{array}{l} du = dz \\ v = \ln(z) \\ dv = \frac{1}{z}dz \\ u = z \end{array}\right\} = z\ln(z) - \int z\frac{1}{z}dz = z\ln(z) - z$$

$\int \ln(z) = z\ln(z) - z$ Evaluo en los límites

$$4\frac{\mu C}{cm}\left(\frac{d}{2}\left(\ln\left(\frac{d}{2}\right)-1\right) - \left.\frac{\ln(z)}{\frac{1}{z}}\right|_{z\to 0} + \cancelto{0}{z}\right) = 4\frac{\mu C}{cm}\,5cm(\ln(5)-1)$$

$$\frac{\ln(z)}{\frac{1}{z}} \to 0 \qquad \xrightarrow{LH} \qquad \frac{\frac{1}{z}}{-\frac{1}{z^2}} = \frac{-z}{1} \to 0$$

$$\boxed{Q = 12.2\,\mu C}$$

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c) En pocas palabras la ley de Lenz. Al poner la carga en movimiento se induce un campo magnético que contrarresta el campo causante de la f.e.m. y que ponía la carga en movimiento