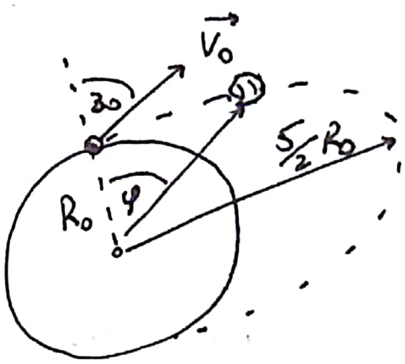
Auxiliar 10



V_0 / $R_{max} = \frac{5}{2}R_0$

Lo primero es notar que solo existe fuerza en $\hat{r}$, la dirección radial, por lo que el torque es nulo y con eso se conserva el momentum angular.

Dado que se conserva el momentum angular podemos trabajar en un plano (coord. cilíndricas a $\dot{z}=0$)

$$\Rightarrow \vec{r} = r\hat{r}$$
$$\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\varphi}\hat{\varphi}$$

$\underline{t=0}$ $\quad r(0) = R_0$

$$\dot{\vec{r}}(0) = V_0\cos(30)\,\hat{r} + V_0\,\text{sen}(30)\,\hat{\varphi}$$
$$= \frac{V_0\sqrt{3}}{2}\hat{r} + \frac{V_0}{2}\hat{\varphi}$$

Forma 1

$$\vec{l} = m\,\vec{r}\times\dot{\vec{r}} = cte \quad (\vec{\tau} = 0 = \dot{\vec{l}})$$
$$= m(r\hat{r})\times(\dot{r}\hat{r} + r\dot{\varphi}\hat{\varphi})$$
$$\Rightarrow \vec{l} = m r^2\dot{\varphi}\,\hat{z}$$

$$\Rightarrow \frac{l}{m} = r^2\dot{\varphi} = r^2(0)\,\dot{\varphi}(0)$$
$$= \frac{R_0 V_0}{2}$$

$$\Rightarrow r^2(t)\,\dot{\varphi}(t) = \frac{R_0 V_0}{2}$$

Forma 2

$$\sum F_\varphi = 0 = \frac{1}{r}\frac{d}{dt}(r^2\dot{\varphi})$$

$$\Rightarrow r^2\varphi = cte \text{ en } t$$

$$r^2(0)\,\dot{\varphi}(0) = r^2(t)\cdot\dot{\varphi}(t)$$

$$\dot{\vec{r}}(0) = \frac{V_0\sqrt{3}}{2}\hat{r} + \frac{V_0}{2}\hat{\varphi} = \dot{r}(0)\hat{r} + r(0)\dot{\varphi}(0)\hat{\varphi}$$

$$\dot{r}(0) = \frac{V_0\sqrt{3}}{2} \;;\quad r(0)\,\dot{\varphi}(0) = \frac{V_0}{2}$$

$$\Rightarrow r^2\dot{\varphi}(t) = \frac{V_0}{2}R_0$$

Ahora aplicamos conservación de Energía en $t=0$ y $t^* \leadsto R=\frac{5R}{2}$

$$\Rightarrow \frac{1}{2}m(\dot{\vec{r}})^2 - \frac{GMm}{r} = E \leadsto cte$$

$$\hookrightarrow (\dot{\vec{r}})^2 = (\dot{r}\hat{r} + r\dot{\varphi}\hat{\varphi})^2 = \dot{r}^2 + r^2\dot{\varphi}^2$$

La cond. de que $r(t^*) = \frac{5R_0}{2}$ sea la dist. maxima, imporemos que $\dot{r}(t^*) = 0$

$$\Rightarrow \frac{1}{2}m\left(\dot{r}^2(0) + r^2(0)\dot{\varphi}^2(0)\right) - \frac{GmM}{r(0)} = \frac{1}{2}m\left(\cancel{\dot{r}(t^*)}^{0} + r^2\dot{\varphi}^2\right) - \frac{GMm}{r(t^*)}$$

$$\Rightarrow \frac{1}{2}m\left(V_0^2\cdot\frac{3}{4} + V_0^2\cdot\frac{1}{4}\right) - \frac{2GMm}{R_0} = \frac{1}{2}m\,\frac{V_0^2R_0^2}{4}\left(\frac{2}{5R_0}\right)^2 - \frac{2GMm}{5R_0}$$

$$V_0^2\left(1-\frac{1}{5}\right) = \frac{2G}{R_0}M\left(1-\frac{2}{5}\right)$$

$$\Rightarrow V_0 = \sqrt{\frac{3}{2}\frac{GM}{R_0}}$$