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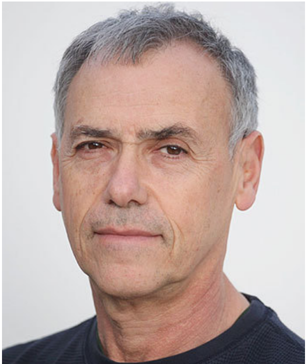
Outline

Introduction

- We saw the Nash bargaining model that is the seminal paper in the bargaining model literature.
- Although it is a very tractable in empirical terms and have solid axiomatic foundations, the NB model does not provide a non-cooperative game to support the result.
- Since it is not a structural game, there is no description of the protocol that players follow to find a solution.
- That's exactly the Rubinstein's model we'll see now.

Introduction

- A key feature of Rubinstein's model of bargaining is that it specifies a standard procedure of bargaining: the players take turns to make offers to each other until agreement is secured.
- This model has much intuitive appeal, since making offers and counteroffers lies at the heart of many real-life negotiations.



Insights

- Insight One: frictionless bargaining processes are indeterminate. A bargaining process may be considered *frictionless* if the players do not incur any costs by haggling (i.e., by making offers and counteroffers) in which case there is nothing to prevent them from haggling forever.
- Insight Two: player's bargaining power depends on the relative magnitude of the players' respective costs of haggling, with the absolute magnitudes of these costs being irrelevant to the bargaining outcome.

Rubinstein Bargaining Model: Setting

Two players, A and B, bargain over the partition of a cake of size π, where $\pi > 0$ according to the following, alternating-offers, procedure.

1. At time 0 player A makes an offer to player B. An offer is a proposal of a partition of the cake.
2. If player B accepts the offer, then agreement is struck and the players divide the cake according to the accepted offer.
3. If player B rejects the offer, then she makes a counteroffer at time $\Delta > 0$.
4. If this counteroffer is accepted by player A, then agreement is struck. Otherwise, player A makes a counter-counteroffer at time 2Δ.
5. This process of making offers and counteroffers continues until a player accepts an offer.

Payoffs

The payoffs are as follows. If the players reach agreement at time $t\Delta$ (with $t = 0, 1, 2, ...$) on a partition that gives player $i(i = A, B)$ a share $x_i, (0 < x_i < \pi)$ of the cake, then player i's payoff is

$$x_i \exp(-r_i t\Delta) = x_i \delta_i^t$$

where $r_i > 0$ is player i's discount rate, and $\delta_i = \exp(-r_i\Delta) \in (0, 1)$ is the discount factor.

On the other hand, if the players perpetually disagree (i.e., each player always rejects any offer made to her), then each player's payoff is zero.

Subgame Perfect Equilibrium (SPE)

The subgame perfect equilibrium (SPE) concept will be employed to characterize the outcome of this game.

Consider a SPE that satisfies the following two properties

- Property *No Delay*: Whenever a player has to make an offer, her equilibrium offer is accepted by the other player.
- Property *Stationarity*: In equilibrium, a player makes the same offer whenever she has to make an offer.

Equilibrium Offer

Given Property *Stationarity*, let x_i^* denote the equilibrium offer that player i makes whenever she has to make an offer.

Consider an arbitrary point in time at which player A has to make an offer to player B.

It follows from Properties *No Delay* and *Stationarity* that player B's equilibrium payoff from rejecting any offer is $\delta_B x_B^*$.

Equilibrium Offer

Why the offer equals $\delta_B x_B^*$?

Because, by Property *Stationarity*, she offers x_B^* after rejecting any offer, which, by Property *No Delay*, is accepted by player A.

Perfection requires that player B accept any offer x_A such that $\pi - x_A > \delta_B x_B^*$, and reject any offer $\pi - x_A < \delta_B x_B^*$.

Furthermore, it follows from Property *No Delay* that $\pi - x_A^* \geq \delta_B x_B^*$. However, $\pi - x_A^*$ cannot be strictly larger than $\delta_B x_B^*$, otherwise player A could increase her payoff slightly.

Hence:

$$\pi - x_A^* = \delta_B x_B^*$$

Equilibrium Offer

Our finding is

$$\pi - x_A^* = \delta_B x_B^*$$

This equation states that player B is indifferent between accepting and rejecting player A's equilibrium offer.

By a symmetric argument (with the roles of A and B reversed), it follows that player A is indifferent between accepting and rejecting player B's equilibrium offer. Thus

$$\pi - x_B^* = \delta_A x_A^*$$

Solving the system yields the unique Subgame Perfect Equilibrium.

Offers in Equilibrium

The uniqueness of the solution to the equation system means that there exists at most one SPE satisfying Properties *No Delay* and *Stationarity*.

