| Aux 8 |

$U(x) = V\cos(\alpha x) - Fx$, V y F const.

ptos. eq. y estabilidad

Lo Li: es pensar que V y F son no nulos

1) $U(x)$ no es idént. nula ($V \neq 0$, $F \neq 0$)

$$U(x) = V\cos(\alpha x) - Fx$$

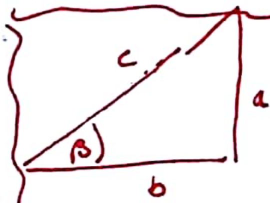
ptos $\Rightarrow$ $U'(x) = -\alpha V \,\text{sen}(\alpha x) - F \stackrel{!}{=} 0$

$$\Rightarrow \text{sen}(\alpha x^*) = \frac{-F}{\alpha V} \quad \therefore \boxed{\frac{\arcsin}{\alpha}\left(\frac{-F}{\alpha V}\right) = x^*}$$

estabilidad $\Rightarrow$ $U''(x) = -\alpha^2 V\cos(\alpha x) + 0$

$$\Rightarrow U''(x^*) = -\alpha^2 V \cos\left(\cancel{\alpha}\,\frac{\arcsin\left(\frac{-F}{\alpha V}\right)}{\cancel{\alpha}}\right)$$

$$\Rightarrow U''(x^*) = -\alpha^2 V\sqrt{1-\left(\frac{F}{V\alpha}\right)^2}$$



$\beta = \arcsin\left(\frac{a}{c}\right)$

$\rightarrow \cos(\beta) = \frac{b}{c}$

(*) $\cos(\alpha) = \sqrt{1-\text{sen}^2(\alpha)}$

Estable > 0
Inestable < 0

$$\Rightarrow U''(x^*) > 0 \iff -\alpha^2 V\sqrt{1-\left(\frac{F}{V\alpha}\right)^2} > 0$$

$$\Rightarrow V < 0 \quad y \quad 1-\left(\frac{F}{V\alpha}\right)^2 > 0$$ Estable

caso cont. a $V<0$ ($V>0$) es inestable pero

si $1-\left(\frac{F}{\alpha V}\right)^2 < 0 \Rightarrow$ no existe sol. en $\mathbb{R}$

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$\Rightarrow$ $V \neq 0$, $F = 0$, $\alpha \neq 0$

pto

$U(x) = V\cos(\alpha x) \Rightarrow U'(x) = -V\alpha\,\mathrm{sen}(\alpha x)$

$k \in \mathbb{Z}$

$\Rightarrow U'(x^*) \overset{!}{=} 0 \Leftrightarrow \mathrm{sen}(\alpha x^*) = 0 \Rightarrow \alpha x^* = k\pi$

$\therefore x^*_k = \dfrac{k\pi}{\alpha}$

$\boxed{\text{1 } x^* \text{ según el } k \text{ (hay infinitos)}}$

Est.

$U''(x) = -V\alpha^2\cos(\alpha x)$

$= \pm 1$ según k es par o impar

$\Rightarrow U''\left(\frac{k\pi}{\alpha}\right) = -V\alpha^2\cos(k\pi)$

$\Rightarrow$ k par $\quad U''(\cdot/\cdot) = -V\alpha^2 \rightsquigarrow$ $V > 0$ inest.

$V < 0$ estable

$\Rightarrow$ k impar $\quad U''\cdot/\cdot = V\alpha^2 \rightsquigarrow$ $V > 0$ estable

$V < 0$ inestable

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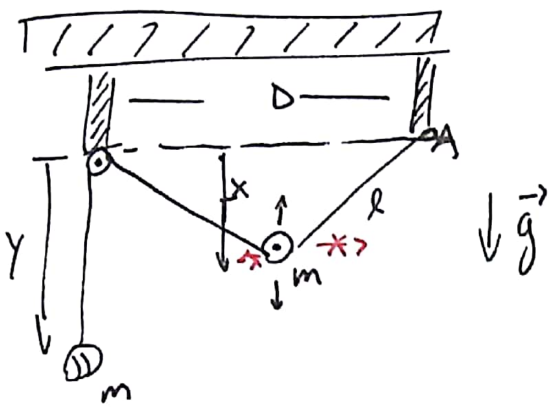
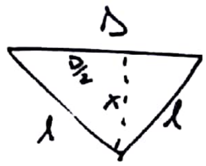


Largo : L

a) $Y \sim x$ y eq.

