Auxiliar Extra C2

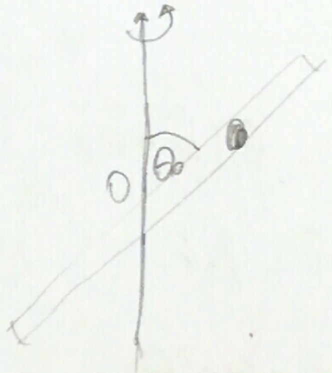
$$\dot r(0) = -\sqrt{\frac{g r_0}{2}}$$
$$r(0) = r_0.$$

P1)

$\vec r = r\hat r$

$\dot{\vec r} = \dot r\hat r$

$\ddot{\vec r} = \ddot r\hat r$



$\vec\Omega = \omega_0\hat k$

$$m\vec a' = m\vec a - m\ddot{\vec R} - 2m\,\vec\Omega\times\vec r' - m\frac{d\vec\Omega}{dt}\times\vec r - m\vec\Omega\times(\vec\Omega\times\vec r)$$

$$m\ddot r\hat r = \vec N - mg\hat k - 0 - 2m\omega_0\dot r\,\hat k\times\hat r - 0 - m\omega_0^2 r\hat k\times(\underbrace{\hat k\times\hat r})$$

$-\text{sen}\theta\hat\theta$

$\hat k\times\hat\theta$

$$= \vec N - mg\hat k - 2m\omega_0\dot r\,\hat\phi - m\omega_0^2 r\,\hat k\times\hat\phi$$

$$m\ddot r\hat r = \vec N - mg\hat k - 2m\omega_0\dot r\,\hat\phi + m\omega_0^2 r\,\text{sen}^2\theta_0\,\hat r$$

$$m\ddot r = -mg\cos\theta_0 + m\omega_0^2 r\,\text{sen}\theta_0.$$

$$\ddot r = -g\cos\theta_0 + \frac{2g}{r_0} r\,\text{sen}^2\theta_0 \quad /\cdot\dot r \quad \int$$

$$\frac{\dot r^2}{2} = -g\cos\theta_0 r + \frac{2g}{r_0}\frac{r^2}{2}\text{sen}^2\theta_0 + C.$$

$$\frac{g r_0}{4} = -g\cos\theta_0 r_0 + g r_0\,\text{sen}^2\theta_0 + C.$$

$$C = g r_0\left(\cos\theta - \text{sen}^2\theta_0 + \frac{1}{4}\right)$$

El mín $\dot r = 0,\ \theta_0 = 60°$

$$\Rightarrow 0 = -\frac{g r}{2} + \frac{g r^2}{r_0}\frac{3}{4} + \cancel{g r_0}\left(\frac{1}{2} - \frac{3}{4} + \frac{1}{4}\right)$$

$\hat{k} \times (\hat{k} \times \hat{r})$ $\quad \hat{k} = \cos\theta\, \hat{r} - \operatorname{sen}\theta\, \hat{\theta}$

$\hat{k} \times (-\operatorname{sen}\theta\, \hat{\theta})$

$-\cos\theta \operatorname{sen}\theta\, \hat{\phi}$

$\hat{k} \times (\hat{k} \times \hat{r}) = \hat{k}(\hat{k} \cdot \hat{r}) - \hat{r}(\hat{k} \cdot \hat{k})$

$(\hat{k} \times (\hat{k} \times \hat{r})) \cdot \hat{r} = \hat{k} \cos\theta_0 - \hat{r} \quad / \cdot \hat{r}$

$= \cos^2\theta_0 - 1 = -\operatorname{sen}^2\theta_0.$

$$0 = -\frac{gr}{2} + \frac{3gr^2}{4r_0} = gr\left(-\frac{1}{2} + \frac{3}{4}\frac{r}{r_0}\right)$$

$$\Rightarrow \quad \frac{3}{4}\frac{r}{r_0} = \frac{1}{2}$$

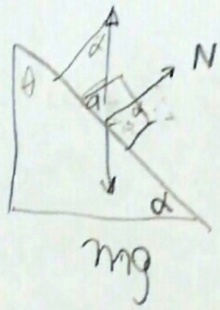
$$\boxed{r = \frac{2}{3} r_0}$$

P2|

$$\vec{N} = \hat{z} N \qquad \hat{z}_0 = \hat{z}\cos\alpha - \hat{x}\,\text{sen}\,\alpha$$

$$\vec{P} = -mg\hat{z}_0$$

$$= -mg\cos\alpha\,\hat{z} + mg\,\text{sen}\,\alpha\,\hat{x}$$



$$m\vec{a}' = m\vec{a} - \cancel{m\ddot{\vec{R}}}^{\,0} - \cancel{m\frac{d\vec{\Omega}}{dt}}^{\,0}\times\vec{r}' - 2m\,\vec{\Omega}\times\vec{v}' - m\,\vec{\Omega}\times(\vec{\Omega}\times\vec{r}')$$

$$\vec{\Omega}\times\vec{v}' = \omega\left(\hat{z}_0\times(\dot{x}\hat{x} + \dot{y}\hat{y})\right)$$

$$= \omega\dot{x}(\hat{z}_0\times\hat{x}) + \omega\dot{y}(\hat{z}_0\times\hat{y})$$

$$= \omega\dot{x}\cos\alpha\,\hat{y} + \omega\dot{y}\left(\cos\alpha(-\hat{x}) - \text{sen}\,\alpha\,\hat{z}\right)$$

