Aux 1

1. $$\Phi = \frac{q/4\pi\varepsilon_0}{|\vec{r}-\vec{r}'|} + \frac{q'/4\pi\varepsilon_0}{|\vec{r}-\vec{r}''|}$$

$$= \frac{q/4\pi\varepsilon_0}{(r^2+r'^2-2\vec{r}\cdot\vec{r}')^{1/2}} + \frac{q'/4\pi\varepsilon_0}{(r^2+r''^2-2\vec{r}\cdot\vec{r}'')^{1/2}} \quad ; \quad |\vec{r}| = a$$

$$= \frac{q/4\pi\varepsilon_0 a}{\left(1+\frac{r'^2}{a^2}-2\frac{\hat{a}\cdot\vec{r}'}{a}\right)^{1/2}} + \frac{q'/4\pi\varepsilon_0 r''}{\left(1+\frac{a^2}{r''^2}-2\frac{\vec{a}\cdot\hat{r}''}{r''}\right)^{1/2}}$$

$$\Rightarrow \frac{q'}{r''} = -\frac{q}{a} \wedge \frac{a}{r''} = \frac{r'}{a} \Rightarrow q' = -q\frac{a}{r'} \wedge r'' = \frac{a^2}{r'}$$

$$\Rightarrow \Phi(|\vec{r}|=a) = 0 \,//$$

2. $$\Phi = \frac{Q/4\pi\varepsilon_0}{(r^2+R^2+2rR\cos(\theta))^{1/2}} - \frac{Q/4\pi\varepsilon_0}{(r^2+R^2-2rR\cos(\theta))^{1/2}}$$

$$- \frac{aQ/4\pi\varepsilon_0}{R\left(r^2+\frac{a^4}{R^2}+\frac{2a^2r}{R}\cos(\theta)\right)^{1/2}} + \frac{aQ/4\pi\varepsilon_0}{R\left(r^2+\frac{a^4}{R^2}-\frac{2a^2r}{R}\cos(\theta)\right)^{1/2}}$$

$$(r^2+R^2+2rR\cos(\theta))^{-1/2} = R^{-1}\left(\overset{0}{\cancel{\frac{r^2}{R^2}}}+1+2\frac{r}{R}\cos(\theta)\right)^{-1/2}$$

$$\simeq R^{-1}\left(1+\frac{2r}{R}\cos(\theta)\right)^{-1/2}$$

$$\simeq \frac{1}{R}\left(1-\frac{r}{R}\cos(\theta)\right)$$

$$(r^2+R^2-2rR\cos(\theta))^{-1/2} \simeq \frac{1}{R}\left(1+\frac{r}{R}\cos(\theta)\right)$$

$$\left(r^2+\frac{a^4}{R^2}+\frac{2a^2r}{R}\cos(\theta)\right)^{-1/2}=r^{-1}\left(1+\cancel{\frac{a^4}{R^2r^2}}^{0}+\frac{2a^2}{rR}\cos(\theta)\right)^{-1/2}$$

$$\simeq r^{-1}\left(1+\frac{2a^2}{rR}\cos(\theta)\right)^{-1/2}$$

$$\simeq \frac{1}{r}\left(1-\frac{a^2}{rR}\cos(\theta)\right)$$

$$\left(r^2+\frac{a^4}{R^2}-\frac{2a^2r}{R}\cos(\theta)\right)\simeq\frac{1}{r}\left(1+\frac{a^2}{rR}\cos(\theta)\right)$$

$$\Rightarrow \Phi \simeq \frac{1}{4\pi\varepsilon_0}\left(-\frac{2Q}{R^2}r\cos(\theta)+\frac{2Q}{R^2}\frac{a^3}{r^2}\cos(\theta)\right)$$

$$=-E_0\left(r-\frac{a^3}{r^2}\right)\cos(\theta)$$

$$\Phi(a)=0 \,//$$

3. $$\Phi=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\|\vec{r}-(b\hat{x}+\frac{a}{2}\hat{y})\|}-\frac{1}{\|\vec{r}-(b\hat{x}-\frac{a}{2}\hat{y})\|}-\frac{1}{\|\vec{r}-(-b\hat{x}+\frac{a}{2}\hat{y})\|}+\frac{1}{\|\vec{r}-(-b\hat{x}-\frac{a}{2}\hat{y})}\right)$$

$$\|\vec{r}-\vec{\omega}\|^{-1}=\left(r^2+\omega^2-2\vec{r}\cdot\vec{\omega}\right)^{-1/2}=r^{-1}\left(1+\frac{\omega^2}{r^2}-2\frac{\hat{r}\cdot\vec{\omega}}{r}\right)^{-1/2}$$

