P1 / $\vec{F} = -\nabla U$ 			Ax 7

o) $U_e(x,y,z) = \frac{K}{2}(x^2+y^2+z^2)$

$$\Rightarrow \vec{F}_e = -\vec{\nabla} U_e = -\left(\frac{\partial U_e}{\partial x}\hat{x} + \frac{\partial U_e}{\partial y}\hat{y} + \frac{\partial U_e}{\partial z}\hat{z}\right)$$

$$= -\frac{K}{2}\left(2x\hat{x} + 2y\hat{y} + 2z\hat{z}\right)$$

$$\Rightarrow \boxed{\vec{F}_e = -K(x\hat{y} + y\hat{y} + z\hat{z})}$$

o) $U_{el}(r) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}$

$$\Rightarrow F_{el}(r) = -\vec{\nabla} U_{el}(r)$$

$$\boxed{\vec{F}_{el}(r) = -\frac{qQ}{4\pi\varepsilon_0 r^2}\hat{r}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r\,\text{sen}(\theta)}\frac{\partial f}{\partial \varphi}\hat{\varphi} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta}$$

Esfericas

o) $U(r) = A e^{-Kr}$ $\Rightarrow \boxed{\vec{F}(r) = AKe^{-Kr}\hat{r}}$

$\boxed{\vec{\nabla}\times\vec{F} = 0 \Rightarrow F\ \text{cons.}}$

En Cartesianas $\nabla \times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$

$$\Leftrightarrow \hat{x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{y}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

∘) $F_1 = -F_0(y\hat{x} + x\hat{y})$

$\Rightarrow \vec{\nabla}\times\vec{F}_1 = \hat{z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) = (-1 - (-1)) = \underline{0}$ ✓✓

$F_{1z} = 0$

$\frac{\partial F_1}{\partial z} = 0$

$\underline{F_1 \text{ es cons.}}$

∘) $\vec{F}_2 = (xy + z)\hat{x} + 2xy\hat{y} + 3x\log\hat{z}$

$\Rightarrow$ ∘) $\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} = 0 - 0 = 0$ ✓✓

∘) $\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} = 1 - 3\log(z)$ xx

∘) $\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 2y - x$ xx

$\underline{F_2 \text{ no es cons.}}$

∘) $\vec{F}_3 = r\hat{r} + r^2\hat{\phi}$

$\nabla\times A = \frac{1}{r^2\sin(\theta)}\left(\left(\frac{\partial(r\sin(\theta)A_\varphi)}{\partial\theta} - \frac{\partial(rA_\theta)}{\partial\varphi}\right)\hat{r} + r\cdot\left(\frac{\partial A_r}{\partial\varphi} - \frac{\partial(r\sin(\theta)A_\varphi)}{\partial r}\right)\hat{\theta} + \left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)r\sin(\theta)\hat{\varphi}\right)$

$\Rightarrow$ ∘) $\underline{\hat{r}}$ | $r^3\cos\theta - 0 \neq 0$ xx

∘) $\underline{\hat{\theta}}$ | $r(0 - 3r^2) \neq 0$ xx

∘) $\underline{\hat{\varphi}}$ | $0 - 0 = 0$ ✓✓

$\underline{F_3 \text{ no es cons.}}$

P2) $V(r) = -\frac{GMm}{r}$

a) Segun ley de Newton

$U(r) = m\,V(r)$

$$\Rightarrow m\ddot{\vec{r}} = -\frac{GmM}{r^2}\hat{r} = -\nabla U$$

$\vec{r} = r\hat{r}$

⊛ Torque : $\vec{\tau} = \vec{r}\times\vec{F} = -\frac{GmM}{r^2}\cdot r\,(\hat{r}\times\hat{r})$

$$\Rightarrow \boxed{\vec{\tau} = 0} \Rightarrow \vec{l} = cte$$

$\theta = \frac{\pi}{2}$

$\dot{\theta} = 0$

$$\Rightarrow \vec{l} = m\vec{r}\times\vec{v} = m(r\hat{r})\times(\dot{r}\hat{r} + r\dot{\varphi}\hat{\varphi})$$

$$\boxed{\vec{l} = mr^2\dot{\varphi}\,\hat{z}} \quad \underline{cte}$$

Dado que $\vec{l}$ es const. todo el mov. se reduce a un plano $\Rightarrow z = 0$

La $\sum \vec{F} = \frac{-GmM}{r^2}\hat{r}$

$$\Rightarrow \underline{\hat{r})} \quad m(\ddot{r} - r\dot{\varphi}^2) = -\frac{GmM}{r^2}\hat{r}$$

$$\underline{\hat{\varphi})} \quad \frac{m}{r}\left(\frac{d}{dt}(r^2\dot{\varphi})\right) = 0 \Rightarrow l = mr^2\dot{\varphi}$$

$$\Rightarrow r\dot{\varphi}^2 = \frac{l^2}{mr^3}$$

$$\therefore m\ddot{r} - m\frac{l^2}{mr^3} = \frac{-GmM}{r^2}$$

$$\Rightarrow m\ddot{r} = \left(\frac{l^2}{r^3} - \frac{GmM}{r^2}\right) = -\frac{d}{dr}\overbrace{\left(\frac{l^2}{2mr^2} - \frac{GmM}{r}\right)}^{U_{eff}}$$

$$\Rightarrow m\ddot{r} = -\frac{d}{dr}U_{eff}$$

b) $E(r,\dot{r})$?