In that SPE, player A always offers

$$x_A^* = \left(\frac{1-\delta_B}{1-\delta_A\delta_B}\right)\pi$$

and player A always accepts an offer x_B if and only if

$$\begin{aligned} \pi - x_B &\geq \delta_A x_A^* \\ \Leftrightarrow x_B &\leq \pi - \delta_A x_A^* \end{aligned}$$

Offers in Equilibrium

Similarly, player B always offers

$$x_B^* = \left(\frac{1 - \delta_A}{1 - \delta_A \delta_B} \right) \pi$$

and player B will always accept an offer x_A if and only if

$$\begin{aligned} \pi - x_A &\geq \delta_B x_B^* \\ \Leftrightarrow x_A &\leq \pi - \delta_B x_B^* \end{aligned}$$

Proposition

The following pair of strategies is a subgame perfect equilibrium of the basic alternating-offers game:

- Player A always offers x_A^* and always accepts an offer x_B if and only if $x_B \leq x_B^*$
- player B always offers x_B^* and always accepts an offer x_A if and only if $x_A \leq x_A^*$

where $x_A^* = \left(\frac{1-\delta_B}{1-\delta_A\delta_B}\right)\pi$ and $x_B^* = \left(\frac{1-\delta_A}{1-\delta_A\delta_B}\right)\pi$.

It can be shown that this SPE is unique.

Implications of the unique SPE

In the unique SPE, agreement is reached at time 0, and the SPE is Pareto efficient.

Since it is player A who starts and makes the offer at time 0, the shares of the cake obtained by players A and B in the unique SPE are $x_A^* = \left(\frac{1-\delta_B}{1-\delta_A\delta_B}\right)\pi$ and $\pi - x_A^* = \left(\frac{\delta_B-\delta_A\delta_B}{1-\delta_A\delta_B}\right)\pi$, respectively.

The equilibrium share to each player depends on both players' discount factors. In particular, the equilibrium share obtained by a player is strictly increasing in her discount factor, and strictly decreasing in her opponent's discount factor.

Bargaining Power and Patience

In the alternating-offers game, if a player does not wish to accept any particular offer and, instead, would like to make a counteroffer, then she is free to do so, but she has to incur in the "cost" of waiting Δ time units.

The smaller is her discount rate, the smaller is this cost. That is why being relatively more patient confers greater bargaining power.

"First-Mover" Advantage

Notice that if the players' discount rates are identical to δ, then player A's equilibrium share $\left(\frac{1}{1+\delta}\right)\pi$ is strictly greater than B's share $\left(\frac{\delta}{1+\delta}\right)\pi$.

This result suggests that there exists a **"first-mover" advantage**.

This advantage disappears as Δ goes to zero (since $\delta \equiv \exp(-r\Delta)$).

Instantaneous Reply ($\Delta \to 0$)

Recall that $\delta_i \equiv \exp(-r_i\Delta))$. Suppose discount rates are different, $r_A \neq r_B$.

Corollary: In the limit, as $\Delta \to 0$, the shares of the players in the unique SPE converge to:

$$\begin{aligned} share_A^* &= \left(\frac{r_B}{r_A + r_B}\right)\pi \\ share_B^* &= \left(\frac{r_A}{r_A + r_B}\right)\pi \end{aligned}$$

Notice that what matter is relative, and not absolute, discount rate. Some impatience is required to have a SPE. If no discount rate then exists a continuum of Nash equilibria.

Properties of the Equilibrium

- Uniqueness if $r_A > 0$ and $r_B > 0$.
- Pareto Efficiency if $r_A > 0$ and $r_B > 0$.

If $r_A = 0$ and $r_B = 0$, then there exist a continuum of subgame perfect equilibria (where some of them are efficient).

Relationship with Nash's Bargaining Solution

It is straightforward to verify that the limiting, as $\Delta \to 0$, SPE payoff pair $(share^*_A, share^*_B)$ is identical to the asymmetric Nash bargaining solution of the bargaining problem (Ω, d) with $\lambda = \left(\frac{r_B}{r_A + r_B}\right)$ where $\Omega = \{(u_A, u_B) : 0 \leq u_A \leq \pi \text{ and } u_B = \pi - u_A\}$ and $d = (0, 0)$.

This remarkable result provides a strategic justification for Nash's bargaining solution.

When and how to use Nash's bargaining solution?

The asymmetric Nash bargaining solution is applicable because the bargaining outcome that it generates is identical to the (limiting) bargaining outcome that is generated by the basic alternating-offers model.

However, it should only be used when Δ is arbitrarily small, which may be interpreted as follows: it should be used in those bargaining situations in which the absolute magnitudes of the frictions in the bargaining process are small.

Furthermore, it should be defined on the bargaining problem where Ω is the set of instantaneous utility pairs obtainable through agreement, and d is the payoff pair obtainable through **perpetual disagreement** in Rubinstein's model.

Horn and Wolinsky (1988)

They study a downstream duopoly in which firms acquire inputs through **bilateral monopoly** relations with suppliers.

Suppose two firms: 1 and 2, that produce related products x_k.

Each firm i uses a single input l_i and pays labor wage of w_i.