$$\simeq\frac{1}{r}\left(1-\frac{1}{2}\left(\frac{\omega^2}{r^2}-2\frac{\hat{r}\cdot\vec{\omega}}{r}\right)+\frac{3}{8}\left(\frac{\omega^2}{r^2}-2\frac{\hat{r}\cdot\vec{\omega}}{r}\right)^2\right)$$; A orden $1/r^2$

$$\simeq\frac{1}{r}\left(1+\frac{\hat{r}\cdot\vec{\omega}}{r}-\frac{\omega^2}{2r^2}+\frac{3(\hat{r}\cdot\vec{\omega})^2}{2r^2}\right)$$

$$\|\vec{r}+\vec{\omega}\|^{-1}\simeq\frac{1}{r}\left(1-\frac{\hat{r}\cdot\vec{\omega}}{r}-\frac{\omega^2}{2r^2}+\frac{3(\hat{r}\cdot\vec{\omega})^2}{2r^2}\right)$$

$$\vec{w} = b\hat{x} + \frac{a}{2}\hat{y} \quad \wedge \quad \vec{w}' = b\hat{x} - \frac{a}{2}\hat{y} \ ; \ w^2 = w'^2$$

$$\Rightarrow \Phi = \frac{q}{4\pi\varepsilon_0 r}\left(\cancel{1} + \cancel{\frac{\hat{r}\cdot\vec{w}}{r}} - \cancel{\frac{w^2}{2r^2}} + \frac{3(\hat{r}\cdot\vec{w})^2}{2r^2} - \cancel{1} - \cancel{\frac{\hat{r}\cdot\vec{w}'}{r}} + \cancel{\frac{w'^2}{2r^2}} - \frac{3(\hat{r}\cdot\vec{w}')^2}{2r^2}\right.$$

$$\left. - \cancel{1} + \cancel{\frac{\hat{r}\cdot\vec{w}'}{r}} + \cancel{\frac{w'^2}{2r^2}} - \frac{3(\hat{r}\cdot\vec{w}')^2}{2r^2} + \cancel{1} - \cancel{\frac{\hat{r}\cdot\vec{w}}{r}} - \cancel{\frac{w^2}{2r^2}} + \frac{3(\hat{r}\cdot\vec{w})^2}{2r^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left(\frac{3}{r^2}\left(\hat{r}\cdot b\hat{x} + \hat{r}\cdot\frac{a}{2}\hat{y}\right)^2 - \frac{3}{r^2}\left(\hat{r}\cdot b\hat{x} - \hat{r}\cdot\frac{a}{2}y\right)^2\right)$$

$$= \frac{3q}{4\pi\varepsilon_0 r^3}\left(\cancel{(\hat{r}\cdot b\hat{x})^2} + 2(\hat{r}\cdot b\hat{x})(\hat{r}\cdot\frac{a}{2}\hat{y}) + \cancel{(\hat{r}\cdot\frac{a}{2}\hat{y})^2} - \cancel{(\hat{r}\cdot b\hat{x})^2} + 2(\hat{r}\cdot b\hat{x})(\hat{r}\cdot\frac{a}{2}\hat{y}) - \cancel{(\hat{r}\cdot\frac{a}{2}\hat{y}}\right.$$

$$= \frac{3q}{\pi\varepsilon_0 r^3}(\hat{r}\cdot\hat{x})(\hat{r}\cdot\hat{y})\frac{ba}{2} = \frac{3q\,ba}{2\pi\varepsilon_0 r^3}(\hat{r}\cdot\hat{x})(\hat{r}\cdot\hat{y})$$

$$= \frac{3q\,ba}{2\pi\varepsilon_0 r^3}\operatorname{sen}^2(\theta)\cos(\varphi)\operatorname{sen}(\varphi)$$

$$\vec{E} = -\nabla\Phi = -\left(\frac{\partial\Phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\hat{\theta} + \frac{1}{r\operatorname{sen}(\theta)}\frac{\partial\Phi}{\partial\varphi}\hat{\varphi}\right)$$

$$= \frac{3qab}{2\pi\varepsilon_0 r^4}\left(\frac{3\operatorname{sen}^2(\theta)\operatorname{sen}(2\varphi)}{2}\hat{r} + \frac{\operatorname{sen}(2\theta)\operatorname{sen}(2\varphi)}{2}\hat{\theta} + \operatorname{sen}(\theta)\cos(2\varphi)\hat{\varphi}\right)$$