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = \vec{\Omega}(\vec{\Omega}\cdot\vec{r}') - \vec{r}'(\vec{\Omega}\cdot\vec{\Omega})$$

$$= \omega^2(z\cos\alpha - x\,\text{sen}\,\alpha)\hat{z}_0 - \omega^2(x\hat{x} + y\hat{y})$$

$$= -\omega^2 x\,\text{sen}\,\alpha\,(\hat{z}\cos\alpha - \hat{x}\,\text{sen}\,\alpha) - \omega^2(x\hat{x} + y\hat{y})$$

$\hat{x}|$
$$\ddot{x} = g\,\text{sen}\,\alpha + 2\omega\cos\alpha\,\dot{y} - \omega^2 x\,\text{sen}^2\alpha + \omega^2 x$$

$$= g\,\text{sen}\,\alpha + 2\omega\cos\alpha\,\dot{y} + \omega^2 x\cos^2\alpha \quad (1)$$

$\hat{y}|$
$$\ddot{y} = -2\omega\dot{x}\cos\alpha + \omega^2 y \quad (2)$$

Derivando (1)

$$\dddot{x} = 2\omega\cos\alpha\,\ddot{y} + \omega^2\dot{x}\cos^2\alpha \quad (3)$$

$$\ddddot{x} = 2\omega\cos\alpha\,\dddot{y} + \omega^2\cos^2\alpha\,\ddot{x} \quad (4)$$

$$\dddot{y} = -2w\ddot{x}\cos\alpha + w^2\dot{y} \quad (5)$$

(5) en (4)

$$\ddddot{x} = 2w\cos\alpha\left(w^2\dot{y} - 2w\cos\alpha\,\ddot{x}\right) + w^2\cos^2\alpha\,\ddot{x}$$

de (1)

$$\frac{\ddot{x} - g\,\text{sen}\alpha - w^2x\cos^2\alpha}{2w.\cos\alpha} = \dot{y}$$

$$\ddddot{x} = w^2\ddot{x} - w^2g\,\text{sen}\alpha - w^4x\cos^2\alpha - 3w^2\cos^2\alpha\,\ddot{x}$$

$$\frac{d^4x}{dt^4} + w^2(3\cos^2\alpha - 1)\ddot{x} + xw^4\cos^2\alpha = -w^2g\,\text{sen}\alpha$$

P31

velocidad orbital

$$mg = \frac{GmM}{R^2}$$

$$g = \frac{GM}{R^2}$$

$$\vec{F} = m\vec{a}$$

$$\frac{G\cancel{m}M}{(3R)^2} = \cancel{m}\frac{v^2}{3R}.$$

$$v_N = \sqrt{\frac{GM}{3R}} = \sqrt{\frac{gR}{3}}$$

sonda.

$$l = m\sqrt{\frac{3}{2}GMR} \rightarrow \text{siguiente página}$$

$$v_A = \frac{l}{mr} = \frac{1}{3R}\sqrt{\frac{3}{2}GMR} = \sqrt{\frac{1}{6}}\sqrt{\frac{GM}{R}} = \sqrt{\frac{gR}{6}}$$

$$\Rightarrow v^* = v_A - v_N = \sqrt{gR}\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\right)$$

relativa a la nave.

$$= \sqrt{1.6 \cdot 1700 \times 1000}\,(0.17) \rightarrow \text{datos no exactos}$$

$$\approx \sqrt{16\times17} \times 10^2 \times 0.17$$

$$\approx 280.5 \text{ m/s} \qquad (284 \text{ m/s}) \rightarrow \text{solución meriam.}$$

3ª Ley de Kepler.

$$T_S = \frac{2\pi}{\sqrt{GM}}a^{3/2} = \frac{2\pi}{R\sqrt{g}}(2R)^{3/2} \rightarrow \text{período sonda}$$

$$T_N = \frac{2\pi}{\sqrt{GM}} \cdot R^{3/2} = \frac{2\pi(3R)^{3/2}}{R\sqrt{g}} \rightarrow \text{período nave}$$

P3

sondo

$$E = \frac{\ell^2}{9 \cdot 2mR^2} - \frac{GmM}{3R} = \frac{\ell^2}{2mR^2} - \frac{GmM}{R}$$

$$\frac{\ell^2}{mR^{\cancel{2}}}\left(\frac{1}{18} - \frac{9}{18}\right) = \frac{GmM}{\cancel{R}}\left(\frac{1}{3} - 1\right)$$

$$-\frac{8}{18}\ell^2 = Gm^2MR \cdot -\frac{2}{3}$$

$$\ell^2 = \frac{6}{4}Gm^2MR$$

$$\ell = m\sqrt{\frac{3}{2}GMR}$$

$$\frac{T_N}{T} = \frac{2\pi}{\theta} \Rightarrow T = \frac{\theta}{\cancel{2\pi}} T_N = \frac{\pi}{\cancel{2\pi}} T_S.$$

$\frac{T_N}{T}$: tiempo que demora en recorre θ

$\frac{\pi}{2\pi} T_S$: en ese tiempo la sonda recorre π.

$$\theta = \pi \frac{T_S}{T_N}$$

$$\theta = \pi \left(\frac{2}{3}\right)^{3/2} = 1.71 \, [rad]$$

$$\frac{\pi}{1.71} = \frac{180}{x} \Rightarrow \theta \approx 98^\circ$$

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