The input is supplied to each firm by a single supplier and the price is determined in bargaining between the firm and its supplier.

Demand and Profits in Horn and Wolinsky

Demand for product i is given by:

$$p_i(x_i, x_j) = a - cx_j - x_i,\ i \neq j = 1, 2$$

where if $0 \leq c \leq 1$, then products are substitutes. Instead, if $-1 \leq c \leq 0$, then products are complements.

Profits of the firm i can be written as:

$$\pi_i(w_i, w_j) = \left[\frac{a(2-c) + cw_j - 2w_i}{4 - c^2}\right]^2$$

Comment the intuition of w_j in profits of firm i.

Bargaining over wages in Horn and Wolinsky

Assuming disagreement payoffs of zero, the wages w_i^* solves the Nash bargaining problem between firm i and the supplier.

$$w_i^* = \arg\max_{w_i} (\pi_i(w_i, w_j) - 0)(w_i l_i(w_i, w_j) - 0)$$

If suppliers are independent, the equilibrium wages are

$$\begin{aligned} w_i^* &= w_j^* = \left[\frac{a(2-c)}{8-c}\right] \\ l_i^* &= l_j^* = \left[\frac{6a}{(2+c)(8-c)}\right] \end{aligned}$$

The authors explores the incentives to merge in this environment for both upstream and downstream firms.

Single Supplier Case

If there is a single supplier for both firms, the supplier objective function will be $\sum_i w_i l_i$.
Applying the Nash solution is not entirely straightforward since the model must account for the interdependencies between the different bargaining problems.
When considering price setting suppliers, a single firm is always better off. When there is bargaining, this is not always the case.

Timing: Symmetric and simultaneous negotiations or asymmetric sequential negotiations?

Bargaining Solution: Nash solutions are the pair of agreed wages (w_1^S, w_2^S), such that all players anticipate them and yet they do not have incentives to deviate.

Disagreement Payoffs

What are the disagreement payoffs for the supplier and firm i?

Option 1) Suppose the state of permanent disagreement with firm i as in Rubinstein's model. Then remaining firm j could act as a monopoly. Thus, the disagreement payoff would be zero for firm i and supplier would get $(a - w_j^S)w_j^S/2$.

Option 2) If the bargaining were modeled as a dynamic process, this choice would correspond to a situation in which, when firm i and the supplier cannot agree, the firm earns zero profit and the other firm, j, operates at the anticipated equilibrium level $l_j(w_1^S, w_2^S)$. This is consistent with symmetric and simultaneous negotiations and **passive beliefs**.

Horn and Wolinsky was the first paper to highlight the Bilateral Bargaining with Externalities (See de Collard-Wexler, Gowrisankaran and Lee (2016) for an updated theoretical review)

Single Supplier Results

Recall that if suppliers are independent, the equilibrium wages are $w_i^* = w_j^* = \left[\frac{a(2-c)}{8-c}\right]$ and $l_i^* = l_j^* = \left[\frac{6a}{(2+c)(8-c)}\right]$
With a single supplier (and option 2), the equilibrium wages are

$$
\begin{aligned}
w_i^S &= w_j^S = \left[\frac{a(2-c)}{8-2c}\right] \\
l_i^S &= l_j^S = \left[\frac{a(6-c)}{(2+c)(8-2c)}\right]
\end{aligned}
$$

As a result, the single supplier is better off ($w^S l^S > w^* l^*$) if products are substitutes ($c > 0$).

If the products are complements ($c < 0$), both of the two independent bargaining suppliers will earn larger profits than the single merged firm.

Novel finding of Horn and Wolinsky

If the suppliers were free to set the prices, internalizing the cross effects is precisely what would make a merged supplier always more profitable than two non-cooperating, independent suppliers.

When prices are determined in bargaining, however, the presence of these effects influences the bargaining leverage of the merged supplier.

In this bargaining setting, a party becomes relatively stronger if it can make a credible commitment that makes concessions more costly than otherwise.

Therefore, when products are substitutes, the bargaining position of the merged upstream supplier is tougher than the position of an independent supplier in the same situation.

The opposite is true for a merged downstream firm.

Exploring Single Downstream firm

If the downstream firms merged, then the unique downstream firm will pay symmetric wages v^S.

The single downstream firm is worse off if products are substitutes $(c > 0)$. Note that $v^S > w^S$ and the profit of the merged firm is smaller than the sum of the two independent downstream firms.

When the products are complements $(c < 0)$, these relations are reversed.

Asymmetric Equilibrium

The paper also explores asymmetric equilibrium in the case products are substitutes ($c > 0$).

If the disagreement payoff of the firm negotiating first is high, then both suppliers are better off relative to symmetric equilibrium.

Once again, externalities matter!!

Conclusions

- We have reached the frontier relative to bargaining models.
- Rubinstein structural model can be linked to Nash Bargaining Model.
- Different alternatives of modelling including disagreement payoffs and believes regarding externalities.
- We are ready to take these models to the data...