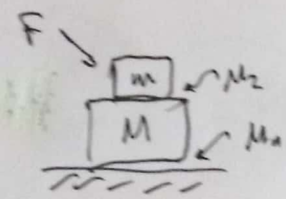
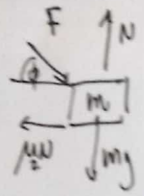
P1) Se tienen un bloque de masa



¿Que inclinación debemos darle a F de modo que la tabla este a punto de moverse cuando el paquete sobre la tabla comience a moverse.

DCL 1

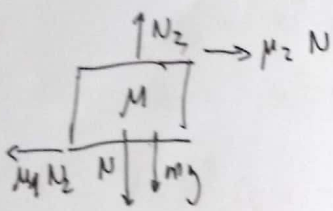


$$\Sigma F_x = F\cos\phi - \mu_2 N = m a_x$$

$$\Sigma F_y = 0 = N - mg - F\,\mathrm{sen}\,\phi$$

$$\boxed{N = mg + F\,\mathrm{sen}\,\phi} \quad (1)$$

DCL 2



$$\Sigma F_x = M\cancel{a_x}^{0} = \mu_2 N - \mu_1 N_2$$

$$\Sigma F_y = 0 = N_2 - N - Mg$$

$$N_2 = N + Mg$$

$$\boxed{N_2 = mg + F\,\mathrm{sen}\,\phi + Mg} \quad (2)$$

$$\mu_2 N = \mu_1 N_2 \quad + (1) + (2)$$

$$\mu_2 (mg + F\,\mathrm{sen}\,\phi) = \mu_1 (mg + F\,\mathrm{sen}\,\phi + Mg)$$

$$mg(\mu_2 - \mu_1) + F\,\mathrm{sen}\,\phi\,(\mu_2 - \mu_1) = \mu_1 Mg$$

$$F\,\mathrm{sen}\,\phi\,(\mu_2 - \mu_1) = \mu_1 Mg - mg(\mu_2 - \mu_1)$$

$$\mathrm{sen}\,\phi = \frac{\mu_1 Mg}{F(\mu_2 - \mu_1)} - \frac{mg(\mu_2 - \mu_1)}{F(\mu_2 - \mu_1)}$$

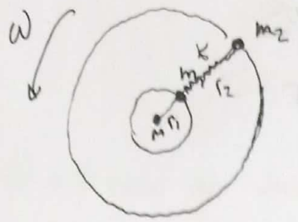
$$\boxed{\mathrm{sen}\,\phi = \frac{g}{F(\mu_2 - \mu_1)} \left(\mu_1 M - m(\mu_2 - \mu_1)\right)}$$

$$\mathrm{sen}\,\phi = \frac{g}{F} \left(\frac{\mu}{\mu_2 - \mu_1} M - m\right)$$

$\mu_2 \neq \mu_1$ y si $\mu_2 = \mu_1 = 0$ no hay roce

Scanned by CamScanner

P2] Dos planetas de masas m_1 y m_2 giran órbitas circulares de radio r_1 y r_2

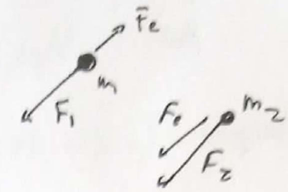


$M \gg m_1 ; m_2$

$$F_1 = G \frac{M m_1}{r_1^2} \hat{r}$$

$$F_2 = G \frac{M m_2}{r_2^2} \hat{r}$$

DCL m_1



$$|F_e| = (r_2 - r_1) K$$

para que giren con el mismo periodo

Si ambos planetas giran en el mismo periodo tienen $= \omega$ (velocidad angular $= \frac{2\pi}{T}$)

$$a_1 = \omega^2 \cdot r_1$$

$$a_2 = \omega^2 \cdot r_2$$

$$m_1 \omega^2 \cdot r_1 = -F_1 + F_e$$

$$m_2 \omega^2 r_2 = -F_2 - F_e$$

$$m_1 a_1 + F_1 = F_e = (r_2 - r_1) K \rightarrow \omega^2 = \frac{(r_2 - r_1) K - F_1}{m_1 r_1}$$

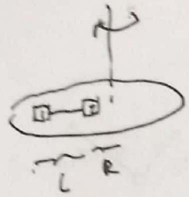
$$m_2 a_2 + F_2 = -F_e = (r_1 - r_2) k \rightarrow \omega^2 = \frac{(r_1 - r_2) k - F_2}{m_2 r_2}$$

$$\left((r_2 - r_1) k - F_1\right)(m_2 r_2) = \left((r_1 - r_2) k - F_2\right) m_1 r_1$$

