P1 $d\vec{B} = \frac{\mu_0}{4\pi} I d\vec{l} \times \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right) \Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \oint d\vec{l} \times \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)$

teorema de Stokes:

$$\oint_C \vec{A}\cdot d\vec{l} = \int_S (\nabla \times \vec{A})\cdot \hat{n}\, da \overset{(*)}{\Longleftrightarrow} \oint_C d\vec{l} \times \vec{P} = \int_S (\hat{n}\, da \times \nabla) \times \vec{P}$$

Ahora: $\vec{B} = \frac{\mu_0 I}{4\pi} \oint d\vec{l}' \times \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right) = \frac{\mu_0 I}{4\pi} \int_S (\hat{n}\, da' \times \nabla') \times \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)$

$$= -\frac{\mu_0 I}{4\pi} \int_S \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right) \times (\hat{n}\, da' \times \nabla') \quad ; \quad \vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$$

$$= -\frac{\mu_0 I}{4\pi} \left[\int_S \left[\nabla' \cdot \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)\right] \cdot \hat{n}\, da' - \nabla \int_S \left[\left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)\cdot \hat{n}\, da'\right] \right]$$

$$\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = \nabla\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) \Rightarrow \nabla\cdot\nabla\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = 4\pi\,\delta(\vec{r}-\vec{r}')$$

$$\Rightarrow \vec{B} = -\frac{\mu_0 I}{4\pi} \left[4\pi \int_S \cancel{\delta(\vec{r}-\vec{r}')}^{\,0} \cdot \hat{n}\, da' - \nabla \int \left[\left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)\cdot \hat{n}\, da'\right] \right]$$

↓

En la superficie S
$\vec{r}$ nunca es $= a\ \vec{r}'$

$$= \frac{\mu_0 I}{4\pi} \nabla \left(\int \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right) \cdot \hat{n}\, da' \right)$$

El ángulo sólido se define como

$$\Omega = \int_S \frac{\vec{R}\cdot d\vec{A}}{R^3}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi} \nabla \Omega_p$$

$(*)$ $\oint_C \vec{A}\cdot d\vec{l} = \int_S (\nabla\times\vec{A})\,\hat{n}da$; tomando $\vec{A} = \vec{\alpha}\times\vec{p}$ con $\vec{\alpha}$ un vector cte.

$$\Leftrightarrow \oint_C (\vec{\alpha}\times\vec{p})\cdot d\vec{l} = \int_S [\nabla\times(\vec{\alpha}\times\vec{p})]\,\hat{n}da$$

$$\Leftrightarrow \oint_C (\vec{p}\times d\vec{l})\cdot\vec{\alpha} = \int_S [\hat{n}da\times\nabla]\cdot(\vec{\alpha}\times\vec{p})$$

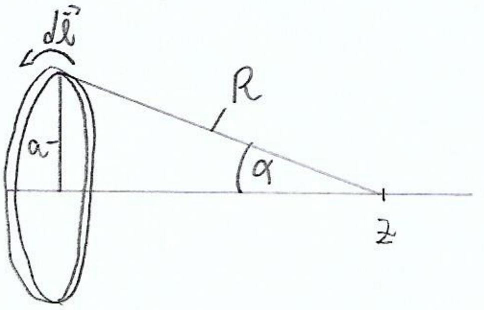
$$\Leftrightarrow -\oint_C (d\vec{l}\times\vec{p})\cdot\vec{\alpha} = \int_S (\vec{p}\times[\hat{n}da\times\nabla])\cdot\vec{\alpha}$$

$$\Leftrightarrow \oint_C (d\vec{l}\times\vec{p})\cdot\vec{\alpha} = \int_S ([\hat{n}da\times\nabla]\times\vec{p})\cdot\vec{\alpha}$$

$$\Leftrightarrow \left[\oint_C d\vec{l}\times\vec{p} - \int_S [\hat{n}da\times\nabla]\times\vec{p}\right]\cdot\vec{\alpha} = 0$$

$$\Rightarrow \oint_C d\vec{l}\times\vec{p} = \int_S [\hat{n}da\times\nabla]\times\vec{p}$$

P2 Calculamos primero un loop, despues superponemos todos para encontrar B_z:



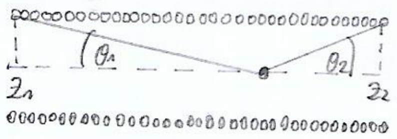
$$B_z = \frac{\mu_0 I}{4\pi}\int \frac{[d\vec{l}\times(\vec{r}-\vec{r}')]_z}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{r}-\vec{r}=\vec{R}=\begin{pmatrix} a\cos(\phi) \\ a\,sen(\phi) \\ z\end{pmatrix} \wedge d\vec{l}=\begin{pmatrix} -a\,sen(\phi) \\ a\cos(\phi) \\ 0\end{pmatrix} d\phi$$

