ORGANIZACIÓN INDUSTRIAL EMPÍRICA
IN7E0

Carlos Noton

Clase 8 - Martes 21 de Agosto
Primavera - 2018

Outline

Introduction

- Recent theoretical and empirical work has highlighted the importance of bargaining.
- Most prices are agreed upon between players breaking the idea of uniform prices and take-it-or-leave-it offers.
- How to deal with this new world is the core issue in the next classes.

Applications in many markets

- The bargaining literature has gained prominence in merger analysis, among other topics, because in sectors with business to business relationships (B2B), standard models of competition, such as Bertrand competition, miss important features of the market.
- Recent work has sought to estimate bilateral bargaining models in a variety of sectors.
- Cable Television in Crawford and Yurukoglu (2012): Channels à-la-carte?
- Medical Devices in Grennan (2013): Uniform prices?
- Health Insurance in Lee and Ho (2016): More competitive health insurance markets?
- Retail in Noton and Elberg (2016): Are supermarkets squeezing small suppliers?

Applications in mergers

- Moreover, structural bargaining models allow for counterfactual exercises that are key for the antitrust authorities to allow or to challenge a merger.
- In a nutshell, the bargaining models are being used to estimate structural parameters and then to simulate the impacts on the upstream and downstream markets of proposed mergers.
- Crawford et al (2015) study foreclosure in cable TV.
- Gowrisankaran, Nevo and Town (2015) study hospital horizontal mergers.
- Elberg, Gowrisankaran and Noton (2017) study an upstream merger in the bottled water industry.
- Cuesta, Noton and Vatter (2017) study vertical integration between healthcare and insurance providers.

Introduction Bargaining

- Initially motivated by bargaining between Unions and Firms.
- Evidence regarding how firms and Union of workers share profits. Analysis using wages as a portion of the profits. It breaks the link with the marginal productivity of labor.
- Upstream and Downstream firms bargain over prices, important issue of externalities and beliefs.
- Crucial role for disagreement payoffs that requires to estimate payoffs in counterfactual scenarios.

Nash Bargaining Model: Setting of Nash (1950)

Two players, A and B, bargain over the partition of a cake of size π, where $\pi > 0$.

The set of possible agreements is

$$X = \{(x_A, x_B) : 0 \leq x_A \leq \pi \text{ and } x_B = \pi - x_A\}$$

where x_i is the share of the cake to player i ($i = A, B$).

Utility Functions

For each $x_i \in [0, \pi]$, $U_i(x_i)$ is player i's utility from obtaining a share x_i of the cake, where player i's utility function $U_i : [0, \pi] \to \mathcal{R}$ is strictly increasing and concave.

Disagreement Payoffs

If the players fail to reach agreement, then player i obtains a utility of d_i, where $d_i \geq U_i(0)$.

The utility pair $d = (d_A, d_B)$ is called the **disagreement point** that is associated with the payoffs in case that there is no agreement between the parties.

There exists an agreement $x \in X$ such that $U_A(x) > d_A$ and $U_B(x) > d_B$, which ensures that there exists a mutually beneficial agreement.

Frontier of Possibilities

In order to define the Nash bargaining solution of this bargaining situation, it is useful to first define the set Ω of *possible utility* pairs obtainable through agreement.

For the bargaining situation described above,

$$\Omega = \{(u_A, u_B) : \exists x \in X | U_A(x_A) = u_A \text{ and } U_B(x_B) = u_B\}$$

Function U_A^{-1}

Fix an arbitrary utility u_A to player A, where $u_A \in [U_A(0), U_A(\pi)]$.

From the strict monotonicity of U_i, there exists a unique share $x_A \in [0, \pi]$ such that $U_A(x_A) = u_A$; i.e., $x_A = U_A^{-1}(u_A)$, where U_A^{-1} denotes the inverse of U_A.

It can be shown that U_A^{-1} is a strictly increasing and convex function.

Function g

Hence

$$g(u_A) \equiv U_B(\pi - U_A^{-1}(u_A))$$

is the utility player B obtains when player A obtains the utility u_A.

It immediately follows that:

$$\Omega = \{(u_A, u_B) : U_A(0) \leq u_A \leq U_A(\pi) \text{ and } u_B = g(u_A)\}$$

that is, Ω is the graph of the function $g : [U_A(0), U_A(\pi)] \rightarrow \mathcal{R}$.