$\Rightarrow$ Sabemos que $L = 2l + Y$, donde



$\Rightarrow l^2 - \left(\frac{D}{2}\right)^2 = x^2$

$l = \sqrt{x^2 + \left(\frac{D}{2}\right)^2}$

$$\Rightarrow L = 2\sqrt{x^2 + \left(\frac{D}{2}\right)^2} + Y$$

$\therefore$

$$\boxed{Y(x) = L - \sqrt{4x^2 + D^2}}$$

Como solo hay energía pot. grav.

$$\Rightarrow U = -m(gx + gY)$$

$$\Rightarrow U(x) = -mg\left(x + L - \sqrt{4x^2 + D^2}\right)$$

$\rightarrow \frac{\partial U}{\partial x}(x_{eq}) \overset{!}{=} 0$

$$= -mg\left(1 + 0 - \frac{4x_{eq}}{\sqrt{D^2+4x_{eq}^2}}\right)$$

$$\rightarrow \boxed{x_{eq} = \pm \frac{D}{2\sqrt{3}}}$$

$*$ $\frac{\partial^2 U}{\partial x^2}(x_{eq}) = \frac{4mg\,D^2}{(D^2+4x_{eq}^2)^{3/2}}$ $\rightarrow$ $\frac{\partial^2 U}{\partial x^2}(x_{eq}) = \frac{3\sqrt{3}\,mg}{2D} > 0$

pos. de eq. estable

b)

$$K = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2$$

$\hookrightarrow$ $\dot{y} = \frac{dy}{dx}\cdot\dot{x} = -\frac{4x\dot{x}}{\sqrt{D^2+4x^2}}$

$$\rightarrow K = \frac{1}{2}\underbrace{m\left(\frac{D^2+20x^2}{D^2+4x^2}\right)}_{\alpha(x)}\dot{x}^2$$

Cuando $\alpha = \alpha(x)$, podemos sup. que cerca de x_{eq}

$\alpha = \alpha(x_{eq}) = cte$

$$\rightarrow K = \frac{1}{2}(2m)\dot{x}^2$$

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$\Rightarrow \quad \frac{d^2 U}{dx^2}(x_{eq}) \Rightarrow \boxed{\omega_{p.o.}^2 = \frac{3\sqrt{3}g}{4D}}$

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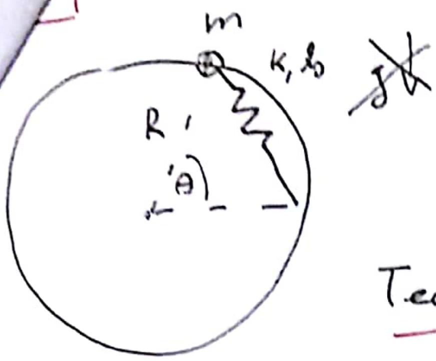

•) pto eq., est. y frec. peq. (pto estable)

Teo. coseno $\quad L^2 = R^2 + R^2 - 2R^2\cos(\theta)$

$$\Rightarrow L = R\sqrt{(1-\cos(\theta))2}$$

* Energía pot. → $U = \frac{k}{2}\left[R\sqrt{2(1-\cos\theta)} - l_0\right]^2$

$$\Rightarrow \frac{\partial U}{\partial \theta} = \frac{2k}{2}\left(R\sqrt{2(1-\cos(\theta))} - l_0\right)\left(\frac{R\,\text{sen}(\theta)}{\sqrt{2(1-\cos(\theta))}}\right) \overset{!}{=} 0$$

$\Rightarrow$ $\text{sen}(\theta_{eq}) = 0$

↳ $\boxed{\theta_{eq} = 0, \pi}$

* Cuando vemos el denominador, este también se va a cero → $\theta_{eq} = 0$ no puede ser un eq.

