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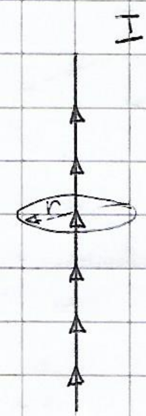
Pauta Aux 7.

P1)



$\hat{r}, \hat{\theta}, \hat{k}.$

$$\vec{B}(r) = \frac{\mu_0}{4\pi} \int_{\Gamma} \frac{I\, d\vec{r}' \times (\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3}$$

$$\vec{r} = r\hat{r}, \quad \vec{r}' = z\hat{k}, \quad d\vec{r}' = dz\,\hat{k}$$

$$\vec{B}(r) = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{dz\,\hat{k} \times (r\hat{r} - z\hat{k})}{(r^2 + z^2)^{3/2}}$$

$$\vec{B}(r) = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{r\,dz}{(r^2+z^2)^{3/2}}\,\hat{\theta}$$

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Cambio de Variable. (muy muy feo)

$r \tan u = z$

$r \sec^2 u \, du = dz$

$\therefore \quad r \tan u = +\infty \Rightarrow u = \pi/2$

$r \tan u = -\infty \Rightarrow u = -\pi/2$

$$\vec{B}(r) = \frac{\mu_0 I}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{r^2 \sec^2 u \, du}{(r^2 \sec^2 u)^{3/2}}$$

$$\vec{B}(r) = \frac{\mu_0 I}{4\pi r} \int_{-\pi/2}^{\pi/2} \cos u \, du = \frac{\mu_0 I}{4\pi r} \left\{ \sin u \Big|_{-\pi/2}^{\pi/2} \right\}$$

$$\boxed{\vec{B}(r) = \frac{\mu_0 I}{2\pi r} \hat{\theta}}$$

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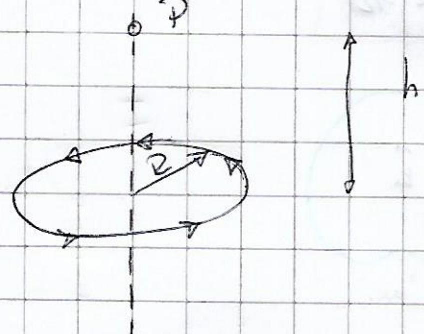
$\hat{r}, \hat{\theta}, \hat{k}$

P2)



a)

$$\vec{B}(r) = \frac{\mu_0}{4\pi} \int_{\Gamma} \frac{I\, d\vec{r}\,' \times (\vec{r} - \vec{r}\,')}{\| \vec{r} - \vec{r}\,' \|^3}$$

$$\vec{r} = h\hat{k} \quad , \quad \vec{r}\,' = R\hat{r} \quad , \quad d\vec{r}\,' = R\, d\theta\, \hat{\theta}$$

$$\vec{B}(r) = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R\, d\theta\, \hat{\theta} \times (h\hat{k} - R\hat{r})}{(h^2 + R^2)^{3/2}}$$

$$\vec{B}(r) = \frac{\mu_0 I}{4\pi} \left\{ \cancelto{0}{\int_0^{2\pi} \frac{Rh\hat{r}}{(h^2+R^2)^{3/2}} d\theta} + \int_0^{2\pi} \frac{R^2\, \hat{k}\, d\theta}{(h^2 + R^2)^{3/2}} \right\}$$

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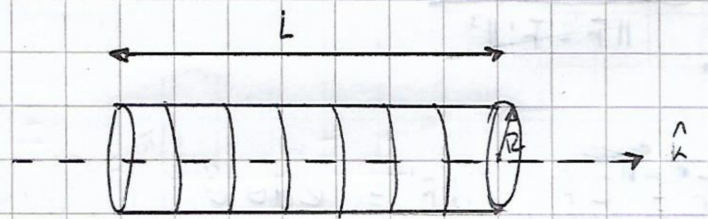
$$\vec{B}(r) = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R^2 \, d\theta}{(h^2+R^2)^{3/2}} \hat{k}$$

$$\boxed{\vec{B}(r) = \frac{\mu_0 I}{2} \frac{R^2}{(h^2+R^2)^{3/2}} \hat{k}}$$

b)



N vueltas.

llamaremos $n = \frac{N}{L}$ (vueltas por unidad de longitud)

$$\vec{B}_{bobina} = \int_0^L \vec{B}_{anillo}(z) \, n \, dz$$

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$$\vec{B}_{bobina} = \frac{\mu_0 I R^2}{2} \int_0^L \frac{n\,dz}{(z^2+R^2)^{3/2}}$$

Cambio de variable.

$R \tg u = z.$ $\qquad u^* = \operatorname{arctg}(L/R)$

$R \sec^2 u \, du = dz$

$$\vec{B}_{bobina} = \frac{\mu_0 I R^2}{2} \int_0^{u^*} \frac{n\, R \sec^2 u \, du}{R^3 \sec^3 u}$$

$$\vec{B}_{bobina} = \frac{\mu_0 I n}{2} \int_0^{u^*} \cos u \, du.$$

$$\vec{B}_{bobina} = \frac{\mu_0 I n}{2} \left\{ \operatorname{Sen} u \Big|_0^{u^*} \right\}$$

$$\vec{B}_{bobina} = \frac{\mu_0 I n}{2} \frac{L}{(R^2+L^2)^{1/2}}$$

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c) $\vec{F} = q\vec{u} \times \vec{B}(r)$

$$\vec{B}(r) = \frac{\mu_0 I}{2} \frac{R^2}{(h^2+R^2)^{3/2}} \hat{k}$$

$$\therefore \quad \vec{F} = q u \hat{j} \times \frac{R^2}{(h^2+R^2)^{3/2}} \hat{k}$$

$$\vec{F} = \frac{q u \mu_0 I R^2}{2(h^2+R^2)^{3/2}} \hat{e}$$

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P3)

$$\frac{1}{2} m v^2 = qV$$

$$\frac{m v^2}{R} = q v B$$

$$\Rightarrow \quad R^2 = \frac{2 m V}{q B^2}$$

$$\therefore \quad \boxed{m = \frac{R^2 q B^2}{2V}}$$

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P4)

$$\mu = abI$$

$$\tau = \mu \times \vec{B}$$

$$|\tau| = abI \operatorname{sen}\theta$$

Considere ahora que lo sacamos del equilibrio $\Rightarrow \theta \neq 0$.
este Torque, es de carácter restitutivo. $\Rightarrow$

$$\tau = -abI \operatorname{sen}\theta . \quad \text{además.}$$

$$\sum \tau = I \ddot{\theta} \quad , \text{asumiendo pequeñas oscilaciones.}$$

$$\ddot{\theta} + \frac{abI}{L}\theta = 0$$

$$\omega_n = \sqrt{\frac{abI}{L}}$$