ORGANIZACIÓN INDUSTRIAL EMPÍRICA
IN7E0

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Outline

Homogenous goods

If homogenous goods (like commodities), then you should expect a single equilibrium price because consumers choose the cheapest producer.

New Assumption: No homogenous goods, instead goods are differentiated.

Does it change the conclusions? A LOT!

Differentiated Goods and Price Dispersion

Differences can be very real (like in cars, clothes or computers).
Nowadays, marketing can differentiate anything.

Differentiated products imply an imperfect substitution.
Prices differ and still expensive products have positive demand.
Also, overwhelming evidence that there is price dispersion out there.

Market Power

Imperfect substitution grants market power to the producers.

Market power implies that prices are higher than marginal cost.

The dream of any producer is to be.... a monopoly.

Can differentiation create a monopoly?

Horizontal Differentiation

Assumptions of the Linear City of Hotelling (1929)

- Linear space (city) of length 1.
- Population uniformly distributed along this line.
- There is a transport cost $t > 0$, factor of the quadratic distance of the shop.
- Unit demand with surplus of s for the consumption of one unit. Suppose that in equilibrium everyone buy the good (i.e. the market is covered.)

Therefore, people are not indifferent between goods. If the prices are the same, they prefer the closest providers in order to save the transport costs. This idea can be literally geographic distance but it could be any other dimension of the product.

Consumers Utility

Suppose there are two shops: one in each extreme of the linear city.
Shop 1 is in coordinate zero and shop 2 is in coordinate 1.
s denotes the surplus enjoyed by each consumer when consuming the good.
Consumer lives in coordinate $x \in [0, 1]$.
Buying from Shop 1 (at distance x and price p_1) yields the following utility:

$$U(Shop_1|x) = s - p_1 - tx^2$$

and buying from shop 2 (at distance $(1 - x)$ and price p_2) yields:

$$U(Shop_2|x) = s - p_2 - t(1 - x)^2$$

Demand Function

The overall demand is determined by the consumer who is indifferent between the two shops. The indifferent consumer lives in coordinate $\widetilde{x} \in [0,1]$ such that:

$$\begin{aligned} U(Shop_1|\widetilde{x}) = U(Shop_2|\widetilde{x}) \quad &\Leftrightarrow \quad s - p_1 - t\widetilde{x}^2 = s - p_2 - t(1-\widetilde{x})^2 \\ &\Rightarrow \quad \widetilde{x}(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t} \end{aligned}$$

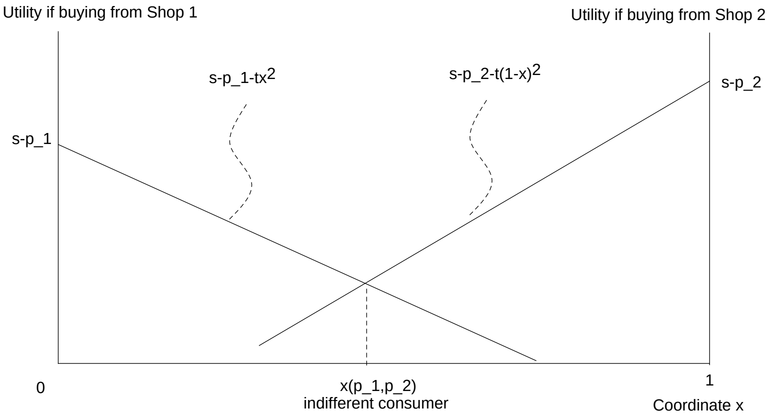
If $p_2 > p_1$ then critical distance $\widetilde{x}$ is closer to 1 and more consumers buy from Shop 1.
The larger the transport cost the closer to one half.

Demand Function

Based on the uniform distribution $[0,1]^1$ of the consumers along the linear city, the fraction at coordinate $\widetilde{x}$ is the same coordinate. Hence the demand D_i of Shop i is given by:

$$\begin{aligned} D_1(p_1,p_2) &= \widetilde{x}(p_1,p_2) \\ D_2(p_1,p_2) &= 1-\widetilde{x}(p_1,p_2) \end{aligned}$$

Hotelling assumes the market is covered.



