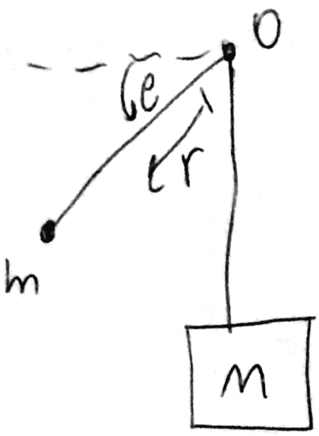
P1] · necesitamos 2 variables para describir el sistema, θ y r. usamos cilíndricos



$$\vec{r}_m = r\hat{\rho}$$

$$\dot{\vec{r}}_m = r\dot{\theta}\hat{\theta} + \dot{r}\hat{\rho}$$

$$\vec{r}_M = -(L-r)\hat{z}$$

$$\dot{\vec{r}}_M = \dot{r}\hat{z}$$

$$T = \frac{m}{2}\left(\dot{r}^2 + (r\dot{\theta})^2\right) + \frac{M}{2}\dot{r}^2$$

$$U = -mgr\,\mathrm{Sen}\,\theta - Mg(L-r)$$

$$\mathcal{L} = \frac{m}{2}\left(\dot{r}^2 + (r\dot{\theta})^2\right) + \frac{M}{2}\dot{r}^2 + mgr\,\mathrm{Sen}\,\theta + Mg(L-r)$$

r]

$$\frac{\partial \mathcal{L}}{\partial \dot{r}} = M\dot{r} + m\dot{r} \quad \longrightarrow (M+m)\ddot{r} = mr\dot{\theta}^2 + mg\,\mathrm{Sen}\,\theta - Mg$$

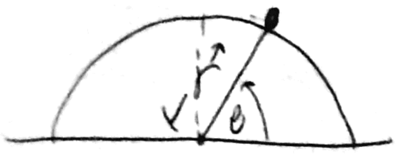
$$\frac{\partial \mathcal{L}}{\partial r} = mr\dot{\theta}^2 + mg\,\mathrm{Sen}\,\theta - Mg$$

θ]

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mr^2\dot{\theta} \quad \longrightarrow m(2r\dot{r}\dot{\theta} + r^2\ddot{\theta}) = mgr\,\mathrm{Cos}\,\theta$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = mgr\,\mathrm{Cos}\,\theta$$

P2] • La restricción de la normal corresponde a $r = R$



• Usamos θ y r para describir el sistema (tomamos r libre si fueramos usar multiplicadores)

$$\vec{r} = r\hat{\rho} \rightarrow \dot{\vec{r}} = \dot{r}\hat{\rho} + r\dot{\theta}\hat{\rho}$$

$$K = \frac{m}{2}(\dot{r}^2 + (r\dot{\theta})^2), \quad V = mgr\,\mathrm{Sen}\,\theta$$

$$\mathcal{L}' = \frac{m}{2}(\dot{r}^2 + (r\dot{\theta})^2) - mgr\,\mathrm{Sen}\,\theta + \lambda(r - R)$$

• Notar que la restricción sea 0 es equivalente a $r = R$

r]

$$\frac{\partial \mathcal{L}'}{\partial \dot{r}} = m\dot{r}$$

$$\frac{\partial \mathcal{L}'}{\partial r} = mr\dot{\theta}^2 - mg\,\mathrm{Sen}\,\theta + \lambda$$

$$\therefore m\ddot{r} = mr\dot{\theta}^2 - mg\,\mathrm{Sen}\,\theta + \lambda$$

$$\rightarrow \lambda = m\ddot{r} - mr\dot{\theta}^2 + mg\,\mathrm{Sen}\,\theta$$

θ]

$$\frac{\partial \mathcal{L}'}{\partial \dot{\theta}} = mr^2\dot{\theta}$$

$$\rightarrow m(\ddot{\theta}r^2 + 2r\dot{r}\dot{\theta}) = -mgr\,\mathrm{Cos}\,\theta$$

$$\frac{\partial \mathcal{L}'}{\partial \theta} = -mgr\,\mathrm{Cos}\,\theta$$

si ponemos $r = R$

$\rightarrow \lambda = -m R \dot{\theta}^2 + m g \operatorname{Sen}\theta$

$\rightarrow m(\ddot{\theta} R^2) = -m g R \operatorname{Cos}\theta$

$\rightarrow \ddot{\theta} = -\frac{g}{R} \operatorname{Cos}\theta$ $\qquad \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta} \cdot \frac{d\theta}{dt} = \dot{\theta} \frac{d\dot{\theta}}{d\theta}$

