Lo que faltó de la auxiliar #6...

P1) Las leyes de conservación del momentum lineal, angular y energía, obtenemos el siguiente sistema de ecuaciones.

Incógnitas: $\{v_f, v_{CM}, \omega\}$

3 ecuaciones
$$\begin{cases} MV = m v_{CM} - M v_f & (1) \\ -MVD = \dfrac{mL^2}{12}\omega + M v_f D & (2) \\ \dfrac{MV^2}{2} = \dfrac{M v_f^2}{2} + \dfrac{m v_{CM}^2}{2} + \dfrac{1}{24} m L^2 \omega^2 & (3) \end{cases}$$

$(1)\cdot D + (2)$ $\quad 0 = m D v_{CM} - \dfrac{mL^2}{12}\omega$

$\hookrightarrow \quad v_{CM} = -\dfrac{L^2\omega}{12D}$; con $\omega < 0$, por ahora $(v_{CM} > 0)$

De (1): $\quad v_f = \dfrac{m v_{CM}}{M} - V$

$$\Rightarrow \frac{MV^2}{\cancel{2}} = \frac{M}{\cancel{2}}\left(\frac{m v_{CM}}{M} - V\right)^2 + \frac{m v_{CM}^2}{\cancel{2}} + \frac{mL^2\omega^2}{\cancel{24}\ 12}$$

$$\rightarrow MV^2 = M\left(\frac{m^2 v_{CM}^2}{M^2} + \frac{2m v_{CM} V}{M} + V^2\right) + m v_{CM}^2 + \frac{mL^2\omega^2}{12}$$

$$\Rightarrow MV^2 = M\left(\frac{m^2}{M^2}\frac{L^4\omega^2}{144D^2} + \frac{2m}{M}\frac{L^2}{12D}\omega V + V^2\right) + \frac{mL^4\omega^2}{144D^2} + \frac{mL^2\omega^2}{12}$$

$$\cancel{MV^2} = \frac{m^2L^4}{144MD^2}\omega^2 + \frac{mL^2V\cancel{\omega}}{6D} + \cancel{MV^2} + \frac{mL^4\omega^{\cancel{2}}}{144D^2} + \frac{mL^2\omega^{\cancel{2}}}{12}$$

$$\Rightarrow \omega\left(\frac{m^{\cancel{2}}L^{4}}{144MD^2} + \frac{\cancel{mL^2}}{12} + \frac{mL^{4}}{144D^2}\right) = -\frac{\cancel{mL^2}V}{6D}$$

$$\Rightarrow \omega\left(\frac{mL^2}{24MD^2}+\frac{1}{2}+\frac{L^2}{24D^2}\right)=\frac{-V}{D}$$

$$\Rightarrow \omega = \frac{\frac{-V}{D}}{\frac{mL^2}{24MD^2}+\frac{1}{2}+\frac{L^2}{24D^2}} = \frac{12D}{L^2}\cdot\frac{\frac{-L^2V}{12D^2}}{\frac{mL^2}{24MD^2}+\frac{1}{2}+\frac{L^2}{24D^2}}$$

$$\Rightarrow \omega = -\frac{12D}{L^2}\,\frac{V}{\frac{m}{2M}+\frac{6D^2}{L^2}+\frac{1}{2}} = -\frac{12D}{L^2}\,\frac{2V}{\frac{m}{M}+\frac{12D^2}{L^2}+1}$$

nos interesa $|\omega|$; $\omega>0$: $\boxed{\omega = \frac{12D}{L^2}\,\underbrace{\frac{2V}{\frac{m}{M}+\frac{12D^2}{L^2}+1}}_{v_{CM}}}$

cambiamos $\omega<0$ por $\omega>0$.

Pero $v_{CM} = \frac{L^2\omega}{12D}$ $\Rightarrow$ esto será v_{CM}

Entonces $\boxed{v_{CM} = \frac{2V}{\frac{m}{M}+\frac{12D^2}{L^2}+1}}$ y con esto:

de (1): $v_f = \frac{m}{M}v_{CM} - V$

$$\Rightarrow \boxed{v_f = \frac{m}{M}\,\frac{2V}{\frac{m}{M}+\frac{12D^2}{12^2}+1} - V}$$

Finalmente:

$$K_{rot} = \frac{6D^2}{L^2}m\left[\frac{2V}{\frac{m}{M}+\frac{12D^2}{L^2}+1}\right]^2$$

$$K_{tras} = \frac{1}{2}m\left[\frac{m}{M}\,\frac{2V}{\frac{m}{M}+\frac{12D^2}{L^2}+1} - V\right]^2$$

P2] Tenemos que μ acaba el combustible en el tiempo $t^* = \dfrac{\mu M_c}{(M_i - M_c)g}$, evaluamos $z(t^*)$:

$$z(t^*) = z_1 = -\frac{g}{2}\left[\frac{\mu M_c}{(M_i - M_c)g}\right]^2 + \frac{\mu M_i}{\alpha}\left(1 - \frac{\alpha\,\mu M_c}{M_i(M_i - M_c)g}\right) \cdot \left[\ln\left(1 - \frac{\alpha \mu M_c}{M_i(M_i - M_c)g}\right) - 1\right]$$

