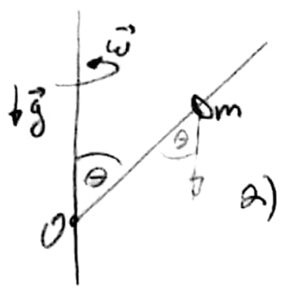
Auxiliar #4

1)



$\dot{\phi} = \omega$

a) ¿Ec. mov.?

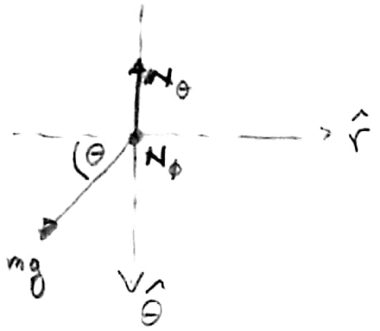
$\vec{r} = r\hat{r}$

$\dot{\vec{r}} = \dot{r}\hat{r} + r(\dot{\phi}\hat{\phi}\sin(\theta) + \cancel{\dot{\theta}\hat{\theta}})$

$\ddot{\vec{r}} = \ddot{r}\hat{r} + 2\dot{r}(\dot{\phi}\hat{\phi}\sin(\theta) + \cancel{\dot{\theta}\hat{\theta}}) + r\ddot{\phi}\hat{\phi}\sin(\theta)$
$+ r\dot{\phi}(-\dot{\phi}(\hat{\theta}\cos(\theta) + \hat{r}\sin(\theta)))\sin(\theta) +$
$\cancel{r\dot{\phi}\hat{\phi}\cos(\theta)\dot{\theta}}$

$\rightarrow \ddot{\vec{r}} = (\ddot{r} - r\dot{\phi}^2\sin^2(\theta))\hat{r} +$
$(r\ddot{\phi}\sin(\theta) + 2\dot{r}\dot{\phi}\sin(\theta))\hat{\phi} +$
$(-r\dot{\phi}^2\cos(\theta)\sin(\theta))\hat{\theta}$

DCL



ec. mov. por eje

$\hat{r}$ | $\cancel{m}(\ddot{r} - r\dot{\phi}^2\sin^2(\theta)) = -\cancel{m}g\cos(\theta)$

$\hat{\phi}$ | $m(\cancel{r\ddot{\phi}\sin(\theta)} + 2\dot{r}\dot{\phi}\sin(\theta)) = N_\phi$

$\hat{\theta}$ | $m(-r\dot{\phi}^2\cos(\theta)\sin(\theta)) = -N_\theta + mg\sin(\theta)$

$$\Rightarrow \boxed{\begin{aligned} &\ddot{r} = r\dot{\phi}^2\sin^2(\theta) - g\cos(\theta) \\ &2\dot{r}\dot{\phi}\sin(\theta) = \frac{N_\phi}{m} \\ &N_\theta = m\sin(\theta)(r\dot{\phi}^2\cos(\theta) + g) \end{aligned}}$$

$\rightarrow$ ec. mov. (r)

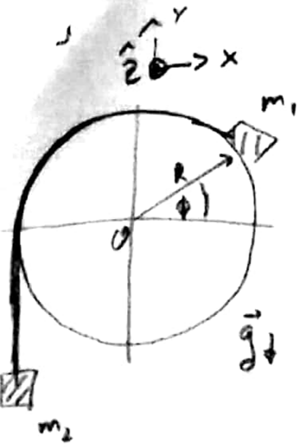
b) ¿ω / r=L ?

↳ $\ddot{r} = \dot{r} = 0$ $\Rightarrow$ $r\,\overset{\omega}{\dot{\phi}}{}^{2}\,\mathrm{sen}^2(\theta) = g\cos(\theta)$ (t)

$$N_\theta = m\,\mathrm{sen}(\theta)\left(r\dot{\phi}^2\cos(\theta) + g\right)$$

con la exp. (t) tenemos $\omega^2 = \dfrac{g\cos(\theta)}{L\,\mathrm{sen}^2(\theta)}$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{g\cos(\theta)}{L\,\mathrm{sen}^2(\theta)}}}$$



$m_2 > m_1$

$l_0 = \pi R$

2) $\dot{\vec{l}}_1 = \vec{\tau}_1 \quad \rightsquigarrow \quad \vec{l} = \vec{r} \times \vec{p}$