$\rightarrow \dot{\theta} \frac{d\dot{\theta}}{d\theta} = -\frac{g}{R} \operatorname{Cos}\theta \Big/\Big/ \int_{\pi/2}^{\theta} d\theta$

$\rightarrow \int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = -\frac{g}{R} \int_{\pi/2}^{\theta} \operatorname{Cos}\theta\, d\theta$

$\frac{\dot{\theta}^2}{2} = -\frac{g}{R} (\operatorname{Sen}\theta - 1)$

$\rightarrow \lambda = -m R \left(-\frac{2g}{R} (\operatorname{Sen}\theta - 1) \right) + m g \operatorname{Sen}\theta$

$= 2mg(\operatorname{Sen}\theta - 1) + m g \operatorname{Sen}\theta$

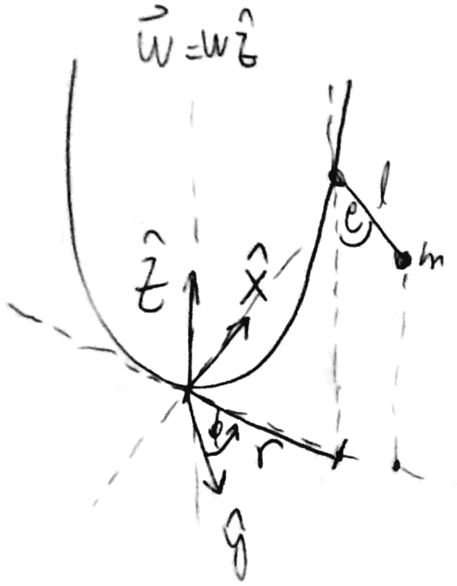
$= mg(3 \operatorname{Sen}\theta - 2)$

así, $\vec{f}_N = + mg(3\operatorname{Sen}\theta - 2) \nabla (r - R)$

$= + mg(3 \operatorname{Sen}\theta - 2)\, \hat{r}$

P3

· La particula tiene 3 grados de libertad



· θ: el angulo de la cuerda con la Vertical

· ϕ: el angulo del alambre con el plano $y=0$

· r: la distancia horizontal del origen a la argolla

$$\vec{r} = (r + l\sin\theta)(\cos\phi\,\hat{y} + \sin\phi\,\hat{x}) + \left(\frac{r^2}{2l} - l\cos\theta\right)\hat{z}$$

$$\rightarrow \dot{\vec{r}} = (\dot{r} + l\cos\theta\,\dot{\theta})(\cos\phi\,\hat{y} + \sin\phi\,\hat{x})$$
$$+ (r + l\sin\theta)(-\sin\phi\, w_0\,\hat{y} + \cos\phi\, w_0\,\hat{x})$$
$$+ \left(\frac{r\dot{r}}{l} + l\sin\theta\,\dot{\theta}\right)\hat{z}$$

$$K = \frac{m\|\dot{\vec{r}}\|^2}{2}, \quad U = mg\left(\frac{r^2}{2l} - l\cos\theta\right)$$

$$= \frac{m}{2}\Big[\left((\dot{r} + l\cos\theta\,\dot{\theta})\sin\phi + (r + l\sin\theta)\cos\phi\, w_0\right)^2$$
$$+ \left((\dot{r} + l\cos\theta\,\dot{\theta})\cos\phi + (r + l\sin\theta)(-\sin\phi\, w_0)\right)^2$$
$$+ \left(\frac{r\dot{r}}{l} + l\sin\theta\,\dot{\theta}\right)^2\Big]$$

$$= \frac{1}{2}m\left((\dot{r}+l\cos\theta\,\dot\theta)^2 + \frac{r^2\dot{r}^2}{l^2} + 2r\dot{r}\dot\theta\sin\theta + l^2\sin^2\theta\,\dot\theta^2 + (r+l\sin\theta)^2\omega_0^2 \right)$$

