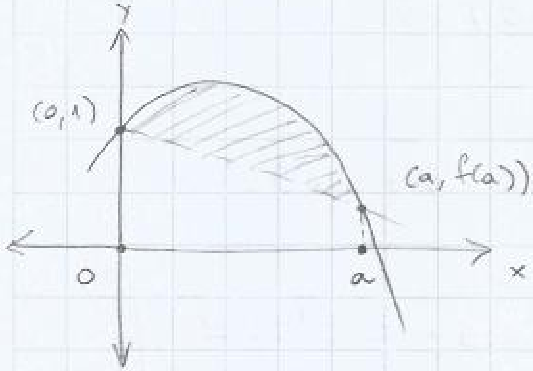
P1| $f(x) = -6x^2 + 5x + 1$. Acá asumimos que tanto $a \geq 0$ como $f(a) \geq 0$.



La recta tiene pendiente

$$\frac{f(a) - 1}{a - 0} = -6a + 5$$

entonces su ec. es

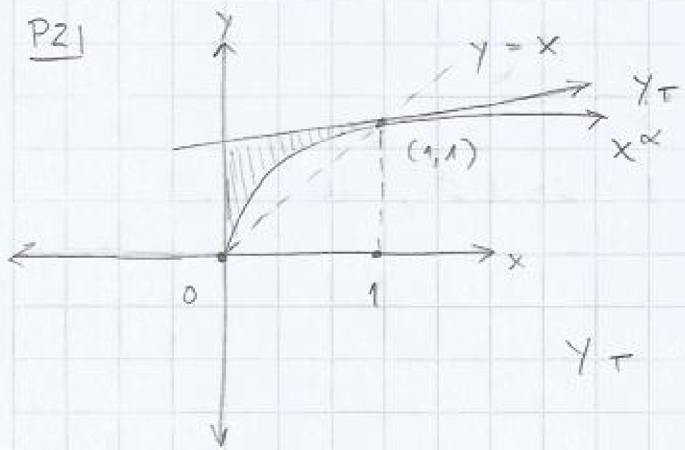
$$y = (-6a + 5)x + 1.$$

El área es $A = \int_0^a (f(x) - y(x))\,dx$

$$= \int_0^a (-6x^2 + \cancel{5x} + \cancel{1} + (6a - \cancel{5})x - \cancel{1})\,dx$$

$$= -\frac{6}{3}x^3 + \frac{6}{2}ax^2 \Big|_0^a = -2a^3 + 3a^3 = a^3$$

P2|



Tenemos $f(x) = x^\alpha$

$\Rightarrow f'(x) = \alpha x^{\alpha - 1}$

La ec de la recta tangente a (1,1)

$$y_T = f(1) + f'(1)(x - 1)$$

$$= \alpha x + (1 - \alpha)$$

El área es $A = \int_0^1 (y_T(x) - f(x))\,dx$

$$= \int_0^1 ((1-\alpha) + \alpha x - x^\alpha)\,dx$$

$$= (1+\alpha) + \frac{\alpha}{2} - \frac{1}{\alpha+1}$$

Álgebra ...

$$\Rightarrow \quad A = \frac{\alpha(1-\alpha)}{2(1+\alpha)} . //$$

El volumen es $V_{OY} = 2\pi \int_0^1 x(y_T - f(x))\,dx$

$$= 2\pi \int_0^1 x((1-\alpha) + \alpha x - x^\alpha)\,dx$$

$$= 2\pi \left(\frac{(1-\alpha)}{2} + \frac{\alpha}{3} - \frac{1}{\alpha+2} \right)$$

Álgebra ...

$$\Rightarrow \quad V_{OY} = \pi \frac{\alpha(1-\alpha)}{3(\alpha+2)} . //$$

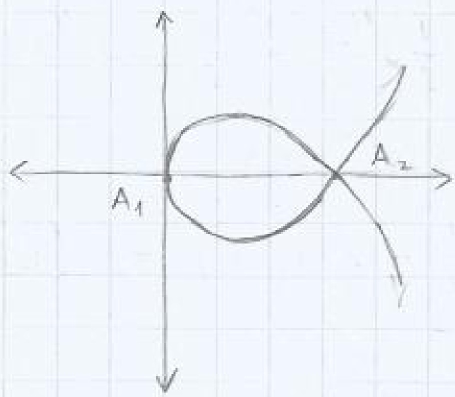
(Está malo el enunciado, para los que estaban en clases el resultado que teníamos está bien)

P3| $a > 0$, $9ay^2 = x(x-3a)^2$

$u = \frac{x}{a}$

$\cdot \frac{1}{a^3} \iff 9v^2 = u(u-3)^2$ $v = \frac{y}{a}$

como $(u-3)^2 > 0$ si $u<0$ no existe v.
Por otro lado $v = 0$ ssi $u=0$ o $u=3$,
luego el gráfico es