La altura, tiempo t después de acabado el combustible es:

$$h(t) = z_1 + \underbrace{v(t^*)}_{v_1}\, t - \frac{g t^2}{2} \quad \Big\}\ \text{sólo actúa la gravedad.}$$

$$v(t^*) = v_1 = -\mu \ln\left(1 - \frac{\alpha \mu M_c}{M_i(M_i - M_c)g}\right) - \frac{\cancel{g}\,\mu M_c}{(M_i - M_c)\cancel{g}}$$

La altura máxima es: $\dot{h}(t) = 0$

$$h_{max} = z_1 + \frac{v_1^2}{2g}$$

$$h_{max} = -\frac{g}{2}\left[\frac{\mu M_c}{(M_i - M_c)g}\right]^2 + \frac{\mu M_i}{\alpha}\left(1 - \frac{\alpha \mu M_c}{M_i(M_i - M_c)g}\right)\left[\ln\left(1 - \frac{\alpha \mu M_c}{M_i(M_i - M_c)g}\right) - 1\right]$$

$$+ \frac{1}{2g}\left[\mu \ln\left(1 - \frac{\alpha \mu M_c}{M_i(M_i - M_c)g}\right) + \frac{\mu M_c}{(M_i - M_c)g}\right]^2$$

$y(x) = L - \sqrt{4x^2+D^2}$

P3 La energía cinética es:

$K = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2$, Tenemos: $y = y(x)$

$\Rightarrow \dot{y} = y'(x)\cdot\dot{x}$

$$\Rightarrow \dot{y} = -\frac{1}{\cancel{2}}(4x^2+D^2)^{-1/2}\cdot\cancel{8}^{\,4}x\cdot\dot{x}$$

$$\Rightarrow \dot{y}^2 = 16x^2(4x^2+D^2)^{-1}\dot{x}^2$$

Con esto, obtenemos $K(x,\dot{x})$:

$$K = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\frac{16x^2}{4x^2+D^2}\dot{x}^2$$

$$K(x,\dot{x}) = \frac{1}{2}m\dot{x}^2\left[1+\frac{16x^2}{4x^2+D^2}\right] \quad //$$

Para expandir en torno a $x_{eq} = \frac{D}{2\sqrt{3}}$, hacemos una Serie de Taylor:

Término a expandir: $\frac{16x^2}{4x^2+D^2}$. Orden: lineal ($ax+b$)

Derivada evaluada en x_{eq}:

$$32x(4x^2+D^2)^{-1} - 16x^2(4x^2+D^2)^{-2}\cdot 8x\Big|_{x_{eq}} =$$

$$\frac{32D}{2\sqrt{3}}\left(4\frac{D^2}{12}+D^2\right)^{-1} - \frac{4\cancel{16}\cdot D^2}{\cancel{12}\,3}\left(\frac{D^2}{3}+\frac{3D^2}{3}\right)^{-2}\cdot\frac{8D}{2\sqrt{3}} =$$

$$4\frac{\cancel{16}D}{\cancel{\sqrt{3}}}\frac{\sqrt{3}}{4D^2} - \frac{\cancel{4D^2}}{\cancel{3}}\cdot\frac{9\sqrt{3}}{\cancel{16}D^4}\cdot\frac{\cancel{4D}}{\cancel{\sqrt{3}}} = \frac{4\sqrt{3}}{D} - \frac{\sqrt{3}}{D} = \frac{3\sqrt{3}}{D}$$

$$\Rightarrow S_t(x) = 1 + \frac{3\sqrt{3}}{D}\left(x - \frac{D}{2\sqrt{3}}\right) = 1 + \frac{3\sqrt{3}}{D}x - \frac{3}{2}$$

$$S_t(x) = \frac{3\sqrt{3}}{D}x - \frac{1}{2}$$

$$S_f(x) = \frac{3\sqrt{3}}{D}x - \frac{1}{2}$$

Con esto, la energía cinética cerca de X_{eq} es:

$$K \approx \frac{1}{2}m\dot{x}^2\left[1 + \frac{3\sqrt{3}}{D}x - \frac{1}{2}\right]$$

$$= \frac{1}{2}m\dot{x}^2\left[\frac{1}{2} + \frac{3\sqrt{3}}{D}x\right] \Big\} \text{ aproximación lineal en } x$$

Notamos que si $x \to X_{eq}$.

$$\Rightarrow K \to \frac{1}{2}m\dot{x}^2\left[\frac{1}{2} + \frac{3\cancel{\sqrt{3}}}{\cancel{D}}\frac{\cancel{D}}{2\cancel{\sqrt{3}}}\right] \Big\} \frac{4}{2} = 2$$

$$\Rightarrow K = m\dot{x}^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{x}^2$$

Es como si fueran dos partículas libres con igual energía cinética. Es válido afirmar

$K \sim m\dot{x}^2$, muy cerca de

$$x = X_{eq} = \frac{D}{2\sqrt{3}}$$

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