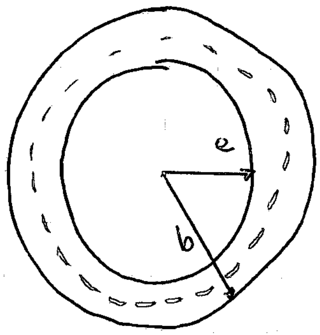
Paulo Alex 10

P1)



$I = I_0 \operatorname{Sen}(\omega t)$

Usando la ley de Ampere.

$$\int_{\Gamma} \vec{B} \cdot d\vec{r} = \mu_0 I_{enc}$$

$I_{enc} = N I$, $d\vec{r} = r\, d\theta\, \hat{\theta}$

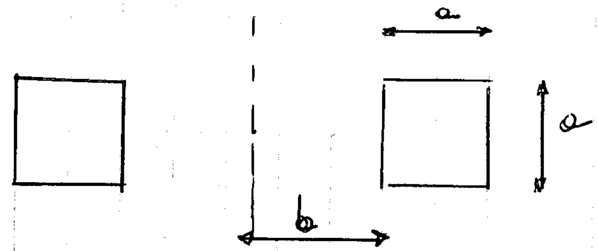
$\vec{B} = B(r)\, \hat{\theta}$

$$\int_0^{2\pi} B(r)\, r\, d\theta = \mu_0 N I_0 \operatorname{Sen}(\omega t)$$

$$2\pi r\, B(r) = \mu_0 N I_0 \operatorname{sen}(\omega t)$$

$$\vec{B}(r) = \frac{\mu_0 N I_0 \operatorname{sen}(\omega t)}{2\pi r}\,\hat{\theta}$$

$$\phi = \int_S \vec{B}\cdot d\vec{s}$$



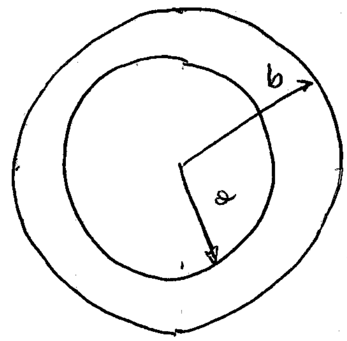
$$d\vec{s} = dr\,dz\,\hat{\theta}$$

$$\phi = \int_0^a \int_b^{a+b} \frac{\mu_0 N I_0 \operatorname{sen}(\omega t)\, dr\,dz}{2\pi r}$$

$$\phi = a\,\frac{\mu_0 N I_0 \operatorname{sen}(\omega t)}{2\pi} \log\left(\frac{b+a}{a}\right)$$

$$\mathcal{E} = -\frac{\partial \phi}{\partial t} = -\frac{a\,\omega\,\mu_0 N I_0 \cos(\omega t)}{2\pi} \log\left(\frac{b+a}{a}\right)$$

P2)



$I_b = I_o \operatorname{Sen}(\omega t)$.

Resistencia R.

$$\phi_a = M_{ab} I_b + L_a I_a .$$

$$M_{ab} I_b = \int_S \vec{B}_b \cdot d\vec{S} = \phi_{A1}$$

$$\vec{B}_b(r) = \frac{\mu_o I_b}{2} \frac{R^2}{(z^2+R^2)^{3/2}} \hat{k} \quad \text{(Según la auxiliar I)}$$

evaluando $R = b$ y $z = 0$.

$$\vec{B}_b = \frac{\mu_o I_b}{2} \frac{b^2}{(b^{3/2})^2} \hat{k} = \frac{\mu_o I_b}{2b} \hat{k}$$

$$\phi_{e1} = \int_0^{a}\int_0^{2\pi} \frac{\mu_0 I b}{2b}\, r\, d\theta\, dr$$

$$\phi_{e1} = \pi a^2 \frac{\mu_0 I b}{2b}$$

$$\phi_{e1} = \frac{\mu_0 \pi a^2}{2b} I_0 \, \text{Sen}(\omega t)$$

$$\mathcal{E} = -\frac{\partial \phi}{\partial t} = -\frac{\partial \phi_{e1}}{\partial t} - L\frac{\partial I_e}{\partial t}.$$

$$-\frac{\partial \phi_{e1}}{\partial t} = -\frac{\mu_0 \pi a^2}{2b}\, \omega I_0 \cos(\omega t)$$

usando ley ohmn.

