Pauta Ej. 7

$$\mathcal{L} = -\rho\phi + \vec{j}\cdot\vec{A} + \frac{\varepsilon_0}{2}\left(\vec{E}^2 - c^2\vec{B}^2\right)$$

$$\vec{E} = -\vec{\nabla}\phi - \dot{\vec{A}} \Rightarrow E^2 = (\vec{\nabla}\phi)^2 + 2\vec{\nabla}\phi\cdot\dot{\vec{A}} + \dot{\vec{A}}^2$$

$$\vec{B} = \vec{\nabla}\times\vec{A} \Rightarrow \vec{B}^2 = (\nabla\times A)^2.$$

$$\Rightarrow \mathcal{L}(\phi,\vec{A},t) = -\rho\phi + \vec{j}\cdot\vec{A} + \frac{\varepsilon_0}{2}\left[(\vec{\nabla}\phi)^2 + 2\vec{\nabla}\phi\cdot\dot{\vec{A}} + \dot{\vec{A}}^2 - c^2(\vec{\nabla}\times\vec{A})^2\right]$$

$$\delta\mathcal{L} = -\rho\delta\phi + \vec{j}\cdot\delta\vec{A} + \frac{\varepsilon_0}{2}\left[2\vec{\nabla}\phi\cdot\vec{\nabla}\delta\phi + 2\vec{\nabla}\delta\phi\cdot\dot{\vec{A}} + 2\vec{\nabla}\phi\cdot\delta\dot{\vec{A}} + 2\dot{\vec{A}}\cdot\delta\dot{\vec{A}} - 2c^2(\vec{\nabla}\times\vec{A})\cdot(\vec{\nabla}\times\delta\vec{A})\right.$$

Integrales por Parte

$$\vec{\nabla}\phi\cdot\vec{\nabla}\delta\phi = \nabla_i\phi\,\nabla_i\delta\phi = -\nabla_i^2\phi\,\delta\phi = -\vec{\nabla}^2\phi\,\delta\phi$$

$$\vec{\nabla}\delta\phi\cdot\dot{\vec{A}} = \dot{A}_i\nabla_i\delta\phi = -\nabla_i\dot{A}_i\,\delta\phi = -(\vec{\nabla}\cdot\dot{\vec{A}})\delta\phi.$$

$$\vec{\nabla}\phi\cdot\delta\dot{\vec{A}} = \nabla_i\phi\,\delta\dot{A}_i = -\nabla_i\dot{\phi}\,\delta A_i = -\vec{\nabla}\dot{\phi}\cdot\delta\vec{A}.$$

$$\dot{\vec{A}}\cdot\delta\dot{\vec{A}} = \dot{A}_i\,\delta\dot{A}_i = -\ddot{A}_i\,\delta A_i = -\ddot{\vec{A}}\cdot\delta\vec{A}.$$

$$\begin{aligned}(\nabla\times\vec{A})\cdot(\vec{\nabla}\times\delta\vec{A}) &= (\nabla\times\vec{A})_i(\nabla\times\delta\vec{A})_i \\ &= (\nabla\times\vec{A})_i\,\varepsilon_{ijk}\nabla_j\delta A_k \\ &= -\varepsilon_{ijk}\nabla_j(\nabla\times A)_i\,\delta A_k \\ &= -\varepsilon_{kij}\nabla_j(\nabla\times A)_i\,\delta A_k \\ &= \varepsilon_{kji}\nabla_j(\nabla\times A)_i\,\delta A_k. \\ &= (\nabla\times(\nabla\times A))_k\,\delta A_k \\ &= (\vec{\nabla}\times(\vec{\nabla}\times\vec{A}))\cdot\delta\vec{A}\end{aligned}$$

$$\Rightarrow \delta\mathcal{L} = -\rho\delta\phi + \vec{j}\cdot\delta\vec{A} + \frac{\varepsilon_0}{2}\left(-2\vec{\nabla}^2\phi\,\delta\phi - 2(\nabla\cdot\dot{\vec{A}})\delta\phi - 2\nabla\dot{\phi}\cdot\delta\vec{A} - 2\ddot{\vec{A}}\cdot\delta\vec{A} - 2c^2(\vec{\nabla}\times(\vec{\nabla}\times\vec{A}))\cdot\delta\vec{A}\right.$$

$$= -\rho\delta\phi + \vec{j}\cdot\delta\vec{A} - \varepsilon_0\left(\vec{\nabla}^2\phi\,\delta\phi + (\nabla\cdot\dot{\vec{A}})\delta\phi + \nabla\dot{\phi}\cdot\delta\vec{A} + \ddot{\vec{A}}\cdot\delta\vec{A} + c^2(\nabla\times(\nabla\times\vec{A}))\cdot\delta\vec{A}\right)$$

a) $$\frac{\delta S}{\delta \phi} = -\rho - \varepsilon_0 (\nabla^2 \phi + (\nabla \cdot \dot{\vec{A}}))$$

Minimizando la acción: $-(\nabla^2 \phi + (\nabla \cdot \dot{\vec{A}})) = \rho / \varepsilon_0$

$$+ \nabla \cdot (-\nabla \phi - \dot{\vec{A}}) = \rho / \varepsilon_0$$

$$+ \nabla \cdot \vec{E} = \rho / \varepsilon_0$$ Ley de Gauss

b) $$\frac{\delta S}{\delta \vec{A}} = \vec{j} - \varepsilon_0 (\nabla \dot{\phi} + \ddot{\vec{A}}) + c^2 (\nabla \times (\nabla \times \vec{A}))$$

Minimizando la acción

$$\varepsilon_0 c^2 (\nabla \times (\nabla \times \vec{A})) = \vec{j} - \varepsilon_0 (\nabla \dot{\phi} + \ddot{\vec{A}})$$

$$(\nabla \times (\nabla \times \vec{A})) = \mu_0 (\vec{j} - \varepsilon_0 (\nabla \dot{\phi} + \ddot{\vec{A}}))$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{j} + \varepsilon_0 \left(\frac{d}{dt}(-\nabla \phi - \dot{\vec{A}})\right)\right)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{j} + \varepsilon_0 \dot{\vec{E}})$$ Ley de Ampere

c) $$\vec{B} = \vec{\nabla} \times \vec{A} \quad / \vec{\nabla} \cdot$$

$$\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A})$$

$$\vec{\nabla} \cdot \vec{B} = 0$$ Inexistencia Monopolos Magnéticos.

$$\vec{E} = -\nabla \phi - \dot{\vec{A}} \quad / \vec{\nabla} \times$$

$$\vec{\nabla} \times \vec{E} = -\vec{\nabla} \times \nabla \phi - \nabla \times \dot{\vec{A}}$$

$$\vec{\nabla} \times \vec{E} = 0 - \frac{d}{dt}(\vec{\nabla} \times \vec{A})$$

$$\vec{\nabla} \times \vec{E} = -\dot{\vec{B}}$$ Ley de Faraday.