Optimal Pricing

Suppose symmetric constant marginal cost c. Thus, profits of Shop 1 are given by:

$$\Pi_1(p_1, p_2) = (p_1 - c)D_1(p_1, p_2) = (p_1 - c)\left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right)$$

The first order conditions give us the reaction function:

$$p_1 = R_1(p_2) = \left(\frac{t + c + p_2}{2}\right)$$

Notice the slope of the reaction function: What's the intuition?

Equilibrium Outcome

Solving for the equilibrium prices, we have:

$$\begin{aligned} p_1^* = p_2^* &= c + t \\ \Pi_1^* = \Pi_2^* &= \frac{t}{2} \end{aligned}$$

Extreme Location: Producers are happy with higher transport costs!
Marginal costs do not affect profits as long as they are symmetric.

Endogenous Location

Suppose that firms choose their location: Shop 1 chooses $a \in [0,1]$ and shop 2 chooses $(1-b) \in [0,1]$ in the linear city.
Maximal differentiation is with $a = b = 0$ (Extreme location as before).
Minimal differentiation is with $a = 1 - b$ (Exactly the same location).
Solving for the indifferent consumer:

$$s - p_1 - t(a - \widetilde{x})^2 = s - p_2 - t(1 - b - \widetilde{x})^2$$

The demand is:

$$D_1(p_1, p_2) = \widetilde{x} = \left(\frac{1 + a - b}{2}\right) + \left(\frac{p_2 - p_1}{2t(1 - a - b)}\right)$$

Computing the equilibrium price yields:

$$p_1^* = c + t(1 - a - b)\left(1 + \frac{a - b}{3}\right)$$

Choosing distance a

Suppose b as given, then the profit function for Shop 1 is:

$$\Pi_1(a) = (p_1(a) - c)D_1(a, p_1(a), p_2(a))$$

The total derivative yields:

$$\frac{d\Pi_1(a, p_1(a), p_2(a))}{da} = \underbrace{\frac{\partial \Pi_1(a)}{\partial p_1}}_{zero} \frac{\partial p_1(a)}{\partial a} + \underbrace{\frac{\partial \Pi_1(a)}{\partial a}}_{direct\ effect} + \underbrace{\frac{\partial \Pi_1(a)}{\partial p_2} \frac{dp_2(a)}{da}}_{strategic\ effect}$$

In this particular setting the result yields:

$$\frac{d\Pi_1}{da} < 0$$

Therefore, the smaller a the larger the profits.

Hence, **maximum differentiation is optimal for both firms and the shops are located in each extreme of the city.**

What is the location that maximizes welfare?

If the Almighty and Good central planner can freely put the firms along the line, Where will she locate them? What is the efficient location?

Assuming the market is covered, every consumer buys the good obtaining the surplus s. In the production side, all units cost the same marginal cost c.

Consumer surplus is maximum, and production cost is minimum

Therefore the central planner only should minimize **transportation costs**.

Optimal locations are $a = \frac{1}{4}$ and $1 - b = \frac{3}{4}$.

Of course, the intermediate points so people minimize traveling!!

Firms tend to differentiate too much in comparison with the social optimum!! (They prefer the extremes!)

Optimal Pricing with Differentiated Products

- Assume that firms set prices (Bertrand competition), we will derive the optimal pricing policy as a function of demand parameters and observable quantities and prices.
- We start with a single product monopolist, then we study a collection of single-product firms.
- We study multi-product monopolist, and then the equilibrium with many competitive multiproduc firms.
- The vector of prices that set all FOC to zero are considered the Nash-equilibria of the statitic-game.

Optimal Pricing for Single Product Monopolist

Assume Bertrand-Nash with differentiated goods, the **single-product monopolist**, with unobservable constant marginal cost c.
Profits of the unique firm are:

$$\Pi(p) = (p - c) \cdot q(p, \theta)$$

where $\mathbf{p}$ is the vector of all prices.
The first order condition yields:

$$(p - c)\frac{\partial q(p, \theta)}{\partial p} + q(p, \theta) = 0 \Leftrightarrow p - c = -\frac{q(p, \theta)}{\frac{\partial q(p,\theta)}{\partial p}}$$

Notice the intuition for the relationship between price sensitivity and markups.

