Auxiliar #9

P1 a) Ocupando la fórmula

$$L_0^x(f) = \int_0^x \sqrt{1+f'(t)^2}\,dt = x^2 + 2x - f(x) \qquad \frac{d}{dx}$$

$$\Rightarrow \sqrt{1+f'(x)^2} = 2x + 2 - f'(x)$$

$$\Rightarrow 1 + f'^2(x) = \left(2(x+1) - f'(x)\right)^2$$

$$\Rightarrow 1 + \cancel{f'^2(x)} = 4(x+1)^2 - 4(x+1)f'(x) + \cancel{f'^2(x)}$$

$$\Rightarrow f'(x) = x + 1 - \frac{1}{4(x+1)}$$

$$\Rightarrow f(x) = \frac{x^2}{2} + x - \frac{1}{4}\ln(x+1) + C$$

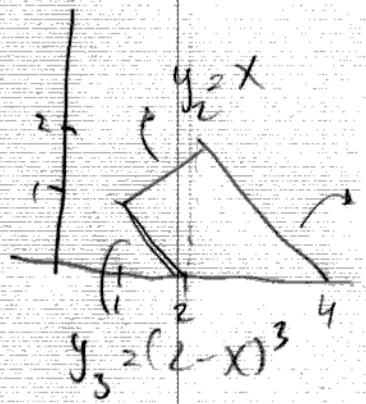
y ocupando que $f(0) = 0$.

$$f(0) = \frac{0^2}{2} + 0 - \cancel{\frac{1}{4}\ln(1)}^{\,0} + C = C$$

$$\rightarrow C = 0$$

$$\therefore f(x) = \frac{x^2}{2} + x - \frac{1}{4}\ln(x+1)$$

P2



$\rightsquigarrow$ Calcularemos el manto que genera cada una y lo sumaremos

(Notar que es distinto del área o volumen ya que aquí no se resta)

$$S_{ox}(y_1) = 2\pi \int_2^4 (4-x)\sqrt{1+(-1)^2}\,dx$$

$$= 2\pi \int_2^4 (4-x)\sqrt{2}\,dx = 2\sqrt{2}\pi \int_2^4 (4-x)\,dx$$

$$= 2\sqrt{2}\pi \left(4x - \frac{x^2}{2}\right)\Big|_2^4 = 2\sqrt{2}\pi\left[16-8-(8-2)\right]$$

$$= 4\sqrt{2}\pi$$

$$S_{ox}(y_2) = 2\pi \int_1^2 x\sqrt{2}\,dx = 2\sqrt{2}\pi\left(\frac{x^2}{2}\right)\Big|_1^2 = 2\sqrt{2}\pi\left(2-\frac{1}{2}\right)$$

$$= 3\sqrt{2}\pi$$

$$S_{ox}(y_3) = 2\pi \int_1^2 (2-x)^3 \sqrt{1+9(2-x)^4}\, dx$$

$$u = 1+9(2-x)^4 \qquad x=1 \Rightarrow u=10$$

$$du = 36(2-x)^3 \cdot -1\, dx \qquad x=2 \Rightarrow u=1$$

$$= \frac{2\pi}{36} \int_{10}^{1} -\sqrt{u}\, du = \frac{\pi}{18} \int_1^{10} \sqrt{u}\, du$$

$$= \frac{\pi}{18} \cdot \frac{2}{3} \left(u^{3/2} \right) \Big|_1^{10}$$

$$= \frac{\pi}{27} \left(10\sqrt{10} - 1 \right)$$

$$\Rightarrow \quad S = S_{ox}(y_1) + S_{ox}(y_2) + S_{ox}(y_3)$$

$$= \pi \left(4\sqrt{2} + 3\sqrt{2} + 10\sqrt{10} - 1 \right)$$

P2 Para dibujar en Polares se aconseja hacer una tabla

θ	r
θ_0	r_0
θ_1	r_1

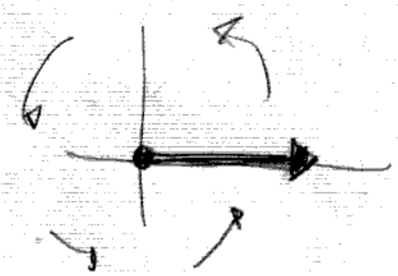
e ir marcando los ptos.

