Pauta Aux 7

P1 $H(t) = \begin{pmatrix} E & v \\ v & -E \end{pmatrix} = \underbrace{\begin{pmatrix} E & 0 \\ 0 & -E \end{pmatrix}}_{H_0} + \underbrace{\begin{pmatrix} 0 & v \\ v & 0 \end{pmatrix}}_{\tilde{V}}$

(a) $\mathbb{P}(|1\rangle \to |2\rangle) = |c_2|^2$. $\dot{c}_2 = \frac{1}{i\hbar}\tilde{V}_{21} e^{i\omega_{21}t} c_1$. A $\mathcal{O}(0)$ en $\tilde{V}_{21}$, $c_1 = 1$, $c_2 = 0$.

A $\mathcal{O}(1)$, tenemos $\dot{c}_2 = \frac{1}{i\hbar}\tilde{V}_{21} e^{i\omega_{21}t} \Rightarrow c_2(\infty) = \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt\, \tilde{V}_{21}(t)\, e^{i\omega_{21}t}$

$\omega_{21} = 2E/\hbar$. $\tilde{V}_{21} = \langle 2|\tilde{V}|1\rangle = (1\ \ 0)\begin{pmatrix} 0 & v \\ v & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = v(t)$

$$\Rightarrow \boxed{\mathbb{P}(|1\rangle \to |2\rangle) = \frac{1}{\hbar^2}\left|\int_{-\infty}^{\infty} dt\, v(t)\, e^{2iEt/\hbar}\right|^2}$$

(b) $H(t) = \begin{pmatrix} 0 & v \\ v & 0 \end{pmatrix}$ $\quad H|\tilde{1}\rangle = \tilde{E}_1|\tilde{1}\rangle$, $H|\tilde{2}\rangle = \tilde{E}_2|\tilde{2}\rangle$.

Autoestados y autovalores son $|\tilde{1}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $|\tilde{2}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ +1 \end{pmatrix}$, $-\tilde{E}_1 = v$, $\tilde{E}_2 = v$

$i\hbar\partial_t|\psi\rangle = H|\psi\rangle$, escribimos $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \Rightarrow i\hbar\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} 0 & v \\ v & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = v\begin{pmatrix} \beta \\ \alpha \end{pmatrix}$

$\Rightarrow i\hbar\dot{\alpha} = v\beta$, $i\hbar\dot{\beta} = v\alpha \Rightarrow i\hbar\ddot{\alpha} = \dot{v}\beta + v\dot{\beta} = \dot{v}\cdot i\hbar\dot{\alpha}/v + (i\hbar)^{-1}v^2\alpha$

Análogamente, $i\hbar\ddot{\beta} = \dot{v}\alpha + \dot{\alpha}v = \dot{v}(i\hbar\dot{\beta})/v + v^2\beta/i\hbar$. Parece no ser tan útil.

Mejor: $|\psi\rangle = \alpha|\tilde{1}\rangle + \beta|\tilde{2}\rangle$ $\quad H|\psi\rangle = \alpha E_1|\tilde{1}\rangle + \beta E_2|\tilde{2}\rangle$

$i\hbar\partial_t|\psi\rangle = i\hbar\left(\dot{\alpha}|\tilde{1}\rangle + \dot{\beta}|\tilde{2}\rangle\right) \Rightarrow i\hbar\dot{\beta} = \beta E_2$, $\quad i\hbar\dot{\alpha} = \alpha E_1$

Tenemos $|\psi_0\rangle = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(|\tilde{2}\rangle - |\tilde{1}\rangle\right) \Rightarrow \alpha(0) = \frac{-1}{\sqrt{2}}$, $\beta(0) = \frac{1}{\sqrt{2}}$

$-\infty$

$|2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(|\tilde{1}\rangle + |\tilde{2}\rangle\right)$

