Auxiliar 2

P1

$\rho(t) = v_0 t$

$\phi(t) = w_0 t$

$z(t) = V_0 t$

a) ¿$\vec{r}(t)$, $\vec{v}(t)$ y $\vec{a}(t)$? en coord. cil.

Por definición sabemos que $r(t)$ en cilíndricos se escribe como:

$$\vec{r}(t) = \rho(t)\,\hat{\rho} + z(t)\,\hat{z}$$

$$\Rightarrow \vec{r}(t) = V_0 t\,\hat{\rho} + V_0 t\,\hat{z} = \boxed{V_0 t(\hat{\rho}+\hat{z}) = \vec{r}(t)}$$

Además sabemos que $\vec{V} = \dot{\vec{r}}$ y $\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$

$\Rightarrow$ Como $\vec{r}(t) = V_0 t(\hat{\rho}+\hat{z})$ (Supondremos que $V_0 =$ cte en t)

$$\Rightarrow \dot{\vec{r}} = V_0\left[(\hat{\rho}+\hat{z}) + t\,\frac{d}{dt}(\hat{\rho}+\cancel{\hat{z}})\right]$$

(las coord. cartesianas no varían en el tiempo)

⊛ Dado que $\hat{\rho} = \hat{x}\cos(\phi) + \hat{y}\,\text{sen}(\phi)$

$$\Rightarrow \frac{d}{dt}\hat{\rho} = -\dot{\phi}\,\hat{x}\,\text{sen}(\phi) + \dot{\phi}\,\hat{y}\cos(\phi) \Rightarrow \dot{\hat{\rho}} = \dot{\phi}\,\underbrace{(-\hat{x}\,\text{sen}(\phi) + \hat{y}\cos(\phi))}^{\hat{\phi}}$$

$$\therefore \boxed{\dot{\hat{\rho}} = \dot{\phi}\,\hat{\phi}}$$

$$\Rightarrow \dot{\vec{r}} = V_0[\hat{\rho}+\hat{z}+t\dot{\phi}\,\hat{\phi}] = \boxed{V_0[\hat{\rho}+w_0 t\,\hat{\phi}+\hat{z}] = \dot{\vec{r}}}$$

•) $$\ddot{\vec{r}} = \frac{d}{dt}\dot{\vec{r}} = V_0\left[\dot{\hat{\rho}} + w_0\hat{\phi} + w_0 t\,\dot{\hat{\phi}} + \cancel{\dot{\hat{z}}}^{\,0}\right]$$

⊛ Dado que $\hat{\phi} = -\hat{x}\,\text{sen}(\phi) + \hat{y}\cos(\phi)$

$$\dot{\hat{\phi}} = -\dot{\phi}\,\hat{x}\cos(\phi) - \dot{\phi}\,\hat{y}\,\text{sen}(\phi) = -\dot{\phi}\,\hat{\rho}$$

$$\boxed{\dot{\hat{\phi}} = -\dot{\phi}\,\hat{\rho}}$$

$$\Rightarrow \quad \ddot{\vec{r}} = v_0\left[\dot{\phi}\hat{\phi} + w_0\hat{\phi} + w_0 t(-\dot{\phi}\hat{\rho})\right]$$

$$= v_0\left[2w_0\hat{\phi} - w_0^2 t\,\hat{\rho}\right]$$

$$\Rightarrow \quad \boxed{\ddot{\vec{r}} = v_0 w_0\left[2\hat{\phi} - w_0 t\hat{\rho}\right]}$$

b) Para coordenadas esferias sabemos que la pos. se escribe en función del radio y su vector unitario

$$\Rightarrow \quad \vec{r}(t) = r(t)\,\hat{r}$$

Donde $r(t) = \sqrt{\rho^2(t) + z^2(t)}$ y $\hat{r} = \hat{\rho}\,sen(\theta) + \hat{z}\cos(\theta)$