Si se deben graficar los ptos

θ	r
$\pi/2$	3
π	-2

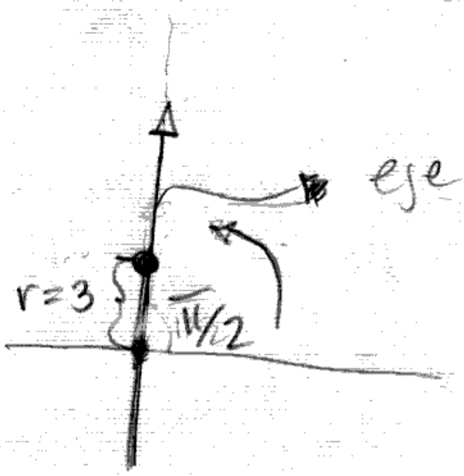
se procederá de la sgte manera

teniendo el plano XY, se parte poniendo el angulo,



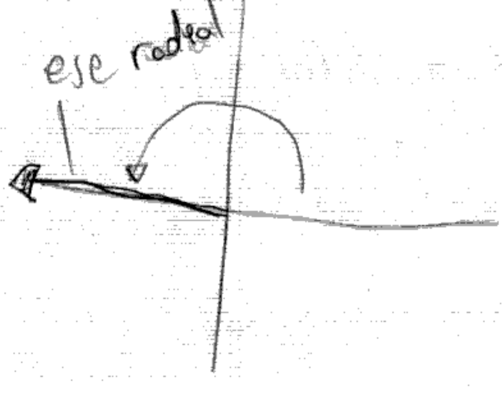
este crecerá en contra del reloj y formará un eje en el cual pondremos el valor del radio