Notar que ϕ es el ángulo que da vuelta a la espira.

$$\Rightarrow B_z = \frac{\mu_0 I}{4\pi}\int_0^{2\pi} d\phi \frac{a^2}{(a^2+z^2)^{3/2}} = \frac{\mu_0 I}{2}\frac{a^2}{(a^2+z^2)^{3/2}}$$

Ahora:



Integramos de z_1 a z_2 por la densidad de espiras.

$$B_z^{TOT} = \int_{z_1}^{z_2} dz\, B_z = \frac{\mu_0 I a^2}{2}\int_{z_1}^{z_2} dz\,(a^2+z^2)^{-3/2}\cdot N$$

Usamos $z = a\tan(\gamma)$; $dz = a\sec^2(\gamma)\,d\gamma$

$$\Rightarrow B_z^{TOT} = \frac{\mu_0 I \cancel{a^2} N}{2}\int_{\tan^{-1}(z_1/a)}^{\tan^{-1}(z_2/a)} d\gamma \cdot \frac{\cancel{a}}{\cos^2(\gamma)}\cdot \frac{1}{\underbrace{\cancel{a^3}\left(1+\frac{sen^2(\gamma)}{\cos^2(\gamma)}\right)^{3/2}}_{\left(\frac{\cos^2(\gamma)+sen^2(\gamma)}{\cos^2(\gamma)}\right)^{3/2} = \frac{1}{\cos^3(\gamma)}}} = \frac{\mu_0 I N}{2}\int_{\tan^{-1}(z_1/a)}^{\tan^{-1}(z_2/a)} d\gamma\,\cos(\gamma)$$

$$\Rightarrow B_z^{tot} = \frac{\mu_0 N I}{2} \operatorname{sen}(\gamma) \Big|_{\tan^{-1}(z_1/a)}^{\tan^{-1}(z_2/a)}$$

$$\operatorname{sen}(\tan^{-1}(x)) = \frac{x}{\sqrt{1+x^2}}$$

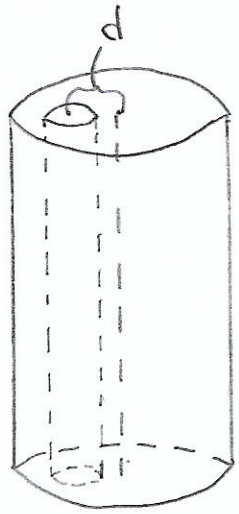
$$\Rightarrow B_z^{TOT} = \frac{\mu_0 N I}{2} \left(\frac{z_2/a}{\sqrt{1+(z_2/a)^2}} - \frac{z_1/a}{\sqrt{1+(z_1/a)^2}} \right)$$

$$= \frac{\mu_0 N I}{2} \left(\frac{z_2}{\sqrt{a^2+z_2^2}} - \frac{z_1}{\sqrt{a^2+z_1^2}} \right)$$

$$\frac{z_2}{\sqrt{a^2+z_2^2}} = \cos(\theta_2) \quad \wedge \quad \frac{z_1}{\sqrt{a^2+z_1^2}} = -\cos(\theta_1)$$

$$\Rightarrow B_z^{TOT} = \frac{\mu_0 N I}{2} \left(\cos(\theta_2) + \cos(\theta_1) \right)$$

P3



La ley de Ampère en forma integral es:

$$\oint_V \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Para simetría cilíndrica se tiene:

$$2\pi B = \mu_0 I_{enc} \iff B = \frac{\mu_0 (j \pi r^2)}{2\pi r} = \frac{\mu_0 j r}{2}$$

Para una corriente que viaja en $\hat{z}$ la dirección se obtiene así:

$$\vec{B} = B\, \hat{z} \times \hat{r} = \frac{\mu_0 j}{2} r\, \hat{z} \times \hat{r} = \frac{\mu_0 j}{2} \hat{z} \times \vec{r}$$

Ahora usando superposición tomamos el cilindro sólido y le sustraemos la parte "faltante".

$$\vec{B} = \frac{\mu_0 j}{2} \hat{z} \times \vec{r} - \frac{\mu_0 j}{2} \left(\hat{z} \times (\vec{r} - \vec{d}) \right) = \frac{\mu_0 j \vec{d}}{2}$$

$\rightarrow$ Constante :0 /