$$\Rightarrow \quad \vec{r} = \sqrt{\rho^2(t) + z^2(t)}\,\left(\hat{\rho}\,sen(\theta) + \hat{z}\cos(\theta)\right)$$

$$\boxed{\vec{r} = v_0 t\sqrt{2}\;\hat{r}}$$

$$\hat{r} = (\hat{x}\cos(\phi) + \hat{y}\,sen(\phi))\,sen(\theta) + \hat{z}\cos(\theta)$$
$$\hat{\theta} = (\hat{x}\cos(\phi) + \hat{y}\,sen(\phi))\cos(\theta) - \hat{z}\,sen(\theta)$$
$$\hat{\phi} = -\hat{x}\,sen(\phi) + \hat{y}\cos(\phi)$$

$$\dot{\vec{r}} = v_0\sqrt{2}\left[\hat{r} + t\frac{d}{dt}\hat{r}\right]$$

$$\Rightarrow \dot{\hat{r}} = (-\dot{\phi}\,\hat{x}\,sen(\phi) + \dot{\phi}\,\hat{y}\cos(\phi))\,sen(\theta) +$$
$$(\hat{x}\cos(\phi) + \hat{y}\,sen(\phi))\cos(\theta)\,\dot{\theta} +$$
$$-\hat{z}\,\dot{\theta}\,sen(\theta)$$

$$\boxed{\dot{\hat{r}} = \dot{\phi}\hat{\phi}\,sen(\theta) + \dot{\theta}\hat{\theta}}$$

$$\Rightarrow \quad \dot{\vec{r}} = v_0\sqrt{2}\left[\hat{r} + t w_0\hat{\phi}\,sen(\theta) + t\dot{\theta}\hat{\theta}\right]$$

Dado que $z(t) = r(t)\cos\theta$ $\Rightarrow$ $\theta = \arccos\left(\frac{z}{r}\right)$

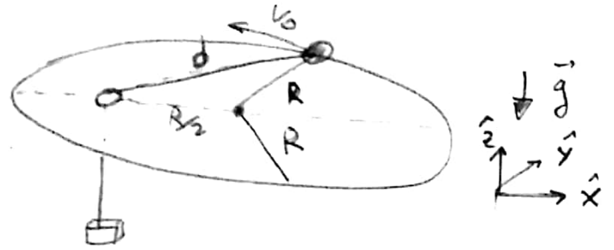
$$= \arccos\left(\frac{v_0 t}{v_0 t\sqrt{2}}\right) = \frac{\pi}{4} = \theta$$

$$\Rightarrow \boxed{\dot{\theta} = 0}$$

$$\Rightarrow \quad \boxed{\dot{\vec{r}} = v_0\sqrt{2}\left[\hat{r} + t w_0\hat{\phi}\right]}$$

Scanned by CamScanner

2)



V_0 suf. pequeño para que la cuerda no se destense

a) ¿$\dot{r}(\theta)$? [del bloque]

Vectorialmente vemos que la posición de la partícula es:

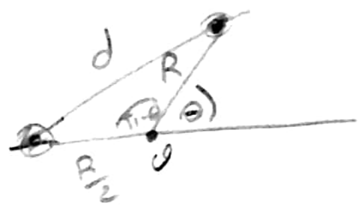
$$\vec{r}(t) = R\hat{\rho} = \frac{R}{2}(-\hat{x}) + \vec{d}$$

Pero como nos piden la rapidez del bloque, vemos que este se mueve junto con la cuerda por lo que lo que la rapidez del bloque será: $\dot{d}$

⇒ Usando el teo. del coseno:

$$d^2 = R^2 + \left(\frac{R}{2}\right)^2 - 2R\cdot\frac{R}{2}\cos(\pi-\theta)$$

$$= \frac{5}{4}R^2 - R^2\underbrace{\cos(\pi-\theta)}_{-\cos(\theta)}$$



$$\Rightarrow d^2 = \frac{5}{4}R^2 + R^2\cos(\theta) \quad / \frac{d}{dt}$$

$$2d\dot{d} = -R^2\,\text{sen}(\theta)\,\dot{\theta}$$

$$\Rightarrow \dot{d} = \frac{-R^2\,\text{sen}(\theta)\,\dot{\theta}}{2R\left(\frac{5}{4}+\cos(\theta)\right)^{\frac{1}{2}}} \leftarrow ?$$