$\theta = \frac{\pi}{2}$, $r = 3$

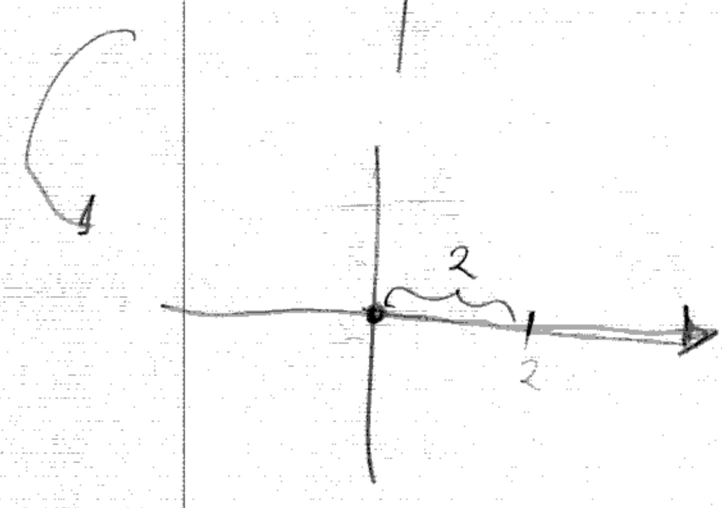


eje radial, aqui en la dirección que veniamos, pondremos el 3

$\theta = \pi$, $r = -2$

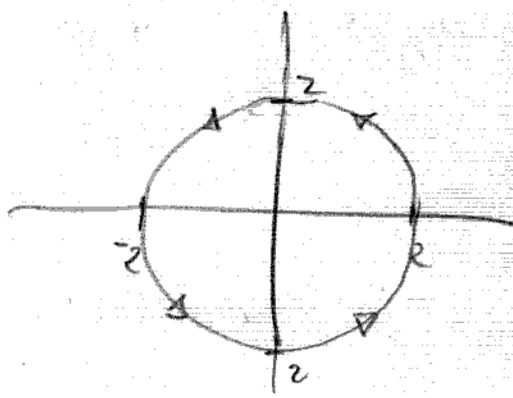


Como $r < 0$, el sentido del eje cambia



P2 a) $r = 2$

θ	r
$\pi/2$	2
π	2
$\vdots$	2
θ	2

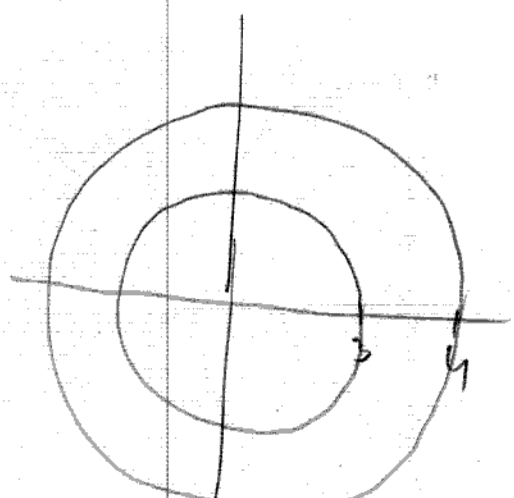


b) $r^2 - r - 12 = 0$

$\Updownarrow$

$(r+3)(r-4) = 0 \Rightarrow r = -3 \lor r = 4$

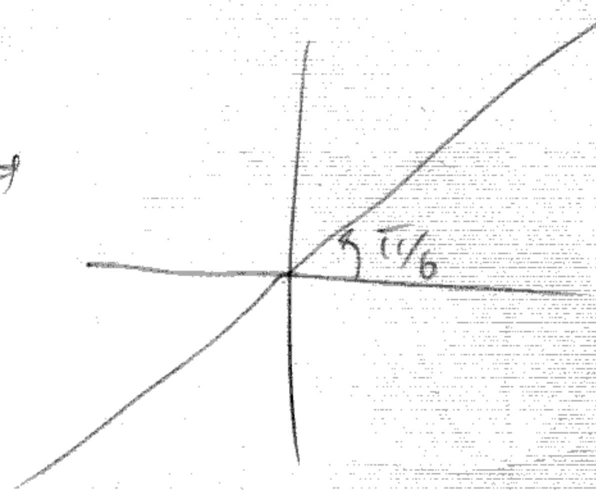
Dos circunferencias



c) $\theta = \frac{\pi}{6}$

θ	r
$\pi/6$	0
$\pi/6$	1
$\vdots$	$\vdots$
$\pi/6$	r

$\Rightarrow$

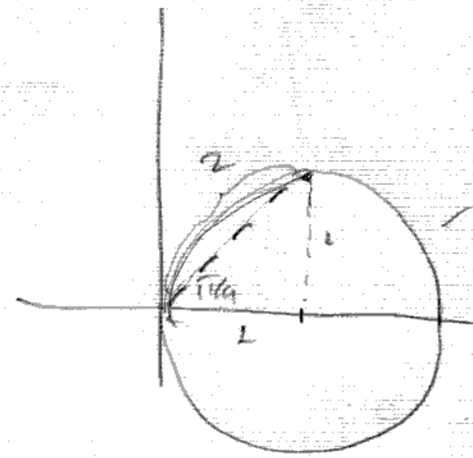


recta

c) $r = 2\cos(\theta)$

r	θ
2	0
$\sqrt{2}$	$\pi/4$
$\sqrt{2}$	$-\pi/4$
-2	π
0	$\pi/2$
1	$-\pi/3$

son equivalentes

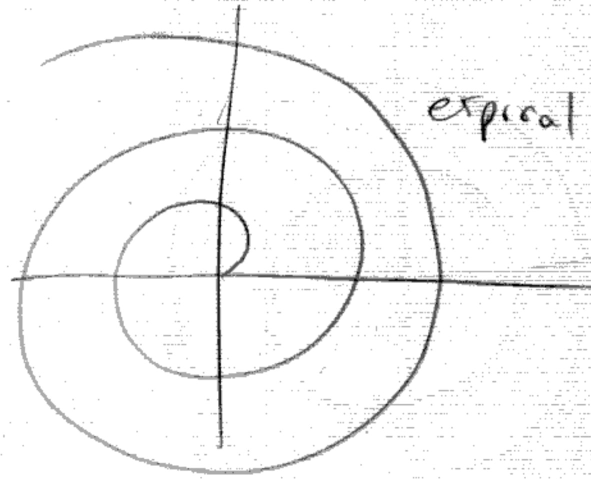


circunferencia

d) $r = \dfrac{\theta}{2\pi}$

si θ crece $\Rightarrow$ r crece

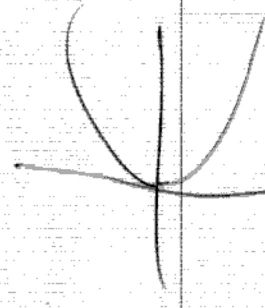
r	θ
0	0
1	2π
2	4π
$1/4$	$\pi/2$
$1/2$	π



espiral

e) $r = \tan(\theta)\sec(\theta) = \dfrac{\operatorname{sen}(\theta)}{\cos^2(\theta)}$ $\Rightarrow$ $r\cos^2(\theta) = \operatorname{sen}(\theta) \quad / \cdot r$