•) $R\sqrt{2(1-\cos(\theta))} - l_0 = 0$

$$\Rightarrow \cos(\theta_{eq}) = 1 - \frac{1}{2}\left(\frac{l_0}{R}\right)^2$$

$\Rightarrow l_0 < 2R$!

o no tendría sentido → $\cos() < -1$

$\circ)$ $$\left.\frac{\partial^2 U}{\partial \theta^2}\right|_{\theta=\pi} = \left.\frac{KR}{\sqrt{2}} \frac{\left(R\sqrt{2(1-\cos(\theta))}-l_0\right)\cos(\theta)}{\sqrt{2(1-\cos(\theta))}}\right|_{\theta=\pi} = \frac{KR}{\sqrt{2}} \cdot \frac{(2R-l_0)(-1)}{2}$$

$$= \frac{KR}{2\sqrt{2}}(l_0-2R)$$

$$\Rightarrow \left.\frac{\partial^2 U}{\partial \theta^2}\right|_{\theta=\pi} = \frac{KR}{2\sqrt{2}}(l_0-2R) \rightsquigarrow \begin{array}{lll} >0 & l_0>2R & \underline{\text{Est.}} \\ <0 & l_0<2R & \underline{\text{Inest.}} \end{array}$$

P.O.

$\circ)$ $$\left.\frac{\partial^2 U}{\partial \theta^2}\right|_{\cos(\theta)=1-\frac{1}{2}\left(\frac{l_0}{R}\right)^2} = \left.\frac{KR\,\text{sen}(\theta)}{\sqrt{2(1-\cos\theta)}} \cdot \frac{R\,\text{sen}(\theta)}{\sqrt{2(1-\cos(\theta))}}\right|_{\cos(\theta)=\ldots}$$

$$= \left.\frac{KR^2 \overset{(1-\cos^2(\theta))}{\text{sen}^2(\theta)}}{2(1-\cos(\theta))}\right|_{\cos(\theta)\ldots} = \left.\frac{KR^2}{2}(1+\cos(\theta))\right|_{\cos\theta=\ldots}$$

$$= \frac{KR^2}{2}\left(1+1-\frac{1}{2}\left(\frac{l_0}{R}\right)^2\right) = KR^2\left(1-\frac{1}{4}\left(\frac{l_0}{R}\right)^2\right)$$

(*) mientras $\cos(\theta_{eq}) = 1-\frac{1}{2}\left(\frac{l_0}{R}\right)^2$ sea eq.

$\Rightarrow$ es estable ✓✓

p.O.

Si podemos escribir la energia como

$$E = \frac{\alpha}{2}\dot{x}^2 + U(x) \rightsquigarrow \left\{ \ddot{x} = -\omega_{po}^2 x \right\}$$

$$\Rightarrow \omega_{p.o.}^2 = \frac{U''(x_{eq})}{\alpha}$$

Para este caso tenemos que $\alpha = mR^2 \rightsquigarrow$ inercia

$$\Rightarrow \omega_{p.o.}^2 = \frac{1}{mR^2} \left.\frac{\partial^2 U}{\partial \theta^2}\right|_{\theta_{eq.}}$$

$$\Rightarrow \theta_{eq} = \frac{\pi}{4} \rightsquigarrow \omega_{po}^2\left(\tfrac{\pi}{4}\right) = \frac{KR}{2\sqrt{2}}(l-2R) \quad , l_0 > 2R$$

$$\Rightarrow \cos(\theta_{eq}) = 1 - \frac{1}{2}\left(\frac{l_0}{R}\right)^2 \rightsquigarrow \omega_{p.o.}^2(\ldots) = KR^2\left(1 - \frac{1}{4}\left(\frac{l_0^2}{R}\right)\right), \; l_0 < 2R$$