Nash Bargaining Solution *(NBS)*

The Nash bargaining solution (NBS) of the bargaining situation described above is the **unique** pair of utilities, denoted by (u_A^N, u_B^N), that solves the following maximization problem:

$$\max_{(u_A,u_B)\in\Theta} \underbrace{(u_A - d_A)(u_B - d_B)}_{\text{Nash Product}}$$

where

$$\begin{aligned}
\Theta &\equiv \{(u_A, u_B) \in \Omega : u_A \geq d_A \text{ and } u_B \geq d_B\} \\
&\equiv \{(u_A, u_B) : U_A(0) \leq u_A \leq U_A(\pi), u_B = g(u_A), u_A \geq d_A, u_B \geq d_B\}
\end{aligned}$$

Uniqueness of the NBS

The maximization problem stated above has a unique solution, because the maximand $(u_A - d_A)(u_B - d_B)$ – which is referred to as the Nash product– is continuous and strictly quasiconcave, g is strictly decreasing and concave, and the set Θ is non-empty.

Graphic Analysis

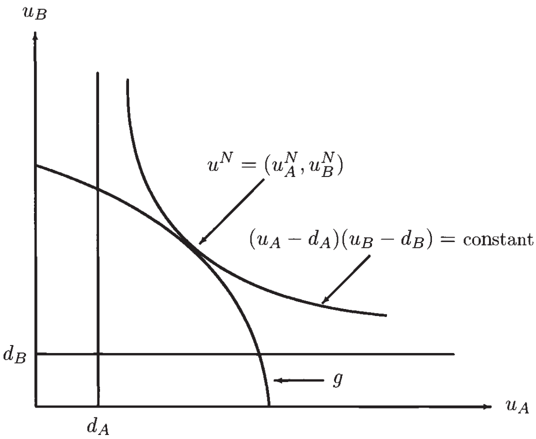

Figure 2.1: u^N is the Nash bargaining solution of the bargaining situation in which the set Ω of possible utility pairs obtainable through agreement is the graph of g, and d is the disagreement point.

Proposition ✠ to Characterize the solution

In the bargaining situation described above, if g is differentiable, then the Nash bargaining solution is the unique solution to the following pair of equations

$$-g'(u_A) = \frac{(u_B - d_B)}{(u_A - d_A)} \text{ and } u_B = g(u_A)$$

where g' denotes the derivative of g.

Proof: Find the value of u_A that maximizes $(u_A - d_A)(g(u_A) - d_B)$ using the first order conditions.

Corollary

In the bargaining situation described above, if g is differentiable, then the share x_A^N of the cake obtained by player A in the Nash bargaining solution is the unique solution to the equation

$$\frac{U_A(x_A) - d_A}{U_A'(x_A)} = \frac{U_B(\pi - x_A) - d_B}{U_B'(\pi - x_A)}$$

and player B's share in the NBS is $x_B^N = \pi - x_A^N$.

Example 1: Split-The-Difference Rule

Suppose utility functions are the identity function. Hence, $U_A(x_A) = x_A$ for all $x_A \in [0, \pi]$ and $U_B(x_B) = x_B$ for all $x_B \in [0, \pi]$.

This means that for each $u_A \in [0, \pi], g(u_A) = \pi - u_A$, and $d_i > 0$ $(i = A, B)$.

Example 1: Split-The-Difference Rule

Applying Proposition ✠ , it follows that

$$
\begin{aligned}
u_A^N &= \frac{1}{2}(\pi - d_B + d_A) \\
u_B^N &= \frac{1}{2}(\pi - d_A + d_B)
\end{aligned}
$$

Thus

$$
\begin{aligned}
x_A^N &= d_A + \frac{1}{2}(\pi - d_A - d_B) \\
x_B^N &= d_B + \frac{1}{2}(\pi - d_A - d_B)
\end{aligned}
$$

Interpretation of Example 1

The players agree first of all to give player i ($i = A, B$) a share d_i of the cake (which gives her a utility equal to the utility she obtains from not reaching agreement), and then they split equally the remaining cake $(\pi - d_A - d_B)$.

Notice that player i's share x_i^N is strictly increasing in d_i and strictly decreasing in $d_j (j \neq i)$.

Example 2: Risk Aversion

Suppose $U_A(x_A) = x_A^\gamma$ for all $x_A \in [0, \pi]$, where $0 < \gamma < 1$, and $U_B(x_B) = x_B$ for all $x_B \in [0, \pi]$.

Suppose $d_A = d_B = 0$. (No disagreement payoffs)

This means that for each $u_A \in [0, \pi], g(u_A) = \pi - u_A^{1/\gamma}$.