Optimal Pricing for Single Product Competitive Firms

Suppose N firms. The vector of prices is $\mathbf{p}$, and the vector of quantities is $\mathbf{q}$ (both of dimension $N \times 1$).
Assume Bertrand-Nash with differentiated goods, a **single-product firm**, with unobservable constant marginal cost c_i.
Profits of the single-product firm i are:

$$\Pi_i(p) = (p_i - c_i) \cdot q_i(\mathbf{p}, \theta)$$

The first order condition yields:

$$(p_i - c_i)\frac{\partial q_i(\mathbf{p}, \theta)}{\partial p_i} + q_i(\mathbf{p}, \theta) = 0 \Leftrightarrow p_i - c_i = -\frac{q_i(\mathbf{p}, \theta)}{\frac{\partial q_i(\mathbf{p}, \theta)}{\partial p_i}}$$

The vector of prices, $\mathbf{p}$, that simultaneously sets to zero all the FOC will be considered the equilibrium prices.

Nash-Bertrand Equilibria

$$\begin{aligned}
p_1 - c_1 &= -\frac{q_1(\mathbf{p},\theta)}{\frac{\partial q_1(\mathbf{p},\theta)}{\partial p_1}} \\
&\vdots \\
p_i - c_i &= -\frac{q_i(\mathbf{p},\theta)}{\frac{\partial q_i(\mathbf{p},\theta)}{\partial p_i}} \Leftrightarrow (\mathbf{p}-\mathbf{c}) = \Omega^{-1}(\mathbf{p},\theta)\cdot \mathbf{q}(\mathbf{p},\theta) \\
&\vdots \\
p_N - c_N &= -\frac{q_N(\mathbf{p},\theta)}{\frac{\partial q_N(\mathbf{p},\theta)}{\partial p_N}}
\end{aligned}$$

where $\Omega_{ii}(p) = -\frac{\partial q_i(\mathbf{p},\theta)}{\partial p_i}$ and $\Omega_{ik}(p) = 0$ if $i \neq k$, ie, in this case Ω is a N by N diagonal matrix.

FOC with two single-product firms

Suppose two firms. Write Ω and the FOC of the market.

Why do we have zeros outside the diagonal?

We have zero outside the diagonal, because firms are competitive and single-product (no cross-price derivatives in competitive FOC).

Multi-product Monopolist

- A multi-product monopolist produces J different products (or varieties). They can be substitutes (Regular Coke, Diet Coke, Coke Zero, etc) or complements (games and consoles).
- Price vector $\mathbf{p} \equiv (p_j)_{j=1}^J$.
- Quantity vector $\mathbf{q} \equiv (q_i)_{j=1}^J$, where quantity of product i is given by $q_j = q_j(p), j = 1, \ldots J$.
- If costs are additive ($C(\mathbf{q}) = \sum_{j=1}^J c_j(q_j)$) and Demands are independent $q_j(p) = q_j(p_j)$, then we can use the single-product monopoly result.

The problem of the Multi-product Monopolist

$$\max_{\mathbf{p}=\{p_j\}_{j=1}^{J}} \sum_{j=1}^{J} p_j q_j(\mathbf{p}) - c(q_1(\mathbf{p}), \ldots, q_J(\mathbf{p}))$$

F.O.C.

$$\underbrace{\left((p_j - c_j)\frac{\partial q_j}{\partial p_j} + q_j(p)\right)}_{\text{Direct Effect of Prices}} + \underbrace{\sum_{k \neq j} (p_k - c_k)\frac{\partial q_k}{\partial p_j}}_{\text{Indirect Effect}} = 0, \quad \forall j$$

Direct Effect: The standard terms that account for the marginal revenue and marginal costs of product j.

Indirect Effect: The firm considers the effects on the other products she owns (indexed by k).The firm internalize the changes on marginal revenues (and eventually marginal costs) of all her products when pricing product j.