$$E = E_c + E_p = \frac{1}{2} m \dot{\vec{r}}^{\,2} - \frac{GmM}{r}$$

$$\Rightarrow \quad E = \frac{m\dot{r}^2}{2} + \frac{\ell^2}{2mr^2} - \frac{GmM}{r}$$

$$\boxed{E = \frac{m\dot{r}^2}{2} + U_{eff}(r)}$$

c) $\left(\frac{dr}{d\varphi}\right)^2 = f(r)$?

$$\dot{\varphi} = \frac{\ell}{mr^4}$$

Desp. de la energía

$$\dot{r}^2 = \frac{2}{m}\left(E - U_{eff}\right) = \left(\frac{d\varphi}{dt} \cdot \frac{dr}{d\varphi}\right)^2$$

$$\Rightarrow \left(\frac{dr}{d\varphi}\right)^2 = \frac{m^2 r^4}{\ell^2} \cdot \frac{2}{m}\left(E - U_{eff}\right)$$

$$= \underbrace{\frac{m^2 r^4}{\ell^2} \cdot \frac{2}{m}\left(E - \frac{\ell^2}{2mr^2} + \frac{GmM}{r}\right)}_{= f(r)}$$

$$\Rightarrow \left(\frac{dr}{d\varphi}\right)^2 = f(r)$$

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d) $r(\theta)$?

{En algún momento cambié de $\varphi \to \theta$ pero son lo mismo ahora y no es el θ de esf.}

Integramos la ec. anterior como $I = \int \frac{dr}{\sqrt{f(r)}} = \int d\theta = \theta(t) - \theta(0)$

$$I = \frac{1}{\sqrt{2M}} \int \frac{\frac{1}{r^2}\, dr}{\sqrt{E - \frac{\ell^2}{2mr^2} + \frac{GmM}{r}}} \quad , \quad u = \frac{1}{r}$$

$$= \frac{-1}{\sqrt{2m}} \int \frac{du}{\sqrt{-\left(u^2 \cdot \frac{\ell^2}{2m}\right) + u\,GMm + E}}$$

Vamos a $\pm \frac{m^3 M^2 G^2}{2\ell^2}$ dentro de la raíz para completar $(\,)^2$

$$= \frac{-1}{\sqrt{2m}} \int \frac{du}{\sqrt{E + \frac{m^3M^2G^2}{2\ell^2} - \left(u\frac{\ell}{\sqrt{2m}} - \sqrt{\frac{m}{2}}\frac{GMm}{\ell}\right)^2}} \quad , \quad v = \frac{u}{2m}$$

$$= -\int \frac{dv}{\sqrt{E + \frac{m^3M^2G^2}{2\ell^2} - \left(v - \sqrt{\frac{m}{2}}\frac{GMm}{\ell}\right)^2}}$$

$$\int \frac{dx}{\sqrt{a^2 - (x-b)^2}} = \arccos\left(\frac{x-b}{a}\right)$$

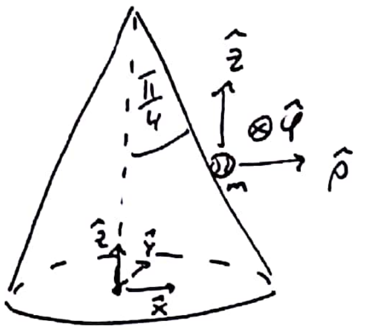
$$a = \sqrt{E + \frac{m^3M^2G^2}{2\ell^2}} \quad , \quad b = \sqrt{\frac{m}{2}} \cdot \frac{GMm}{\ell}$$

$$\Rightarrow \quad I = \arccos\left(\frac{\frac{1}{r} - b}{a}\right) = \theta + c$$

$\uparrow$ Cond. Inicial de r y θ

$$\Rightarrow \quad \boxed{r(\theta) = \frac{1}{\sqrt{\frac{m}{2}}\frac{GMm}{\ell} + \left(\sqrt{E + \frac{M^2m^3G^2}{2\ell^2}}\right)\cos(\theta + c)}}$$

P31



$\dot\varphi(0) = \omega_0$

$\vec f = -\frac{B\hat\rho}{\rho^2}$

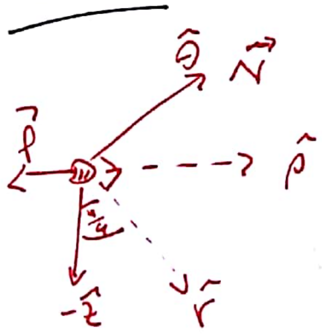
a) $\vec f$ en cart ⇝ $\vec f = \frac{-B(\hat x\cos(\varphi) + \hat y\,\text{sen}(\varphi))}{x^2+y^2}$ ⇝ $\cos(\varphi) = \frac{x}{\sqrt{x^2+y^2}}$