$\Rightarrow \mathbb{P}(|1\rangle \to |2\rangle) = \frac{1}{2}\left|\alpha(\infty) + \beta(\infty)\right|^2 \qquad \frac{\dot{\alpha}}{\alpha} = \frac{+E_1}{i\hbar} = \frac{-v}{i\hbar} \Rightarrow \alpha(t) = \alpha(-\infty)\exp\left[-\frac{1}{i\hbar}\int_{-\infty}^{t} v(t)\,dt\right]$

$\alpha(t) = -\frac{1}{\sqrt{2}}\exp\left[-\frac{1}{i\hbar}\int_{-\infty}^{t} v(t)\,dt\right]$, $\quad \beta(t) = \frac{1}{\sqrt{2}}\exp\left[\frac{1}{i\hbar}\int_{-\infty}^{t} v(t)\,dt\right]$

$$\mathbb{P}(|1\rangle \to |2\rangle) = \frac{1}{4}\left|\exp\left[\frac{i}{\hbar}\int_{-\infty}^{\infty} v(t)\,dt\right] - \exp\left[\frac{-i}{\hbar}\int_{-\infty}^{\infty} v(t)\,dt\right]\right|^2 = \boxed{\sin^2\left(\frac{1}{\hbar}\int_{-\infty}^{\infty} v(t)\,dt\right)}$$

Con teo de perturbaciones, $\mathbb{P} = \frac{1}{\hbar^2}\left|\int_{-\infty}^{\infty} v(t)\,dt\right|^2$. Calzan al usar $\sin x = x$, necesitamos $\frac{1}{\hbar}\int_{-\infty}^{\infty} v(t)\,dt$ pequeño.

P2

$$\vec{E}(t) = \begin{cases} 0, & t<0 \\ \vec{E}_0 e^{-\gamma t}, & t\geq 0 \end{cases} \qquad H = H_0 + V(t), \quad V = -Eez$$

$$C_2 = -\frac{i}{\hbar}\int_0^t V_{21}\, e^{i\omega_{21}t'}\, dt' \qquad V_{21} = \langle 2lm|-Eez|100\rangle \qquad \omega_{21} = \frac{E_2-E_1}{\hbar}$$

z es impar $\Rightarrow \langle 200|z|100\rangle = 0$. $Y_l^m \sim e^{im\phi}$, y $\int_0^{2\pi} e^{im\phi}\, d\phi = 0$ para $m\neq 0$

$\Rightarrow \langle 2lm|z|100\rangle \neq 0$ sólo para $l=1$, $m=0$.

$$\langle 210|z|100\rangle = 2\pi \cdot \frac{1}{4\pi a_0^3\sqrt{2}} \cdot \frac{1}{a_0}\int_0^\infty r^4 e^{-3r/2a_0}\, dr \int_0^\pi \cos^2\theta \sin\theta\, d\theta = \frac{256}{243\sqrt{2}}\, a_0$$

$$\Rightarrow C_2 = -\frac{i}{\hbar}\int_0^t \left(-eE_0 a_0 \cdot \frac{256}{243\sqrt{2}}\right) e^{-\gamma t'} e^{i\omega_{21}t'}\, dt' \equiv i\beta\int_0^t e^{t'(i\omega_{21}-\gamma)}\, dt'$$

$$= i\beta\cdot\left\{\frac{e^{t(i\omega_{21}-\gamma)}-1}{i\omega_{21}-\gamma}\right\} \Rightarrow P = \beta^2\cdot\frac{\left(e^{t(i\omega_{21}-\gamma)}-1\right)\left(e^{t(-i\omega_{21}-\gamma)}-1\right)}{(i\omega_{21}-\gamma)(-i\omega_{21}-\gamma)}$$

$$= \beta^2\cdot\left\{\frac{e^{-2\gamma t}+1-e^{-\gamma t}\left(e^{i\omega_{21}t}+e^{-i\omega_{21}t}\right)}{\gamma^2+\omega_{21}^2}\right\} = \boxed{\beta^2\left\{\frac{1+e^{-2\gamma t}-2e^{-\gamma t}\cos\omega_{21}t}{\gamma^2+\omega_{21}^2}\right\}}$$

Cosas razonables:

- Para $t=0$, $P=0$
- Para t grande, $P = \frac{\beta^2}{\gamma^2+\omega_{21}^2}$; esto aumenta si β (i.e., la perturbación) aumenta, disminuye si γ (el damping de $\vec{E}$) aumenta, y disminuye si la diferencia E_2-E_1 aumenta (se requiere un "mayor salto").