$$\mathcal{E} = R I_e$$

$$-\frac{\mu_0 \pi a^2}{2b}\omega I_0 \cos(\omega t) - L\frac{\partial I_a}{\partial t} = R I_a .$$

$$L\frac{\partial I_a}{\partial t} + R I_a = -\frac{\mu_0 \pi a^2 \omega I_0}{2b}\cos(\omega t)$$

$$\frac{\partial I_a}{\partial t} + \frac{R}{L} I_a = -\frac{\mu_0 \pi a^2 \omega I_0}{2b}\cos(\omega t)$$

$$\frac{\partial I_a}{\partial t} + \frac{R}{L} I_a = E_0 \cos(\omega t) \quad / \; \text{Factor integrante.}$$

$$\overset{\circ}{\left(I_a e^{(R/L)t} \right)} = E_0 \cos(\omega t)\, e^{(R/L)t} \quad / \int dt$$

$$I_a e^{(R/L)t} = E_0 \left[\frac{L \cos(\omega t) R}{R^2 + L^2\omega^2} + \frac{L^2 \operatorname{sen}(\omega t)\omega}{R^2 + L^2\omega^2} \right] e^{(R/L)t} + C$$

$$I_e(t) = E_0\left[\frac{L\cos(\omega t)R}{R^2+L^2\omega^2} + \frac{L^2\,\mathrm{Sen}(\omega t)\,\omega}{R^2+L^2\omega^2}\right] + c\,e^{-(R/L)t}$$

$$I_e(t=0) = 0$$

$$E_0\left[\frac{LR}{R^2+L^2\omega^2}\right] + c = 0$$

$$I_e(t) = E_0\left[\frac{L\cos(\omega t)R}{R^2+L^2\omega^2} + \frac{L^2\,\mathrm{Sen}(\omega t)\,\omega}{R^2+L^2\omega^2} - \frac{LR}{R^2+L^2\omega^2}e^{-(R/L)t}\right.$$

P3) Usamos la ley de Faraday.

$$\phi = \int \vec{B} \cdot d\vec{S} \qquad d\vec{S} = r\,dr\,d\theta\,\hat{k}.$$

$$\mathcal{E} = -\frac{\partial \phi}{\partial t}.$$

$$\phi = \int_0^{\theta}\int_0^{R} B_0 \cos(\Omega t)\,\frac{R}{\cancel{r}}\,\cancel{r}\,dr\,d\theta$$

$$\phi = \int_0^{\theta}\int_0^{R} B_0 \cos(\Omega t) R\,dr\,d\theta = B_0 R^2 \cos(\Omega t)\,\theta.$$

$$\theta = \omega t.$$

$$\frac{\partial \phi}{\partial t} = -\Omega B_0 R^2 \sin(\Omega t)\,\omega t + B_0 R^2 \cos(\omega t)\,\omega$$

$$\mathcal{E} = -\frac{\partial \phi}{\partial t} \Rightarrow B_0 R^2 \left(\Omega \sin(\Omega t)\,\omega t - \cos(\Omega t)\,\omega\right)$$

Notamos que las resistencias estan conectadas en paralelo.

usando la ley ohmn.

$$\mathcal{E} = I_1 R_1 \Rightarrow I_1 = \frac{B_0 R^2}{R_1}\left(\operatorname{Sen}(\Omega t)\,\Omega \omega t - \operatorname{Cos}(\Omega t\right.$$

$$I_2 = \frac{B_0 R^2}{R_2}\left(\operatorname{Sen}(\Omega t)\,\Omega\omega t - \operatorname{Cos}(\Omega t)\,\omega\right)$$

b) ahora $\omega = \omega_0 e^{-t/\tau}$

$$\phi = B_0 R^2 \operatorname{Cos}(\Omega t)\,\omega_0 e^{-t/\tau}\, t$$

$$\frac{\partial \phi}{\partial t} = -B_0 R^2 \Omega \operatorname{Sen}(\Omega t)\,\omega_0 e^{-t/\tau}\, t - B_0 R^2 \operatorname{Cos}(\Omega t)\,\frac{\omega_0}{\tau} e^{-t/\tau}$$

$+$

b) ahora $\omega = \omega_0 e^{-t/\tau}$

$$\omega = \frac{d\theta}{dt}.$$

$$\theta = \int_0^t \omega_0 e^{-t/\tau}\, dt$$

$$\theta = -\omega_0 \tau \left(e^{-t/\tau} - 1 \right)$$

$$\frac{\partial \phi}{dt} = B_0 r^2 \left\{ \operatorname{sen}(\Omega t)\, \Omega\, \omega_0 \tau \left(e^{-t/\tau} - 1\right) + \cos(\Omega t)\, \omega_0 e^{-t/\tau} \right\}.$$

$$I_1 = -\frac{B_0 r^2}{R_1} \left\{ \operatorname{sen}(\Omega t)\, \Omega\, \omega_0 \tau \left(e^{-t/\tau} - 1\right) + \cos(\Omega t)\, \omega_0 e^{-t/\tau} \right\}$$

$$I_2 = -\frac{B_0 r^2}{R_2} \left\{ \operatorname{sen}(\Omega t)\, \Omega\, \omega_0 \tau \left(e^{-t/\tau} - 1\right) + \cos(\Omega t)\, \omega_0 e^{-t/\tau} \right\}$$