⇝ $\text{Sen}(\varphi) = \frac{y}{\sqrt{x^2+y^2}}$

(*) $\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}$ ⟹ $\vec f = \frac{-B(\hat x x + \hat y y)}{(x^2+y^2)^{\frac{3}{2}}}$

↳ $\frac{\partial f_y}{\partial x} = \frac{3}{2}\frac{By\cdot 2x}{(x^2+y^2)^{\frac{5}{2}}} = \frac{\partial f_x}{\partial y}$ ✓

b) DCL



⇝ $\hat\varphi$ | $\sum F_\varphi = 0$

⇝ $\frac{d}{dt}(r^2\dot\varphi\,\text{sen}^2(\theta)) = 0$

$t=0$, $r(0) = r_0$, $\dot\varphi(0) = \omega_0$, $\theta(0) = \frac{\pi}{4}$ } $\dot\varphi = \frac{r_0^2\omega_0}{r^2}$

c)

$$E_m = \frac{1}{2} m |\vec{v}|^2 + U(r, \dot{r})$$

$$\vec{f} = \frac{-B\,\hat{\rho}}{\rho^2} \Rightarrow \vec{f} = -\nabla U_f$$

$$\Rightarrow \frac{-B\,\hat{\rho}}{\rho^2} = -\frac{\partial U_f}{\partial \rho}\hat{\rho} - \frac{1}{\rho}\frac{\partial U_f}{\partial \varphi}\hat{\varphi} - \frac{\partial U_f}{\partial z}\hat{z}$$

∘) $\frac{B}{\rho^2} = \frac{\partial U_f}{\partial \rho} \Rightarrow U_f(\rho) - U_f(\rho_1) = -B\left(\frac{1}{\rho} - \frac{1}{\rho_1}\right)$

∘) $0 = \frac{\partial U_f}{\partial \varphi}$

∘) $0 = \frac{\partial U_f}{\partial z}$

$\Rightarrow U_f(\rho) = \frac{-B}{\rho}$

Usamos un ρ_1 como referencia
dado que en $\rho = 0 \to U_f \to \infty$
$\Rightarrow$ tenemos la libertad de usar
$\rho_1 \to \infty \Rightarrow U(\rho_1) = 0$
⊛ con esto fijamos el pto de pot. cero

⊛ por geometría $\to$ $r\,\mathrm{sen}\left(\frac{\pi}{4}\right) = \rho \Rightarrow \rho = \frac{r}{\sqrt{2}}$

$$\vec{v} = \dot{r}\,\hat{r} + r\dot{\varphi}\,\hat{\varphi}\,\mathrm{sen}(\theta) + \underbrace{r\dot{\theta}}_{=0}\,\hat{\theta}$$

$$\Rightarrow |\vec{v}|^2 = \dot{r}^2 + \left(\frac{r_0^2 \omega_0 \sqrt{2}}{2r}\right)^2$$

$$\therefore\ E_{mec.}(r, \dot{r}) = \frac{1}{2} m \dot{r}^2 + \frac{1}{4} m \frac{r_0^2 \omega_0^2}{r^2} - \frac{B\sqrt{2}}{r}$$

d) Como la energía se conserva

$$E_M(r_0, \dot{r}=0) = E_M(r_f, \dot{r}=0) \quad , \quad r_f \rightarrow r_{max}, r_{min}$$

$$\Rightarrow \frac{1}{4} \frac{m r_0^4 w_0^2}{r_0^2} - \frac{B\sqrt{2}}{r_0} = \frac{1}{4} m \frac{r_0^4 w_0^2}{r_f^2} - \frac{B\sqrt{2}}{r_f}$$

$$\Rightarrow r_f = \frac{-r_0^3 w_0^2 b\sqrt{2} \pm \sqrt{(r_0^3 w_0^2 b\sqrt{2})^2 + 4 r_0^2 w_0^2 (\frac{1}{4} - b\sqrt{2}) r_0^4 \frac{w_0^2}{4}}}{2 r_0^2 w_0^2 (\frac{1}{4} - b\sqrt{2}}$$

{Donde pasamos de B a b tal que $B = b \cdot C$, sin perdida de generalidad con C las const. necesarias para factorizar }

$$\Rightarrow r_f = \frac{-r_0^3 w_0^2 b\sqrt{2} \pm (r_0^3 w_0^2 b\sqrt{2} - \frac{r_0^3 w_0^2}{2})}{2 r_0^2 w_0^2 (\frac{1}{4} - b\sqrt{2})}$$

$$\Rightarrow r_f = r_0 \ , \ \frac{r_0}{4b\sqrt{2} - 1}$$

$$\hookrightarrow \boxed{b > \frac{1}{4\sqrt{2}}}$$