$\Rightarrow$ $r^2\cos^2(\theta) = r\operatorname{sen}(\theta)$

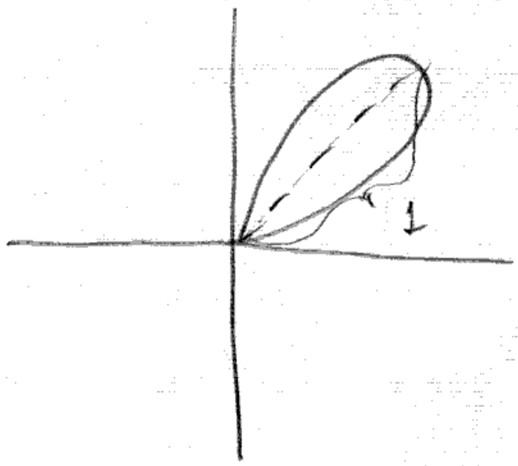


$y = x^2$

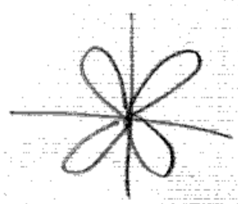
$\Rightarrow$ $x^2 = y$

g) $r = \text{sen}(2\theta)$

r	θ
0	0
$\sqrt{3}/2$	$\pi/6$
1	$\pi/4$
$\sqrt{3}/2$	$\pi/3$
0	$\pi/2$



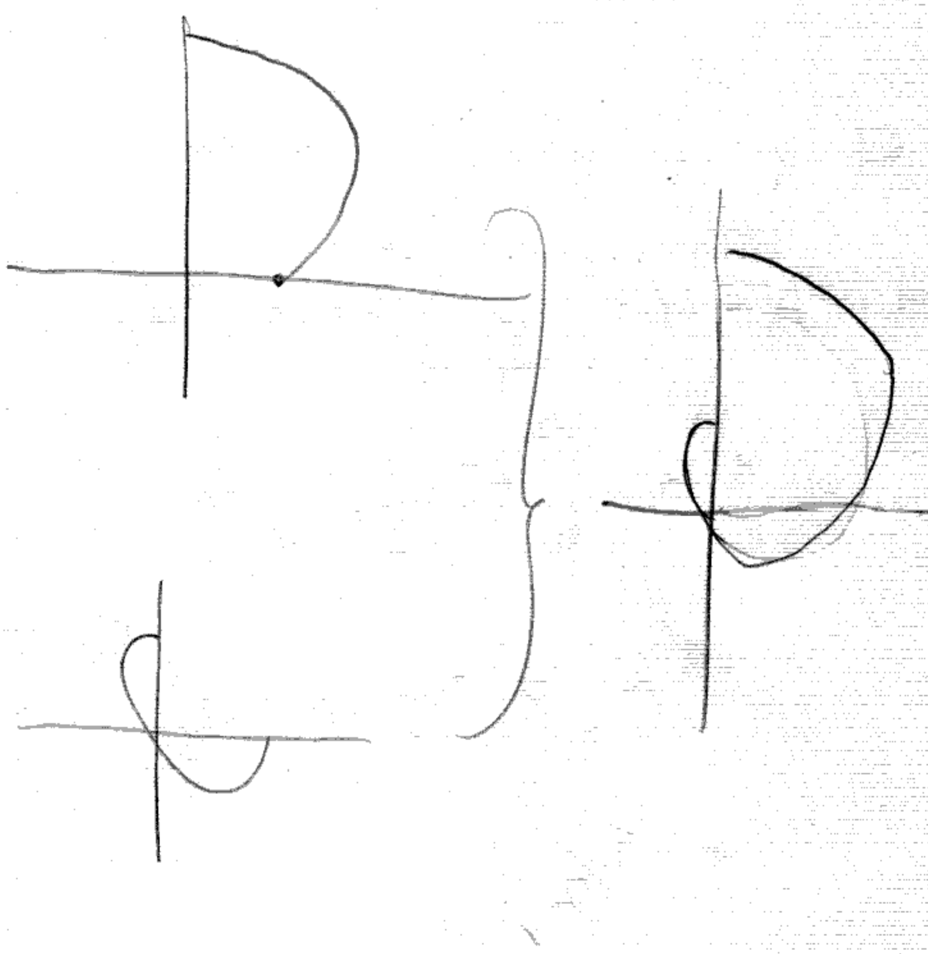
Si continuan quedarán 4 petalos



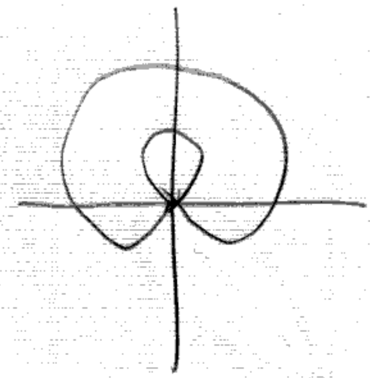
h) $r = 1 + 2\,\text{sen}\theta$

r	θ
1	0
2	$\pi/6$
$1+\sqrt{2}$	$\pi/4$
$1+\sqrt{3}$	$\pi/3$
3	$\pi/2$

r	θ
0	$-\pi/6$
$1-\sqrt{2}$	$-\pi/4$
$1-\sqrt{3}$	$-\pi/3$
-1	$-\pi/2$

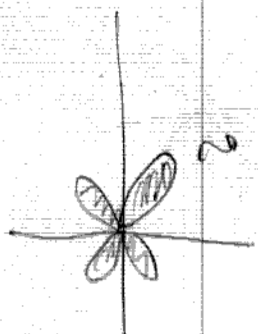


Si siguen



P3 a) Area de $\rho(\theta) = \operatorname{Sen}(2\theta)$

↳ 4 petalos



no necesitamos calcular 1 petalo y multiplicamos por 4

$\Rightarrow$ El primer petalo se forma entre $\theta = 0$ y $\theta = \frac{\pi}{2}$

$$\Rightarrow A = 4 \cdot \frac{1}{2} \int_0^{\pi/2} (\operatorname{Sen}(2\theta))^2 \, d\theta$$

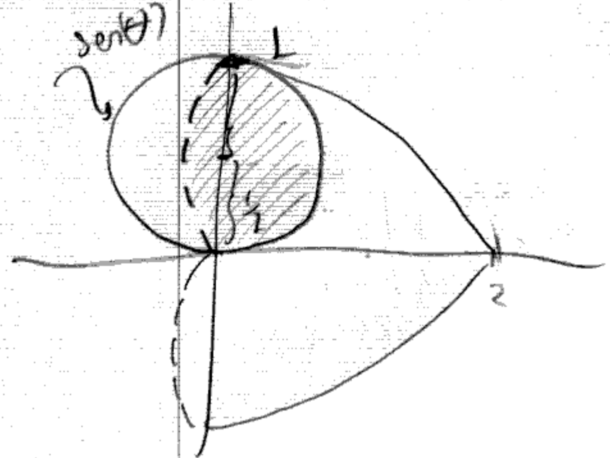
$$\operatorname{Cos}(2\theta) = \operatorname{Cos}^2(\theta) - \operatorname{Sen}^2(\theta)$$
$$= 1 - 2\operatorname{Sen}^2(\theta)$$

