Pauta Control #1

P.1

a) $\operatorname{div}(\vec{A}) = \frac{1}{\rho}\left\{ \frac{\partial}{\partial\theta}(\operatorname{sen}(\theta)) + \frac{\partial}{\partial z}(\rho z)\right\}$ (0,5)

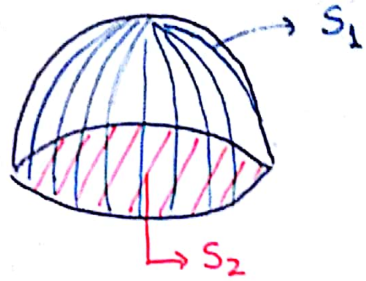
$= \frac{1}{\rho}\cos(\theta) + 1$ (1,0)

b) Calcular flujo saliendo de Ω:

- El borde tiene 2 partes:

S_1: Mitad superior esfera.

S_2: Disco / Tapa inferior.



(1°) flujo en S_2:

$\hat{n} = -\hat{z}$ $\quad z = 0 \Rightarrow \vec{A} = \operatorname{sen}(\theta)\hat{\theta} + 0\hat{z}$.

$\vec{A}\cdot\hat{n} = (\operatorname{sen}(\theta)\hat{\theta} + 0\hat{z})(-\hat{z}) = 0 \Rightarrow \left[\text{Flujo}_{S_2} = 0\right]$ (0,5)

(2°) flujo en S_1:

$\hat{n} = \hat{r}$ $\quad \hat{r} = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{x} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{y} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{z}$

$\vec{A}\cdot\hat{n} = \operatorname{sen}(\theta)\underbrace{\hat{\theta}\cdot\hat{r}}_{0.} + z\hat{z}\cdot\hat{r}$

$\left[\vec{A}\cdot\hat{n} = \frac{z^2}{\sqrt{x^2+y^2+z^2}}\right]$ (0,5)

- Para calcular el flujo si $(r,\varphi,\theta) \xrightarrow{\Phi} (x(r,\varphi,\theta), y(r,\varphi,\theta), z(r,\varphi,\theta))$

$$\frac{\partial\Phi}{\partial\varphi} * \frac{\partial\Phi}{\partial\theta} = h_\varphi \hat{\varphi} \times h_\theta \hat{\theta} = r^2 \operatorname{sen}(\varphi)\,\hat{r}$$

$$\text{Flujo en } S_1 : \int_0^{\pi/2}\int_0^{2\pi} \frac{z^2}{r} \cdot r^2 \operatorname{sen}(\varphi)\, d\theta\, d\varphi$$

donde $r = R$ y $z = r\cos(\varphi)$

$$F_{S1} : \int_0^{\pi/2}\int_0^{2\pi} R^2\cos^2(\varphi)\operatorname{sen}(\varphi) \cdot R\, d\theta\, d\varphi$$

$$= 2\pi R^3 \int_0^{\pi/2} \operatorname{sen}(\varphi)\cos^2(\varphi)\, d\varphi$$

$$F_{S1} = 2\pi R^3 \left(-\frac{\cos^3(\varphi)}{3}\right)\Bigg|_0^{\pi/2} = \frac{2}{3}\pi R^3 . \qquad (0,5)$$

c) $\vec{A}$ en coordenadas cartesianas.

① $\hat{\theta} = \frac{1}{\rho}(-y, x, 0) = \frac{1}{\sqrt{x^2+y^2}}(-y, x, 0)$ (0,5)

$\hat{z} = \hat{k}$

② $x = \rho\cos(\theta)$
$y = \rho\,\text{sen}(\theta) \rightarrow \text{sen}(\theta) = \frac{y}{\rho} = \frac{y}{\sqrt{x^2+y^2}}$ (0,5)
$z = z$

$$\vec{A} = \frac{y}{x^2+y^2}(-y, x, 0) + z(0,0,1)$$

$$\vec{A} = \left(\frac{-y^2}{x^2+y^2}, \frac{xy}{x^2+y^2}, z\right)$$ (0,5)

d) sin tensor

$(w_1, w_2, w_3) \times (x_1, x_2, x_3)$

$= (w_2x_3 - w_3x_2, w_3x_1 - w_1x_3, w_1x_2 - w_2x_1)$ (0,5)

$\text{rot}(F_1, F_2, F_3) = \left(\frac{\partial}{\partial 1}, \frac{\partial}{\partial}, \frac{\partial}{\partial}\right) \times (F_1, F_2, F_3)$

$\text{rot}(\vec{w} \times \vec{r}) = (w_1 - (-w_1), w_2 - (-w_2), w_3 - (-w_3))$

$= 2\vec{w}$. (1,0)

- CON TENSOR:

$$\vec{\omega} \times \vec{r} = \sum_{ijk} \varepsilon_{ijk}\, \omega_i x_j\, \hat{e}_k$$

$$(\vec{\omega} \times \vec{r})_k = \sum_{ij} \varepsilon_{ijk}\, \omega_i x_j$$

$$\mathrm{rot}(\vec{\omega} \times \vec{r}) = \sum_{pkq} \varepsilon_{pkq} \frac{\partial}{\partial x_p} (\omega \times r)_k\, \hat{e}_q$$

cte.

$$= \sum_{pkq} \varepsilon_{pkq} \frac{\partial}{\partial x_p} \left(\sum_{ij} \varepsilon_{ijk}\, \omega_i\, x_j \right) \hat{e}_q \qquad (0,3)$$

$$= \sum_{ijpkq} \varepsilon_{pkq}\, \varepsilon_{ijk}\, \omega_i \frac{\partial x_j}{\partial x_p}\, \hat{e}_q \qquad \left(\frac{\partial x_j}{\partial x_p} = \delta_{jp} \right) (0,2)$$

Cambio índice

$$= \sum_{ijpkq} - \varepsilon_{pqk}\, \varepsilon_{ijk}\, \omega_i\, \delta_{jp}\, \hat{e}_q \qquad (0,3)$$

$$= \sum_{ijpq} - (\delta_{pi}\delta_{qj} - \delta_{pj}\delta_{qi})\, \underline{\underline{\delta_{jp}}}\, \omega_i\, \hat{e}_q$$

$j \to p$

$$= \sum_{ipq} (\delta_{pp}\delta_{qi} - \delta_{pi}\delta_{qp})\, \omega_i\, \hat{e}_q \qquad (+0,2)$$

$$= \sum_{ipq} \delta_{qi}\, \delta_{pp}\, \omega_i\, \hat{e}_q - \delta_{pi}\, \delta_{qp}\, \omega_i\, \hat{e}_q$$

$p = q$

$$= \sum_{iq} \delta_{qi}\, 3\, \omega_i\, \hat{e}_q - \sum \delta_{qi}\, \omega_i\, \hat{e}_q \qquad (0,2)$$

$$= 2 \sum \omega_q\, \hat{e}_q = 2\vec{\omega}. \qquad (0,3)$$

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