Rewriting the FOC

- Assuming additive costs $C(\mathbf{q}) = \sum_{j=1}^{J} c_j(q_j)$ (This assumption kills any synergy or economies of scale), we can write the FOC as the following:

$$\boxed{\frac{p_j - c_j}{p_j} = \frac{1}{|\epsilon_{jj}|} + \frac{1}{p_j q_j |\epsilon_{jj}|} \sum_{k \neq j} (p_k - c_k) q_k \epsilon_{jk}}$$

- $\epsilon_{jk} = (\partial q_k / \partial p_j)(p_j / q_k)$: cross-price elasticity, ie, percentage change of demand for good k due to a marginal percentage change of price j.
- Substitutes: $\epsilon_{jk} > 0, \; k \neq j$.
- Complements: $\epsilon_{jk} < 0, \; k \neq j$.

Substitutes

- Intuition: Suppose you set the prices of soft-drinks that are close substitutes: would you set larger o lower prices than an independent manufacturer?
- Substitutes: an increase in price i will **increase** the demand of product j, $\epsilon_{ij} > 0$, $i \neq j$.
- The result is that the multiproduct producer of substitutes will set higher prices than a collection of single-product monopolies because the multiproduct firm will benefit from a softer competition between all his products.

Complements

- Intuition: Suppose you sell a certain non-compatible console: would you set prices of games cheaper or more expensive than an independent game developer?
- Complements: an increase in price i will **decrease** the demand of product j, $\epsilon_{ij} < 0$, $i \neq j$.
- The result is that the multiproduct producer of complement goods will set lower prices than a collection of single-product monopolies, because the multiproduct producer will benefit from the sales of all complements.

FOC with one firm producing two products

Suppose a monopolist producing two products. Write Ω and the FOC of the market.

Why do we have non-zeros outside the diagonal?

Because firms internalize consumer substitution when maximizing profits.

Multi-Product Firms

Assume N **multi-product firms** competing a la Nash-Bertrand with differentiated goods. Suppose constant marginal costs, c_j. Profits of firm i are given by:

$$\Pi_i = \sum_{j \in \mathcal{F}_i} (p_j - c_j) \cdot q_j(\mathbf{p}, \theta)$$

where $\mathbf{p}$ is the vector of all prices.
The first order condition with respect to price j yields:

$$q_j + \sum_{r \in \mathcal{F}_i} (p_r - mc_r) \frac{\partial q_r}{\partial p_j} = 0$$

The **set of J equations** for firm i implies the price and margins for each of her products.

FOC for Multi-Product Firm

Define

$$\Omega_{jr}(p) \begin{cases} -\frac{\partial q_j(\mathbf{p})}{\partial p_r}, & \text{if } \exists f : (r,j) \subset \mathcal{F}_i; \\ 0, & \text{Otherwise.} \end{cases}$$

The first order conditions of all $J \times N$ products in vectorial notation can be written as:

$$\mathbf{q}(\mathbf{p}) - \Omega(\mathbf{p})(\mathbf{p} - \mathbf{c}) = 0 \quad \Leftrightarrow \quad \mathbf{p} - \mathbf{c} = \Omega^{-1}(\mathbf{p})q(\mathbf{p})$$

where $\mathbf{p}$ is the vector of J prices, $\mathbf{s}$ is the J vector of market shares and $\mathbf{c}$ is the J vector of marginal costs.
This **set of J equations** hold in equilibrium. A price change in one product may imply a change in all prices.

FOC with one firm producing two products and another single product firm

Suppose one firm producing two products and another single product firm.
Write Ω and the FOC of the market.

How the diagonal terms looks like?

Conclusions

- Introducing Differentiated Products change the results in IO dramatically.
- Prices are dispersed and always above marginal cost.
- Market outcomes are always away from social optimum.
- In empirical terms this new setting impose a huge challenge: If you have $N = 4$ firms and J=5 products each; then you need 20^2 elasticities...!!
- The discrete choice models are going to be our best friends in this task of demand estimations with differentiated goods.