$\vec{\tau} = \vec{r} \times \vec{F}$

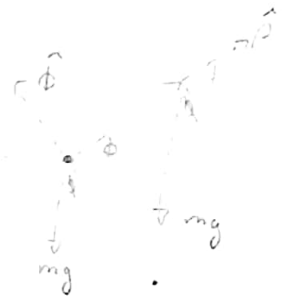
) Dado que $\vec{l} = \vec{r} \times \vec{p}$ donde $\vec{p} = m\dot{\vec{r}}$

$$\Rightarrow \vec{l} = m\,\vec{r} \times \dot{\vec{r}}$$

o) $\vec{r}_1 = R\hat{\rho}$, $\dot{\vec{r}}_1 = R\dot{\phi}\hat{\phi}$, $\ddot{\vec{r}}_1 = R\ddot{\phi}\hat{\phi} - R\dot{\phi}^2\hat{\rho}$

$$\Rightarrow \vec{l} = m(R\hat{\rho}) \times (R\dot{\phi}\hat{\phi}) = mR^2\dot{\phi}\hat{z}$$

$$\Rightarrow \dot{\vec{l}} = mR^2\ddot{\phi}\hat{z}$$



o) $\vec{\tau} = \vec{r} \times \vec{F}$

$\hat{\rho} \times \hat{\rho} = 0$, $\hat{\rho} \times \hat{\phi} = \hat{z}$

$$\Rightarrow \vec{\tau} = (R\hat{\rho}) \times \left(-mg\left(\sin(\phi)\hat{\rho} + \cos(\phi)\hat{\phi}\right) + T\hat{\phi}\right)$$

$$= -mgR\cos(\phi)\hat{z} + RT\hat{z}$$

$$\Rightarrow \cancel{m}R^{\cancel{2}}\ddot{\phi}\hat{z} = -\cancel{m}g\cancel{R}\cos(\phi)\hat{z} + \cancel{R}\frac{T}{m}$$

$$\Rightarrow \boxed{\ddot{\phi} = -\frac{g}{R}\cos(\phi) + \frac{T}{m}}$$

b) ¿Ec. mov 1 y 2?

•) $\vec{r}_1 = R\hat{\rho}_1$, $\dot{\vec{r}}_1 = R\dot{\phi}\hat{\phi}_1$, $\ddot{\vec{r}}_1 = R\ddot{\phi}\hat{\phi}_1 - R\dot{\phi}^2\hat{\rho}_1$

•) $\vec{r}_2 = -R\hat{x} - R\phi\hat{y} = -R(\hat{x} + \phi\hat{y})$

↳ $\dot{\vec{r}}_2 = -R(0 + \dot{\phi}\hat{y})$, $\ddot{\vec{r}}_2 = -R\ddot{\phi}\hat{y}$

$\hat{y} = \cos(\phi)\hat{\phi} + \operatorname{sen}(\phi)\hat{\rho}$

$\Rightarrow \sum F_1 = m_1 g(-\hat{y}) + N\hat{\rho}_1 = m_1(R\ddot{\phi}\hat{\phi}_1 - R\dot{\phi}^2\hat{\rho}_1)$

$\Rightarrow$ $\underline{\hat{\rho}_1 /}$ $-m_1 R\dot{\phi}^2 = N - m_1 g\operatorname{sen}(\phi)$

$\underline{\hat{\phi}_1 /}$ $m_1 R\ddot{\phi} = -m_1 g\cos(\phi) + T$

$\Rightarrow \sum F_2 = m_2 g(-\hat{y}) + T = m_2 - R\ddot{\phi}\hat{y}$

$\Rightarrow N = m_1(g\operatorname{sen}(\phi) - R\dot{\phi}^2)$

$T = m_1(R\ddot{\phi} + g\cos(\phi))$

$T = m_2(g - R\ddot{\phi})$ ✓

c) ¿$\dot{\phi}(\phi)$?

$T = T$

$\Rightarrow m_2(g - R\ddot{\phi}) = m_1(R\ddot{\phi} + g\cos(\phi))$

$(m_1 + m_2)R\ddot{\phi} = g(m_2 - m_1\cos(\phi))$ $/\cdot\dot{\phi}$

$(m_1 + m_2)R\frac{d}{dt}\left(\frac{\dot{\phi}^2}{2}\right) = g(m_2 - m_1\cos(\phi))\frac{d\phi}{dt}$ $/\int_0^t dt$

$(m_1 + m_2)R\frac{\dot{\phi}^2}{2} = g(m_2\phi(t) - m_1\operatorname{sen}(\phi))$