Applying the Corollary, it follows that

$$\begin{aligned} x_A^N &= \frac{\gamma\pi}{1+\gamma} \\ x_B^N &= \frac{\pi}{1+\gamma} \end{aligned}$$

Interpretation of Example 2

As γ decreases, x_A^N decreases and x_B^N increases. In the limit, as γ goes to 0, x_A^N goes to 0 and x_B^N goes to π.

Player B may be considered risk neutral (since her utility function is linear), while player A risk averse (since her utility function is strictly concave), where the degree of her risk aversion is decreasing in γ.

Given this interpretation of the utility functions, it has been shown that player A's share of the cake decreases as she becomes more risk averse. She is willing to sacrifice the share of the pie to avoid the risk of disagreement.

Risk of Disagreement

Importantly: so far there is no randomness in the outcomes.

The only risk is the risk of disagreement.

Alvin Roth has a nice paper discussing the difference and the extensions of the model with random outcomes.

Axiomatic Foundation 1

We can show that NBS is the only bargaining solution that satisfies the following four properties (or, axioms).

This axiomatization provides a justification for using the NBS.

- *Invariance to Equivalent Utility Representations*: The agreements will remain the same if considering affine transformations of the utility functions for a given bargaining problem. Preferences matter, not utility representations of it.

Axiomatic Foundation 2 and 3

- *Pareto Efficiency*: For a given bargaining problem, there is no another allocation that leaves one player indifferent and the other strictly better off.
- *Symmetry*: If disagreement payoffs are the same, and payoffs are symmetric, then the solutions are also symmetric. In other words, identities of the players are irrelevant.

Axiomatic Foundation 4

- *Independence of Irrelevant Alternatives*: Assuming fixed disagreement payoffs. If one particular solution s was agreed when many alternatives were available, then s must be agreed when a strict subset of these alternatives are available. Irrelevant alternatives should not change the final agreement.

Proposition ♣ by Nash (1950)

Axioms:

1. *Invariance to Equivalent Utility Representations*
2. *Pareto Efficiency*
3. *Symmetry*
4. *Independence of Irrelevant Alternatives*

For any $(\Omega, d) \in \Sigma$, a bargaining solution $f : \Sigma \to \mathcal{R}^2$ satisfies Axioms 1 to 4 **if and only if** $f = f^N$, the Nash's bargaining solution.

Asymmetric Nash Bargaining Solutions

The NBS depends upon the set Ω of possible utility pairs and the disagreement point d.

However, the outcome of a bargaining situation may be influenced by other forces/skills (or, variables), such as the tactics employed by the bargainers, the procedure through which negotiations are conducted, the information structure and the players' discount rates.

None of these forces seem to affect the two objects upon which the NBS is defined, and yet it seems reasonable not to rule out the possibility that such forces may have a significant impact on the bargaining outcome.

Definition

For each $\lambda \in (0,1)$, an asymmetric (or, generalized) Nash bargaining solution is a function $f_\lambda^N : \Sigma \rightarrow \mathcal{R}^2$, defined as follows.

For each $(\Omega, d) \in \Sigma$, $f_\lambda^N(\Omega, d)$ is the unique solution to the following maximization problem

$$\max_{(u_A, u_B) \in \Theta} (u_A - d_A)^\lambda (u_B - d_B)^{1-\lambda}$$

where

$$\Theta \equiv \{(u_A, u_B) \in \Omega^e : u_A \geq d_A \text{ and } u_B \geq d_B\}$$

where Ω^e is the pareto frontier of Ω.

Axiomatic Results?

For each $\lambda \in (0, 1)$, an asymmetric NBS f_λ^N satisfies Axioms 1,2 and 4.

Furthermore, any bargaining solution that satisfies Axioms 1, 2 and 4 is an asymmetric NBS for some value of λ.

Unless $\lambda = 1/2$, an asymmetric NBS does not satisfy Axiom 3.

Proposition ✠ in the Assymetric Case

For each $\lambda \in (0,1)$, and any bargaining problem $(\Omega, d) \in \Sigma$ such that g is differentiable, and $d_i \geq \underline{u}_i (i = A, B)$, the asymmetric NBS is the unique solution to the following pair of equations:

$$-g'(u_A) = \left(\frac{\lambda}{1-\lambda}\right) \frac{(u_B - d_B)}{(u_A - d_A)} \text{ and } u_B = g(u_A)$$

where g' denotes the derivative of g.

Conclusions

1. We have covered the cornerstone of the bargaining models.
2. This model is very tractable in empirical terms and have solid axiomatic foundations.
3. However, it is not a structural game. There is no description of the protocol that players follow to find a solution.
4. That's exactly the Rubinstein's model we'll see next class.