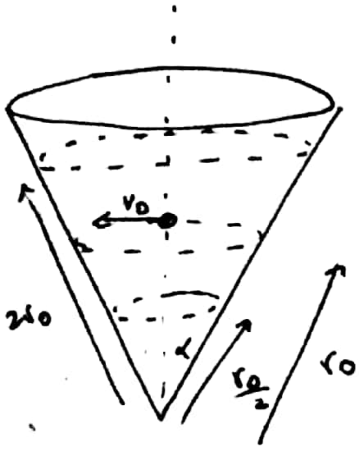
Ejercicio 2



$t=0$ $\quad \vec{r}(0) = r_0 \hat{r}$, $\theta = \alpha = cte$

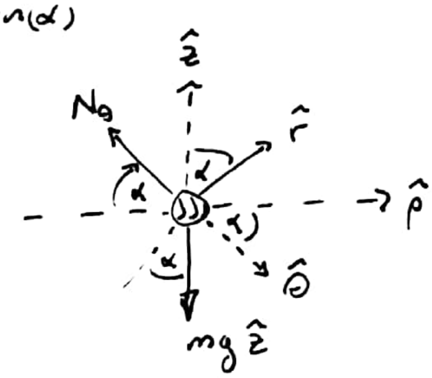
$\vec{v}(0) = v_0 \hat{\phi}$

$$\Rightarrow \vec{r} = r(t)\hat{r}$$

$$\dot{\vec{r}} = \dot{r}\hat{r} + r\hat{\phi}\dot{\phi}sen(\theta) + \cancel{r\dot{\theta}\hat{\theta}}$$

$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\phi}^2 sen^2(\theta))\hat{r} - r\dot{\phi}^2 sen(\alpha)\cos(\alpha)\hat{\theta} + \frac{(r^2\dot{\phi}\, sen^2(\alpha))^{\cdot}}{r\, sen(\alpha)}\hat{\phi}$$

$$\dot{\hat{r}} = \dot{\phi}\hat{\phi}\, sen(\theta) + \dot{\theta}\hat{\theta}$$
$$\dot{\hat{\theta}} = \dot{\phi}\hat{\phi}\cos(\theta) - \dot{\theta}\hat{r}$$
$$\dot{\hat{\phi}} = -\dot{\phi}(\hat{\theta}\cos(\theta) + \hat{r}\, sen(\theta))$$



a) ¿Ec. Mov.?

$$\sum \vec{F} = \vec{N}_\theta + mg\hat{z} = m\vec{a}$$
$$= -N\hat{\theta} + mg(-\hat{r}\cos(\alpha) + \hat{\theta}\, sen(\alpha))$$

$\Rightarrow$ $\hat{r}]$ $\quad m(\ddot{r} - r\dot{\phi}^2 sen^2(\alpha)) = -mg\cos(\alpha)$ $\quad (1)$

$\hat{\theta}]$ $\quad m(-r\dot{\phi}^2 sen(\alpha)\cos(\alpha)) = -N_\theta + mg\, sen(\alpha)$ $\quad (2)$

$\hat{\phi}]$ $\quad m(2\dot{r}\dot{\phi}\, sen(\alpha) + r\ddot{\phi}\, sen(\alpha)) = 0$ $\quad (3)$

$$m\left(\frac{sen(\alpha)}{r}\frac{d}{dt}(r^2\dot{\phi})\right) \quad , \quad sen(\alpha) \neq 0, \quad r \neq 0$$

$$\rightarrow \frac{d}{dt}(r^2\dot{\phi}) = 0$$

$$\Rightarrow \underline{r^2\dot{\phi} = cte}$$

En $t=0 \rightsquigarrow$ $r(0) = r_0$

$$\dot{\vec{r}}(0) = V_0\,\hat{\phi} = \dot{r}(0)\,\hat{r} + r_0 \,\mathrm{sen}(\alpha)\,\dot{\phi}_0\,\hat{\phi}$$

$$\Rightarrow \dot{r}(0) = 0$$

$$r_0\,\mathrm{sen}(\alpha)\,\dot{\phi}_0 = V_0 \Rightarrow \dot{\phi}_0 = \frac{V_0}{r_0\,\mathrm{sen}(\alpha)} \quad (4)$$

$\Rightarrow$ como $r^2\dot{\phi}$ es const.

$$r^2(t)\,\dot{\phi}(t) = r^2(0)\,\dot{\phi}(0)$$

Reemplazando ec. (4) $\Rightarrow$

$$\dot{\phi}(t) = \frac{r_0 V_0}{r^2\,\mathrm{sen}(\alpha)} \quad (5)$$

Usando la ec. de mov. en $(\hat{r})$

$$\Rightarrow m\left(\ddot{r} - r\,\dot{\phi}^2\,\mathrm{sen}^2(\alpha)\right) = -mg\cos(\alpha) \quad (6)$$

Reemplazando (5) en (6)

$$\ddot{r} - \frac{r_0^2}{r^3}V_0^2 = -g\cos(\alpha)$$

$$\circledast \quad \ddot{r} = \frac{d}{dt}\frac{dr}{dt} = \frac{dr}{dt}\cdot\frac{d}{dr}(\dot{r}) \Rightarrow \ddot{r} = \dot{r}\,\frac{d\dot{r}}{dr}$$

$$\Rightarrow \dot{r}\,\frac{d\dot{r}}{dt} - \frac{r_0^2V_0^2}{r^3} = -g\cos(\alpha) \quad \Big/ \int_{r(t=0)}^{r(t)} dr$$

$$\int_0^{\dot{r}(t)} \dot{r}\,d\dot{r} - \int_{r_0}^{r(t)} \frac{r_0^2V_0^2}{r^3}\,dr = -g\int_{r_0}^{r(t)} \cos(\alpha)\,dr$$

$$\Rightarrow \frac{1}{2}\left(\dot{r}^2(t) - 0\right) + \frac{1}{2}\left(r_0^2V_0^2\right)\left(\frac{1}{r^2(t)} - \frac{1}{r_0^2}\right) = -g\cos(\alpha)\,(r-r_0)$$

$$\Rightarrow \boxed{\dot{r}^2 = r_0^2V_0^2\left(\frac{1}{r_0^2} - \frac{1}{r}\right) - 2g\cos(\alpha)(r-r_0)} \quad (7)$$

c) Para que no se escape por arriba, necesitamos que en

$r = 2r_0$ y $\dot{r}(2r_0) = 0$

$$\Rightarrow 0 = (V_0 \cancel{r_0})^2 \left(\frac{1}{\cancel{r_0}^2} - \frac{1}{(2\cancel{r_0})^2} \right) - 2g\cos(\alpha)(2r_0 - r_0)$$

$$= \frac{3V_0^2}{4} - 2g\cos(\alpha) r_0$$

$$\Rightarrow \boxed{V_0 = \sqrt{\frac{8}{3} g \cos(\alpha) r_0}}$$

d) Para que no escape por abajo, necesitamos que en

$r = \frac{r_0}{2}$ y $\dot{r}\left(\frac{r_0}{2}\right) = 0$

$$\Rightarrow 0 = (V_0 \cancel{r_0})^2 \left(\frac{1}{\cancel{r_0}^2} - \frac{1}{\left(\frac{\cancel{r_0}}{2}\right)^2} \right) - 2g\cos(\alpha)\left(\frac{r_0}{2} - r_0\right)$$

$$= -3V_0^2 + g r_0 \cos(\alpha)$$

$$\Rightarrow \boxed{V_0 = \sqrt{\frac{g r_0 \cos(\alpha)}{3}}}$$