Pauta Aux 11

P1)

$$E = \frac{1}{2} L I_0^2 .$$

$$L = \frac{\phi}{I_0}$$

Usando la ley de Ampere.

$$\int \vec{H} \cdot d\vec{l} = I_{enc}$$

$$I_{enc} = N I_0$$

$$H_1 r \pi + 2 H_0 h + H_1 r \pi = N I_0$$

Usando las condiciones de borde $B_{1N} = B_{2N} = B_{0N}$

$$\vec{H} = \frac{\vec{B}}{\mu}$$

$$\frac{Br\pi}{\mu_1} + \frac{2Bh}{\mu_0} + \frac{Br\pi}{\mu_2} = N I_0$$

Vamos a aproximar $r = \frac{a+b}{2}$

$$\vec{B} = \frac{N I_0 \ \hat{\theta}}{\frac{2h}{\mu_0} + \left(\frac{a+b}{2}\right)\pi \frac{\mu_1+\mu_2}{\mu_1 \mu_2}}$$

$$\phi = \int_S \vec{B} \cdot d\vec{S}$$

$$\phi = \frac{N I_0}{\frac{2h}{\mu_0} + \left(\frac{a+b}{2}\right)\pi \frac{\mu_1+\mu_2}{\mu_1 \mu_2}} \quad \frac{(a-b)^2}{4}\pi$$

$$L = \frac{\phi}{I_0}$$

$$L = \frac{N}{\frac{2h}{\mu_0} + \pi \frac{(\mu_1 + \mu_2)}{\mu_1 \mu_2} \frac{(a+b)}{2}} \frac{(a-b)^2 \pi}{4}$$

$$E = \frac{1}{2} \frac{N I_0^2}{\frac{2h}{\mu_0} + \frac{\pi (\mu_1 + \mu_2)}{\mu_1 \mu_2} \frac{(a+b)}{2}} \frac{(a-b)}{4} \pi$$

$$\vec{F} = - \frac{\partial E}{\partial h}$$

$$\vec{F} = \frac{N I_0^2 (a-b)^2 / 4 \ \pi}{\mu_0 \left[\frac{2h}{\mu_0} + \frac{\mu_1 + \mu_2}{2 \mu_1 \mu_2} \pi (a+b) \right]}$$

P2)

$$\vec{M} = (M_0 + M z)\,\hat{k}$$

$$\nabla \times \vec{M} = \vec{J}_m$$

En Coordenadas ~~esfericas~~ Cilindricas Tenemos,

$$\nabla \times \vec{M} = \left(\frac{1}{r}\frac{\partial M_z}{\partial \theta} - \frac{\partial M_\theta}{\partial z} \right)\hat{r} + \left(\frac{\partial M_r}{\partial z} - \frac{\partial M_z}{\partial r} \right)\hat{\theta}$$

$$+ \frac{1}{r}\left(\frac{\partial (r M_\theta)}{\partial r} - \frac{\partial M_r}{\partial \theta} \right)\hat{k}.$$

Luego $\vec{J}_m = \vec{0}$

$$\vec{M} \times \hat{n} = \vec{K}_m.$$

Viendo el dibujo, $\hat{n} = \cos\phi\,\hat{k} + \operatorname{Sen}\phi\,\hat{r}$

$$\vec{K}_m = \vec{M} \times \hat{n}$$

$$\vec{K}_m = (Mz + M_0) \sin\phi \, \hat{\theta}.$$

b) $$\vec{A}(r) = \frac{\mu_0}{4\pi} \int_{S'} \frac{K_m \, ds'}{\|\vec{r} - \vec{r}'\|}$$

$\vec{r} = 0$

$\vec{r}' = R\hat{n}.$

$ds' = R^2 \sin\phi \, d\phi \, d\theta.$

$$\vec{A}(r) = \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_0^{\pi} \frac{(Mz + M_0)}{R} R^2 \sin\phi^2 \hat{\theta} \, d\phi \, d\theta$$

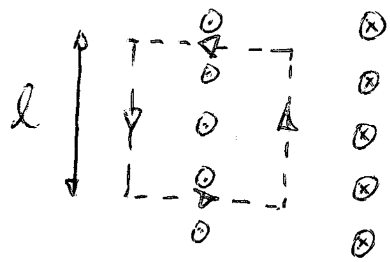
$$\vec{A}(r) = \frac{\mu_0}{4\pi}\left[\cancelto{0}{\int_0^{2\pi}\hat{\theta}\,d\theta}\int_0^{\pi}(rz + r_0)\,z\,\sin^2\phi\,d\phi\right]$$

$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\vec{A}(r) = 0$$

P3)

Usando la ley de Ampere.



$$\oint \vec{H}\cdot d\vec{l} = n l I$$

$$H l = n l I \longrightarrow \boxed{\vec{H} = n I \hat{k}}$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{B} = \mu_0 \left(1 + \frac{\mu_1}{\mu_0} f(r) \right) n I \hat{k}$$

$$\chi_m = \frac{\mu}{\mu_0} - 1$$

$$\chi_m = \frac{\mu'}{\mu_0} f(r)$$

$$\vec{M} = \frac{\mu'}{\mu_0} f(r) \, I_m \, \hat{k}$$

b) $$\vec{K}_m = \vec{M} \times \hat{n} \quad , \quad \hat{n} = \hat{r}$$

$$\vec{K}_m = \frac{\mu'}{\mu_0} f(r) \, I_m \, (\hat{k} \times \hat{r})$$

$$\vec{K}_m = \frac{\mu'}{\mu_0} f(r) \, I_m \, \hat{\theta} .$$

$$\vec{J}_m = \nabla \times \vec{M} . = - \frac{\partial M_z}{\partial r} \hat{\theta} .$$

$$\vec{J}_m = -\frac{\mu_1}{\mu_0} I_m \frac{\partial f}{\partial r} \hat{\theta}.$$

c) $$I = \int_{\Gamma} \vec{K}_m \, d\vec{\ell} + \int_S \vec{J} \cdot d\vec{S} \overset{!}{=} 0$$

$$\int_0^L \frac{\mu_1}{\mu_0} f(R) I_m \, dz + (-) \int_0^L \int_0^R \frac{\mu_1}{\mu_0} I_m \frac{\partial f}{\partial r} \, dr \, dz \overset{!}{=} 0$$

$$\frac{\mu_1}{\mu_0} f(R) I_m L - \frac{\mu_1}{\mu_0} I_m L \left(f(R) - f(0) \right) \overset{!}{=} 0$$

$$\frac{\mu_1}{\mu_0} I_m L f(0) = 0 \Rightarrow \boxed{f(0) = 0}$$