$\underline{r|}$

$$\frac{\partial \mathcal{L}}{\partial \dot r} = \frac{1}{2}m\left(2(\dot r + l\cos\theta\dot\theta) + 2\frac{\dot r r^2}{l^2} + 2r\dot\theta\sin\theta \right)$$

$$= m\left(\dot r\left(1+\frac{r^2}{l^2}\right) + \dot\theta(l\cos\theta + r\sin\theta)\right)$$

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{1}{2}m\left(2r\frac{\dot r^2}{l^2} + 2\dot r\dot\theta\sin\theta + \omega_0^2\cdot 2(r+l\sin\theta)\right) - mg\frac{r}{l}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot r}\right) - \frac{\partial \mathcal{L}}{\partial r} = 0 \rightarrow m\left(\ddot r\left(1+\frac{r^2}{l^2}\right) + \dot r\left(\frac{2r\dot r}{l^2}\right) + \ddot\theta(l_0\cos\theta + r\sin\theta) + \dot\theta(-l_0\sin\theta\dot\theta + \dot r\sin\theta + r\dot\theta\cos\theta)\right)$$

$$- mr\frac{\dot r^2}{l^2} - m\dot r\dot\theta\sin\theta - m\omega_0^2(r+l\sin\theta) + \frac{mgr}{l} = 0$$

$$\rightarrow \ddot r\left(1+\frac{r^2}{l^2}\right) + \frac{\dot r^2 r}{l^2} + \ddot\theta(l_0\cos\theta + r\sin\theta) + \dot\theta(-l_0\sin\theta\dot\theta + \dot r\sin\theta + r\dot\theta\cos\theta) - \dot r\dot\theta\sin\theta - \omega_0^2(r+l\sin\theta) + \frac{gr}{l} = 0$$

(1)

θ]

$$\frac{\partial L}{\partial \dot\theta} = \frac{m}{2}\Big(2(\dot r + l\cos\theta\,\dot\theta)\,l\cos\theta + 2\dot r\,\dot r\sin\theta$$

$$+2l^2\sin^2\theta\,\dot\theta\Big)$$

$$= m(\dot r(l_0\cos\theta + \sin\theta) + l^2\dot\theta)$$

$$\frac{\partial L}{\partial \theta} = \frac{m}{2}\Big(2(\dot r + \cancel{l\cos\theta\dot\theta})(-l\sin\theta\,\dot\theta)$$

$$+2\dot r\dot r\,\dot\theta\cos\theta + \cancel{l^2\dot\theta^2(2\sin\theta\cos\theta\dot\theta)}$$

$$+2(r+l\sin\theta)\,l\cos\theta\,\omega_0^2\Big) - mgl\sin\theta$$

$$\rightarrow \frac{m}{2}\Big(\ddot r(l_0\cos\theta + \sin\theta) + \dot r(\cancel{-l_0\sin\theta\dot\theta} + \dot r\sin\theta + \cancel{r\cos\theta\dot\theta})$$

$$+l^2\ddot\theta\Big) - m\Big(\cancel{\dot r\dot\theta(-l\sin\theta + r\cos\theta)}$$

$$+\omega_0^2(r+l\sin\theta)\,l\cos\theta - gl\sin\theta\Big) = 0$$

$$\rightarrow l^2\ddot\theta + \ddot r(l\cos\theta + r\sin\theta) + \dot r^2\sin\theta$$

$$-\omega_0^2(r+l\sin\theta)\,l\cos\theta + gl\sin\theta = 0$$

// (2)

• en el equilibrio

$$\dot{r} = \ddot{r} = \dot{\theta} = \ddot{\theta} = 0$$

$$(2) \rightarrow -w_0^2 (r + l \operatorname{Sen}\theta) l \cos\theta + g l \operatorname{Sen}\theta = 0$$