$$= 2 \int_0^{\pi/2} \frac{1 - \operatorname{Cos}(4\theta)}{2} \, d\theta$$

$$= 2 \left[\theta - \frac{1}{4} \operatorname{Sen}(4\theta) \right] \Bigg|_0^{\pi/2}$$

$$= \pi/2$$

b) $\rho_1 = \text{sen}(\theta) \qquad \rho_2 = 1+\cos(\theta)$



$$\text{Area} = \quad + \quad$$

Semicírculo

Sección cardiode de 2^{do} cuadrante

$\pi/2$ a π

$$\text{Area} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{8}$$

$$\text{Area} = \frac{1}{2}\int_{\pi/2}^{\pi} (1+\cos(\theta))^2 = \frac{3\pi}{8} - 1$$

hagan6, yo use wolfram

$$\Rightarrow \text{Area} = \frac{\pi}{8} + \frac{3\pi}{8} - 1 = \pi/2 - 1$$

P4 $A_n = \frac{1}{2}\int_{2(n-1)\pi}^{2n\pi} (e^{-\theta})^2\, d\theta = \frac{1}{2}\int_{(2n-2)\pi}^{2n\pi} e^{-2\theta}\, d\theta$

$$= -\frac{1}{4}\left[e^{-2\theta}\right]\Big|_{(2n-2)\pi}^{2n\pi}$$

$$= -\frac{1}{4}\left[e^{-4n\pi} - e^{-(4n-4)\pi}\right]$$

$$= \underbrace{\frac{1}{4}\left[e^{-4(n-1)\pi} - e^{-4n\pi}\right]}_{\text{Area n-esima vuelta}}$$

$$A_1 = \frac{1}{4}\left(1 - e^{-4\pi}\right)$$

$$\Rightarrow A_n = \frac{1}{4} e^{-4(n-1)\pi}\left(1 - e^{-4\pi}\right)$$

$$= A_1 e^{-4(n-1)\pi}$$

$\blacksquare$