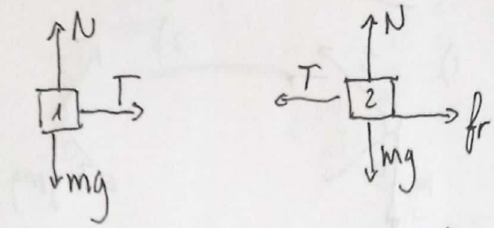
$$2 (r_2 - r_1) k (m_2 r_2 + m_1 r_1) = F_1 m_2 r_2 - F_2 m_1 r_1$$

$$\boxed{k = \frac{F_1 m_2 r_2 - F_2 m_1 r_1}{2(r_2 - r_1)(m_2 r_2 + m_1 r_1)}}$$

P3)



DCL's



a) para el caso en donde está estático $\qquad fr \le \mu_{est} N$

$$\sum F_x = 0 = N - mg \Rightarrow N = mg$$

$$\sum F_y = a_c m = T = m\omega^2 (L+R)$$

2)

$$\sum F_x = fr - T = a_c m \Rightarrow fr - T = m\omega^2 (R)$$

$$\sum F_y \Rightarrow N = mg$$

$$fr - m\omega^2(L+R) = m\omega^2(R)$$ en el caso crítico $fr = \mu_{est} N$

$$fr = \mu_{est}\, mg$$

$$\mu_{est}\, \cancel{m}g = \cancel{m}\omega^2 (R + L+R)$$

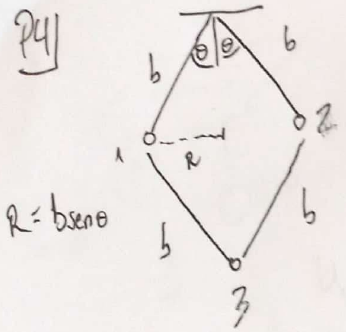
$$\frac{\mu_{est} - g}{2R+L} = \omega$$

$$\omega = \pm\sqrt{\frac{\mu_{est} - g}{2R+L}}$$

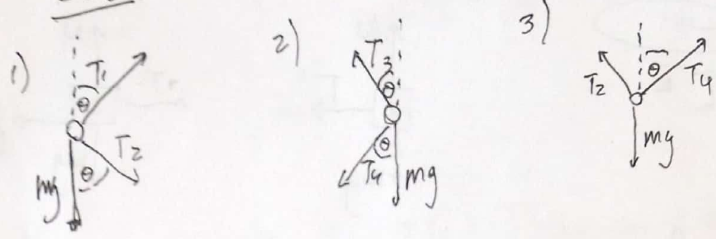
según el dibujo del aux el disco gira en sentido horario por lo que consideramos la solución negativa.

$$\omega = -\sqrt{\frac{\mu_{est}\, g}{2R+L}}$$

P4



DCL's



$$1)\ \Sigma F_x = (T_1+T_2)\,\text{sen}\theta = m\vec{a}_c = m\omega^2 \cdot b\,\text{sen}\,\theta \Rightarrow T_1+T_2 = m\omega^2 b \quad (1)$$

$$\Sigma F_y\ (T_1-T_2)\cos\theta - mg = 0 \Rightarrow T_1 - T_2 = \frac{mg}{\cos\theta} \quad (2)$$

$$2)\ \Sigma F_x = -(T_3+T_4)\,\text{sen}\theta = m\vec{a}_c = -m\omega^2 \cdot b\,\text{sen}\theta \Rightarrow T_3+T_4 = m\omega^2 b \quad (3)$$

$$\Sigma F_y = (T_3-T_4)\cos\theta - mg = 0 \Rightarrow \boxed{(T_3-T_4) = \frac{mg}{\cos\theta}} \quad (6)$$

$$3)\ \Sigma F_y = (T_4+T_2)\cos\theta - mg = 0 \Rightarrow T_4+T_2 = \frac{mg}{\cos\theta} \quad (4)$$

$$\Sigma F_x = (T_4-T_2)\,\text{sen}\,\theta = 0 \Rightarrow T_4 = T_2 \quad \text{con } \theta \neq 0 \quad (5)$$

de (5) y (4) $\Rightarrow T_4 = T_2 = \dfrac{mg}{2\cos\theta} \quad (7)$

de (6) y (2) $\Rightarrow T_1 - \cancel{T_2} = T_3 - \cancel{T_4}$ y que $T_1 = T_3$

(2) y (7) $\quad T_1 - T_2 = \dfrac{mg}{\cos\theta} \Rightarrow \boxed{T_3 = T_1 = \dfrac{3mg}{2\cos\theta}}$

