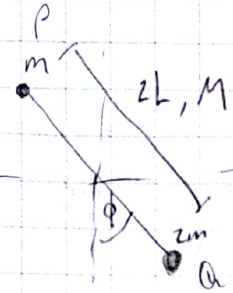
Aux #4 - 5 sistemas

P1)



$I_{barra} = \frac{1}{12} M \cdot (2L)^2 \cdot \frac{1}{12} \cdot 4ML^2 \cdot \frac{1}{3} ML^2$

$I_p = L^2 \cdot m$, $I_{Q} = 2mL^2$

$I_{sist} = \frac{1}{3} ML^2 + mL^2 + 2mL^2 \left(\frac{1}{3} ML^2 + 3mL^2\right)$

$I_{sist} = \frac{L^2}{3}(M^2 + 9m^2)$ $\quad T_{rot} = \frac{1}{2} I_{sist} \omega^2 = \frac{1}{2} I_0 \omega^2 + \frac{3mL^2\omega^2}{2}$

$\therefore E_{tot} = \frac{1}{2} I_{sist} \omega^2 - 2mgL\cos\phi + mgL\cos\phi \;/\frac{d}{dt}$

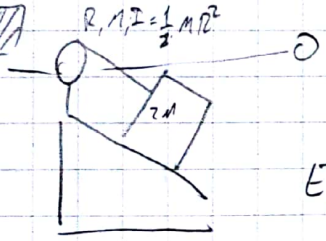
$\dot{E} = 0 = I_{sist} \dot{\phi}\ddot{\phi} + mgL\sin\phi\,\dot{\phi}$ $\quad \ddot{\phi} + \frac{mgL\sin\phi}{\frac{L^2}{3}(M^2+9m^2)} = 0$ Taylor a 1er orden

$\ddot{\phi} = \frac{-mgL\sin\phi}{I_0 + 3mL^2} = \frac{-mgL\sin\phi}{\frac{L^2}{3}(M^2+9m^2)} = \frac{-mgL\sin\phi}{\frac{L^2m^2}{3}\left(\frac{M^2}{m^2}+1\right)}$ $\quad \sin\phi \approx \phi$

$\phi(t) = A\cos(\omega t)$

$\ddot{\phi} = \frac{-3gL}{L^2 m} \frac{\sin\phi}{\left(\frac{M^2}{m^2}+1\right)}$

P2)



$E_i = 0$; $v = \omega R \Rightarrow \omega = \frac{v}{R}$

$E_f = \frac{1}{2} I \omega^2 + \frac{1}{2} 2M v^2 - 2MgH$

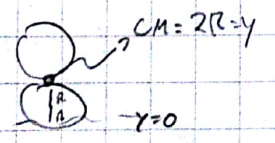
$E_i = E_f \Rightarrow 2MgH = \frac{1}{2}\left(\frac{1}{2} MR^2 \frac{v^2}{R^2} + 2Mv^2\right)$

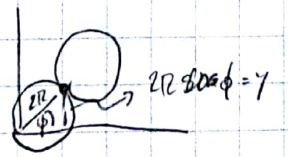
$4MgH = \frac{5}{2} Mv^2$

$v = \sqrt{\frac{8}{5} gH}$ //

P3)

E se conserva



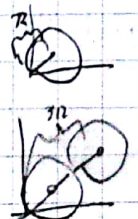


$E_i = K_i + U_{g_i} = 2Mg\,2R$

$E(\theta) = \frac{1}{2} I_{sist} \omega^2(\theta) + 2Mg\,2R\cos\theta$

$I_1 = \frac{2MR^2}{5} + MR^2 = \frac{7}{5} MR^2$

$I_2 = \frac{2MR^2}{5} + M(3R)^2 = \frac{47}{5} MR^2$



$I_{sist} = I_1 + I_2 = \frac{54}{5} MR^2$

$\therefore \frac{54}{10} MR^2\omega^2 + 4MgR\cos\theta = 4MgR$

$\omega^2(\theta) = \frac{20}{27} \frac{g}{R}(1 - \cos\theta)$ //