P3 Caso particular de la P2, para $\gamma=0$.

$P = \frac{\beta^2}{\omega_{21}^2}\cdot 2(1-\cos\omega_{21}t)$. Calza con lo que puse en la pauta de la P1 del C1 en el límite $I\to 0$ (despreciar el splitting dentro del condensador)

$\boxed{PA}$ $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$

(a) $\gamma_n = i\int \langle \psi_n | \frac{\partial \psi_n}{\partial R} \rangle dR = i\int_{L_1}^{L_2} \langle \psi_n | \frac{\partial \psi_n}{\partial L} \rangle dL$

$$\frac{\partial \psi_n}{\partial L} = -\frac{1}{\sqrt{2L^3}} \sin\left(\frac{n\pi x}{L}\right) - \sqrt{\frac{2}{L}}\left(\frac{n\pi x}{L^2}\right)\cos\left(\frac{n\pi x}{L}\right)$$

$$\langle \psi_n | \frac{\partial \psi_n}{\partial L} \rangle = \int_0^L \frac{-1}{\sqrt{2L^3}} \sin^2\left(\frac{n\pi x}{L}\right)\sqrt{\frac{2}{L}} - \frac{2}{L}\left(\frac{n\pi x}{L^2}\right)\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right) dx \qquad \cos 2y = 1 - 2\sin^2 y$$

$$= \frac{-1}{L^2}\int_0^L \frac{1}{2} - \frac{1}{2}\cancelto{0}{\cos\left(\frac{2n\pi x}{L}\right)} + \frac{n\pi x}{L}\sin\left(\frac{2n\pi x}{L}\right) dx = \frac{-1}{2L} + \frac{-n\pi}{L^3}\int_0^L x \sin\left(\frac{2n\pi x}{L}\right) dx$$

$$= -\frac{1}{2L} + \frac{-n\pi}{L^3}\left[-x \cdot \frac{L}{2n\pi}\cos\left(\frac{2n\pi x}{L}\right)\Big|_0^L + \int_0^L \frac{L}{2n\pi}\cancelto{0}{\cos\left(\frac{2n\pi x}{L}\right)} dx\right] = \boxed{0}$$

Tiene que ver con que $\gamma_n \in \mathbb{R}$, y $\psi_n \in \mathbb{R} \Rightarrow \gamma_n$ debe ser cero.

(b) $\theta_n(T) = -\frac{1}{\hbar}\int_0^T E_n(t')\,dt' \qquad v = \frac{L_2 - L_1}{T} = \text{constante}$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad L = L_1 + vt \Rightarrow \theta_n(T) = -\frac{1}{\hbar}\int_0^T \frac{n^2\pi^2\hbar^2}{2m(L_1+vt)^2} dt \qquad \begin{matrix} u = L_1 + vt \\ dt = du/v \end{matrix}$$

$$= \frac{-n^2\pi^2\hbar}{2m}\cdot\frac{1}{v}\int_{L_1}^{L_2}\frac{du}{u^2} = \frac{-n^2\pi^2\hbar}{2mv}\left[\frac{-1}{L_2} + \frac{1}{L_1}\right] = \boxed{\frac{n^2\pi^2\hbar}{2mv}\left[\frac{1}{L_2} - \frac{1}{L_1}\right]}$$

(c) De (a), sabemos que $\gamma_n = 0$ siempre $\Rightarrow$ para el ciclo igual $\gamma = 0$.