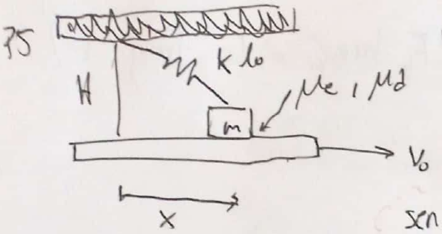
luego por (1) $\quad T_1 + T_2 = m\omega^2 b$

$$\frac{3mg}{2\cos\theta} + \frac{mg}{2\cos\theta} = m\omega^2 b$$

$$\frac{2\cancel{m}g}{\cos\theta} = \cancel{m}\omega^2 b$$

$$\boxed{\frac{2g}{\omega^2 b} = \cos\theta}$$

35



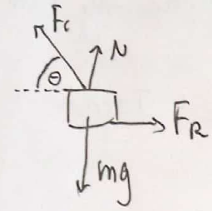
Ecs

$\sum F_x = F_R - F_e \cos\theta = 0$ (V cte)

$\sum F_y = N + F_e \operatorname{sen}\theta - mg = 0$ (reposo)

$F_R \leq \mu_{est} \cdot N$ ya que el bloque se mueve con la superficie

DCL



$\operatorname{sen}\theta = \frac{H}{L}$

$\cos\theta = \frac{x}{L}$

$tg\theta = \frac{H}{L}$

$L = \sqrt{x^2 + H^2}$

$F_e = k(L - l_0)$ donde $L = \frac{H}{\operatorname{sen}\theta} = \frac{x}{\cos\theta}$

$F_e = k\left(\frac{H}{\operatorname{sen}\theta} - l_0\right)$

1) Normal de $\sum F_y \Rightarrow N = mg - k\left(\frac{H}{\operatorname{sen}\theta} - l_0\right)\operatorname{sen}\theta$

Fricción de $\sum F_x \Rightarrow F_R = k\left(\frac{H}{\operatorname{sen}\theta} - l_0\right)\cos\theta$

Notar que solo se cumple si $\underline{k(L - l_0)\cos\theta \leq \mu_e \cdot N}$

2) $l_0 = 2H$ y $kH > mg$ Condición de despegue $N = 0$

de $\sum F_y = \overset{0}{\cancel{N}} + F_e \operatorname{sen}\theta - mg = 0 \Rightarrow mg = \operatorname{sen}\theta \cdot k\left(\frac{H}{\operatorname{sen}\theta} - 2H\right)$

$mg = kH - 2Hk \operatorname{sen}\theta$

$2Hk \operatorname{sen}\theta = kH - mg \Rightarrow kH > mg \Rightarrow$ es positivo

$\operatorname{sen}\theta = \frac{kH - mg}{2Hk}$

$\frac{H}{\sqrt{x^2 + H^2}} = \frac{kH - mg}{2Hk}$

$\frac{2H^2 k}{kH - mg} = \sqrt{x^2 + H^2}$

$x^2 = \frac{4H^4 k^2}{(kH - mg)^2} - H^2$

$$\boxed{x = \sqrt{\frac{4H^4 k^2}{(kH - mg)^2} - H^2}}$$

3) $L_0 = 0$ comienza a deslizar caso crítico donde $F_R = \mu_{est} \cdot N$

$\mu_{est} \cdot N = k(L - l_0)\cos\theta$

$\mu_{est}\left(mg - k\left(\underbrace{\frac{H}{\operatorname{sen}\theta}}_{L} - \overset{0}{\cancel{l_0}}\right)\operatorname{sen}\theta\right) = k(L - \overset{0}{l_0})\cos\theta$

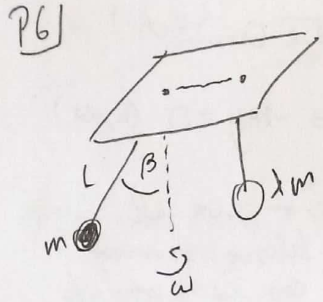
$\mu_{est}\left(mg - k \underbrace{L \operatorname{sen}\theta}_{H}\right) = k \underbrace{L \cos\theta}_{x}$

$\mu_{est}(mg - kH) = kx$

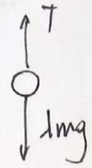
$$\boxed{\frac{\mu_{est}(mg - kH)}{k} = x}$$

* la restricción $kH > mg$ era necesaria para que el cuerpo no se deslice antes de llegar al punto donde pueda levantarse.