Para $0<u<3$ y $v>0$ tenemos

$$v = \frac{1}{3}(3-u)u^{1/2},$$

luego podemos calcular el largo del lazo cerrado como el doble del arco desde $u=0$ a $u=3$ de esta función (multiplicada por a)

$$\Rightarrow \ell = 2\int_{A_1}^{A_2}\left(\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right)^{1/2}dt$$

$U = x/a$

$v = y/a$

$$= 2a\int_{A_1}^{A_2}\left(\left(\frac{du}{dt}\right)^2+\left(\frac{dv}{dt}\right)^2\right)^{1/2}dt$$

P4) Con $\rho \cos\theta = x$, $\rho \sin\theta = y$

$$\rho^2 = x^2 + y^2 ,$$

la ec.

$$(x^2 + y^2 - c^2)(x^2 + y^2) = 4a^2 x^2$$

es

$$(\rho^2 - c^2)\rho^2 = 4a^2 \rho^2 \cos^2\theta$$

$$\Leftrightarrow \quad (\rho^2 - c^2) = 4a^2 \cos^2\theta$$

o sea $\rho^2 = (2a\cos\theta)^2 + c^2 \Rightarrow \theta \in [0, \pi]$

El área es

$$A = \frac{1}{2}\int_0^{2\pi} \rho^2 \, d\theta = \frac{1}{2}\int_0^{2\pi} (2a\cos\theta)^2 \, d\theta + \pi c^2$$

$$= 2a^2 \int_0^{2\pi} \cos^2\theta \, d\theta + \pi c^2$$

J //

sea $I = \int \cos^2\theta \, d\theta = \sin\theta\cos\theta + \int \sin^2\theta \, d\theta$,

pero

$$I + J = \int 1 \, d\theta = \theta ,$$

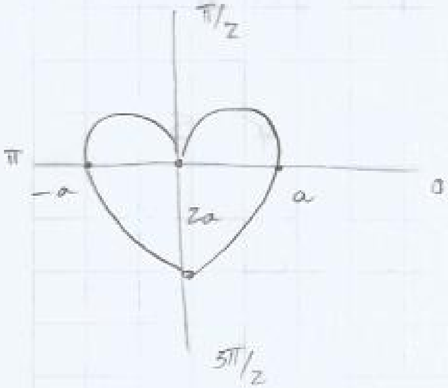
entonces $I = \frac{1}{2}(\theta + \sin\theta\cos\theta) + \text{cte}$ y

$$A = a^2(\theta + \sin\theta\cos\theta)\Big|_0^{2\pi} + \pi c^2 = 2\pi a^2 + \pi c^2 \quad //$$

$$\rho = a(1-\sin\theta) \quad , \quad A = \frac{1}{2}\int \rho^2 \, d\theta$$

PS|

(i) $$A = 2 \cdot \frac{1}{2} \int_{-\pi/2}^{\pi/2} a^2 (1-\sin\theta)^2 \, d\theta$$

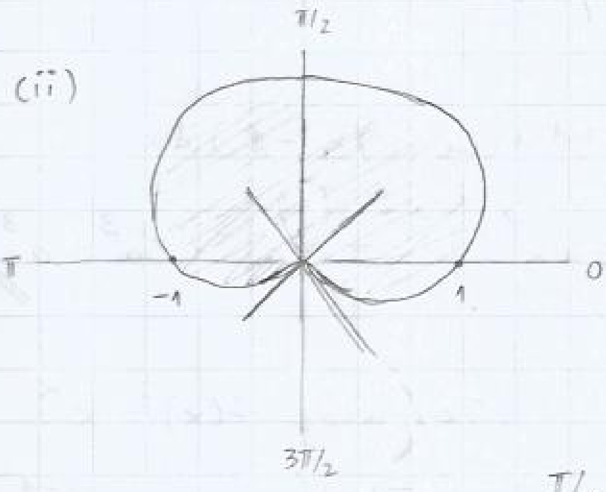


$$= a^2 \int_{-\pi/2}^{\pi/2} (1 - 2\sin\theta + \sin^2\theta) \, d\theta$$

$$= a^2 \left(1 + 2\cos\theta + \frac{\theta - \sin\theta\cos\theta}{2} \right) \Bigg|_{-\pi/2}^{\pi/2}$$

$$= a^2 \left(\pi + \frac{\pi}{2} \right) = \frac{3}{2}\pi a^2 . \ //$$

(ii)



$$\rho = 1 + 2\sin\theta$$

$$\sin\theta \geq -\frac{1}{2}$$

$$\Rightarrow \quad \theta = -\pi/6 \ , \ 7\pi/6$$

$$\Rightarrow \quad A = 2 \cdot \frac{1}{2} \int_{-\pi/6}^{\pi/2} (1 + 2\sin\theta)^2 \, d\theta \ \ldots$$