Para encontrar $\dot{\theta}$, usamos que se mueve a vel const. (la partícula)

$$\Rightarrow \vec{V}_0 = R\dot{\theta}\hat{\theta} \Rightarrow \dot{\theta} = \frac{V_0}{R}$$

$\vec{r} = R\hat{\rho}$
$\dot{\vec{r}} = R\dot{\theta}\hat{\theta} = \vec{V}_0$

$$\Rightarrow \dot{d} = \frac{-V_0\,\text{sen}(\theta)}{2\left(\frac{5}{4}+\cos(\theta)\right)^{\frac{1}{2}}}$$

b) ¿ $\dot{d}_{max}$? $\rightsquigarrow$ $\ddot{d} = 0$

$$\Rightarrow \dot{d} = -\frac{V_0 \operatorname{sen}(\theta)}{2\left(\frac{5}{4}+\cos(\theta)\right)^{\frac{1}{2}}} \Rightarrow \ddot{d} = -\frac{V_0}{2} \cdot \frac{\left(2\cos(\theta)\left(\frac{5}{4}+\cos(\theta)\right)^{\frac{1}{2}} + \operatorname{sen}(\theta)\frac{+\operatorname{sen}(\theta)}{\left(\frac{5}{4}+\cos(\theta)\right)^{\frac{1}{2}}}\right)}{\left(\frac{5}{4}+\cos\theta\right)^{\frac{1}{2}}}$$

$$\Rightarrow \ddot{d}_{max} \stackrel{!}{=} 0 \Rightarrow 2\cos(\theta)\left(\frac{5}{4}+\cos(\theta)\right)^{\frac{1}{2}} + \frac{\operatorname{sen}^2(\theta)}{\left(\frac{5}{4}+\cos(\theta)\right)^{\frac{1}{2}}} = 0$$

$$\Rightarrow 2\cos(\theta)\left(\frac{5}{4}+\cos(\theta)\right) + \operatorname{sen}^2(\theta)$$

$$= \frac{5}{2}\cos(\theta) + \cos^2(\theta) + \underbrace{\cos^2(\theta) + \operatorname{sen}^2(\theta)}_{=1} = 0$$

$\rightarrow$ Usando $x = \cos(\theta)$

$$\Rightarrow x^2 + \frac{5}{2}x + 1 = 0 \iff x = \frac{-\frac{5}{2} \pm \sqrt{\frac{25}{4} - 4}}{2} = \frac{-\frac{5}{2} \pm \frac{1}{2}\sqrt{25-16}}{2}$$

$$\Rightarrow x = -\frac{5}{4} \pm \frac{3}{4} \quad \begin{cases} x_1 = -2 & \times \\ x_2 = -\frac{1}{2} & \checkmark \end{cases}$$

$$\rightarrow \cos(\theta) = -\frac{1}{2}$$

$$\rightarrow \underline{\theta = \frac{2}{3}\pi}$$

$$\rightarrow \dot{d} = \frac{-V_0 \operatorname{sen}\left(\frac{2\pi}{3}\right)}{2\left(\frac{5}{4} + -\frac{1}{2}\right)^{\frac{1}{2}}} = \frac{V_0 \cancel{\frac{\sqrt{3}}{2}}}{2\cancel{\frac{\sqrt{3}}{2}}} \rightarrow \boxed{\dot{d} = \frac{V_0}{2}}$$

c) Evaluando $\ddot{d}(0) = -\frac{V_0}{2} \frac{\left(2\left(\frac{5}{4}+1\right)^{\frac{1}{2}} + 0\right)}{\left(\frac{5}{4}+1\right)^{\frac{1}{2}}} = -V_0$