$\Rightarrow \boxed{\dot{\phi}^2 = \frac{2g(m_2\phi(t) - m_1\operatorname{sen}(\phi))}{(m_1 + m_2)R}}$ $/\sqrt{\ }$

como $\dot{\phi} = \dfrac{2g}{(m_1+m_2)R}\left(m_2\,\phi - m_1\,\mathrm{sen}(\phi)\right)$

y $\ddot{\phi} = \dfrac{g\left(m_2 - m_1\cos(\phi)\right)}{(m_1+m_2)R}$

$$\Rightarrow \quad T = m_2\,g - \frac{m_2\cancel{R}\,g\left(m_2 - m_1\cos(\phi)\right)}{(m_1+m_2)\cancel{R}}$$

$$= \frac{m_2\,g}{m_1+m_2}\left(m_1 + \cancel{m_2} - \cancel{m_2} - m_1\cos\phi\right)$$

$$\Rightarrow \quad \boxed{T = \frac{m_1 m_2\,g}{(m_1+m_2)}\left(1-\cos\phi\right)}$$

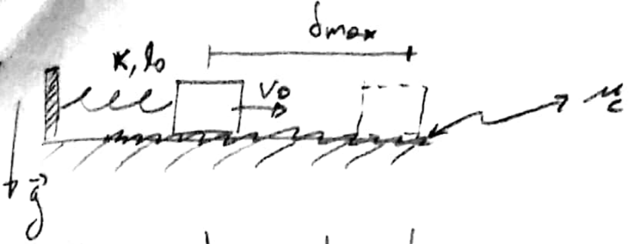
Sabemos que la cond de despegue es $N=0$

$$\Rightarrow \quad N = 0 = m_1\left(g\,\mathrm{sen}(\phi) - R\,\dot{\phi}^2\right)$$

$$\Rightarrow \quad \cancel{g}\,\mathrm{sen}(\phi) = \frac{\cancel{R}\cdot 2\cancel{g}}{(m_1+m_2)\cancel{R}}\left(m_2\,\phi - m_1\,\mathrm{sen}\,\phi\right)$$

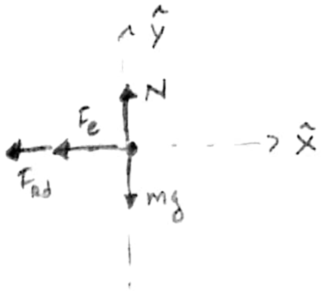
$$(m_1+m_2)\,\mathrm{sen}(\phi) = 2\left(m_2\,\phi - m_1\,\mathrm{sen}\,\phi\right)$$

$$(3m_1+m_2)\,\mathrm{sen}(\phi) = 2m_2\,\phi$$



En este prob tenemos una fuerza no conservativa

$\Rightarrow \Delta E_M = W$, con W el trabajo de la fuerza de roce



$\Rightarrow \underline{\hat{y}}\,|\; m\ddot{y} = 0 = -mg + N \;\Rightarrow\; \underline{N = mg}$

$\underline{\hat{x}}\,|\; m\ddot{x} = -F_e - F_{Rd}$

Sabemos que $F_{Rd} = \mu_c N \Rightarrow \boxed{\vec{F}_{Rd} = -\mu_c mg\,\hat{x}}$

$$\therefore\; W_{F_{rd}} = \int_{l_0}^{l_0+\delta_{max}} \vec{F}_{Rd} \cdot d\vec{l} = \int_{l_0}^{l_0+\delta_{max}} -\mu_c mg\,\hat{x} \cdot dx\,\hat{x}$$

$$= -\mu_c mg\,(l_0 + \delta_{max} - l_0) = \boxed{-\mu_c mg\,\delta_{max} = W}$$

Retomando la energía: $E_M = E_c + U$

podemos decir que $h = 0$ (pt. ref. la sup horizontal)

$\Rightarrow$ inicialmente $\Rightarrow E_c^i = \frac{1}{2} m V_0^2$, $E_g^i = mgh = 0$, $E_e^i = 0$

$\Rightarrow$ final $\Rightarrow E_c^f = 0$, $E_g^f = 0$, $E_e^f = \frac{1}{2}\delta_{max}^2 K$

no esta estirado

$$\Rightarrow E_M^f - E_M^i = \frac{1}{2}\delta_{max}^2 K - \frac{1}{2} m V_0^2 = -\mu_c mg\,\delta_{max}$$

$$\Rightarrow \boxed{\mu_c = \frac{mV_0^2 - K\delta_{max}^2}{2mg\,\delta_{max}}}$$