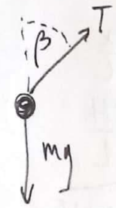
P6)



DCL's



$$\Sigma F_y = \cancel{m a_y}^{0} = T - \lambda m g \quad (1)$$



$$\Sigma F_y \Rightarrow T\cos\beta - mg = m \cancel{a_y}^{0}$$

$$\Sigma F_x = m \cdot a_c = T \operatorname{sen}\beta$$

En condición de equilibrio

$$a_y = 0$$

por lo que

$$T = \lambda m g$$

Y $T\cos\beta = mg$

$$\lambda \cancel{mg} \cos\beta = \cancel{mg}$$

$$\frac{1}{\lambda} = \cos\beta$$

$$\boxed{\beta = \arccos\left(\frac{1}{\lambda}\right)}$$

$$m \cdot a_c = \lambda m g \operatorname{sen}\beta$$

$$\cancel{m}\,\omega^2 \cdot r = \lambda \cancel{m} g \operatorname{sen}\beta$$

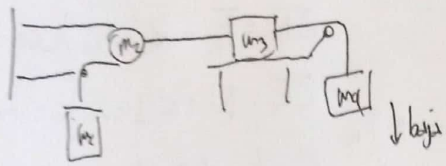
$$\omega^2 = \frac{\lambda g \operatorname{sen}\beta}{r}$$

pero por trigonometría $r = L \cdot \operatorname{sen}\beta$

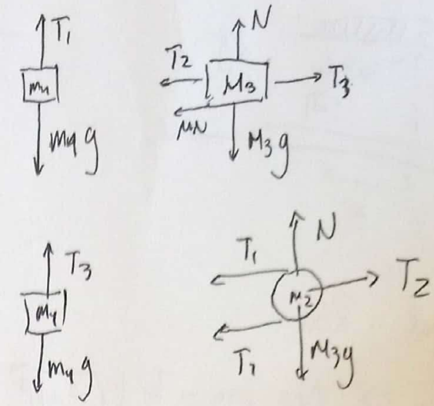
$$\omega^2 = \frac{\lambda g \operatorname{sen}\beta}{L \cdot \operatorname{sen}\beta}$$

$$\boxed{\omega = \pm\sqrt{\frac{\lambda g}{L}}}$$

como según el dibujo gira en sentido antihorario nos quedamos con la solución positiva $\omega = \sqrt{\frac{\lambda g}{L}}$



DCL's



$\rightarrow \underbrace{2a_3 = 2a_4 = 2a_2}_{} = a_1$

por que las cuerdas son inextensibles

$2a_2 = a_1$ ya que cada vez que la polea se mueve Δx la masa m_1 se mueve $2\Delta x$

$$\Sigma F_1 \Rightarrow m_1 \vec{a}_1 = T_1 - m_1 g$$

$$\Sigma F_2 \Rightarrow m_2 \vec{a}_2 = -2T_1 + T_2$$

$$\Sigma F_3 \Rightarrow m_3 \vec{a}_3 = T_3 - T_2 - \mu N$$

$$\Sigma F_4 \Rightarrow m_4 \vec{a}_4 = T_3 - m_4 g$$

$$-m_4 a_4 = T_3 - m_4 g$$

$$a_3 = +a_4 = a_2$$

$$2a_2 = a_1$$

$$\Sigma F_{3y} \Rightarrow m_3 g = N$$

notando que $\boxed{\vec{a_4} = -a_4}$ ya que va hacia abajo.

Despejando todo en función de a_2

$$T_1 = m_1(2a_2 + g)$$

$$T_2 = m_2 a_2 + 2T_1$$

$$-T_2 = m_3 a_2 + \mu N - T_3$$

$$T_3 = m_4(g - a_2)$$

$$T_2 = m_2 a_2 + 2T_1 = -m_3 a_2 - \mu N + T_3$$

$$m_2 a_2 + 2m_1(2a_2 + g) = -m_3 a_2 - \mu m_3 g + m_4(g - a_2)$$

$$m_2 a_2 + 2m_1 \cdot 2a_2 + 2m_1 g = -m_3 a_2 - \mu m_3 g + m_4 g - m_4 a_2$$

$$a_2(m_2 + 4m_1 + m_3 + m_4) = g(m_4 - 2m_1 - \mu m_3)$$

$$a_4 = a_3 = a_2 = \frac{g(m_4 - 2m_1 - \mu m_3)}{(m_2 + 4m_1 + m_3 + m_4)}$$

$$a_1 = \frac{2g(m_4 - 2m_1 - \mu m_3)}{(m_2 + 4m_1 + m_3 + m_4)}$$

$$T_1 = m_1(2a_2 + g)$$

$$T_3 = m_4(g - a_2)$$

$$T_2 = m_2 a_2 + 2T_1$$

$$T_2 = m_2 a_2 + 2m_1(2a_2 + g)$$

$$T_2 = m_2 a_2 + 4m_1 a_2 + 2m_1 g$$

$$T_2 = (m_2 + 4m_1)a_2 + 2m_1 g$$