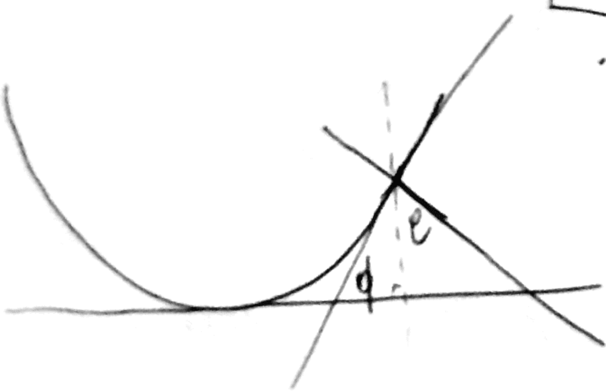
$$\rightarrow w_0^2 (r + l \operatorname{Sen}\theta) = g \operatorname{tg}\theta$$

$$(4) \rightarrow -w_0^2 (r + l \operatorname{Sen}\theta) + \frac{g r}{l} = 0$$

$$\rightarrow \boxed{\operatorname{Sen}\theta = \frac{r}{l}\left(\frac{g}{l w_0^2} - 1\right)}$$

$$\rightarrow w_0^2 \left(r + l \left(\frac{r}{l}\left(\frac{g}{l w_0^2} - 1\right)\right)\right) = g \operatorname{tg}\theta$$

$$\rightarrow \boxed{\operatorname{tg}\theta = \frac{r}{l}}$$



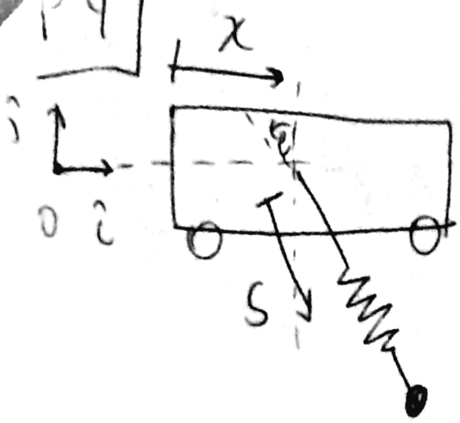
• Para que sea perpendicular, basta que $\phi = \theta$

Pero $\operatorname{tg}\phi = \dfrac{d\left(\frac{r^2}{2l}\right)}{dr} = \dfrac{r}{l}$

∴ $\phi = \theta$, la cuerda es perpendicular, y r puede ser despejado

Notemos que esto tiene sentido, pues en el equilibrio la tensión se cancela con la normal.

P4



• 3 coordenadas generalizadas, x, s y θ

$$\vec{r}_{car} = x\hat{i} \rightarrow \dot{\vec{r}}_{car} = \dot{x}\hat{i}$$

$$\vec{r}_m = x\hat{i} + s(\sin\theta\,\hat{i} - \cos\theta\,\hat{j})$$

$$= (x + s\sin\theta)\hat{i} - s\cos\theta\,\hat{j}$$

$$\rightarrow \dot{\vec{r}}_m = (\dot{x} + \dot{s}\sin\theta + s\dot{\theta}\cos\theta)\hat{i}$$
$$-(\dot{s}\cos\theta - s\dot{\theta}\sin\theta)\hat{j}$$

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left((\dot{x} + \dot{s}\sin\theta + s\dot{\theta}\cos\theta)^2 + (\dot{s}\cos\theta - s\dot{\theta}\sin\theta)^2\right)$$

$$U = -mgs\cos\theta + \frac{1}{2}k(s-l)^2$$

x]

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = M\dot{x} + m(\dot{x} + \dot{s}\sin\theta + s\dot{\theta}\cos\theta), \quad \frac{\partial \mathcal{L}}{\partial x} = 0$$

$$\rightarrow M\ddot{x} + m(\ddot{x} + \ddot{s}\sin\theta + \dot{s}\dot{\theta}\cos\theta + \dot{s}\dot{\theta}\cos\theta + s\ddot{\theta}\cos\theta$$
$$- s\dot{\theta}^2\sin\theta) = 0$$

$$(M+m)\ddot{x} + m(\ddot{s}\sin\theta + 2\dot{s}\dot{\theta}\cos\theta + s\ddot{\theta}\cos\theta - s\dot{\theta}^2\sin\theta) = 0$$

θ]

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = m\left[(\cancel{\dot{s}\cos\theta} - s\dot{\theta}\sin\theta)(-s\sin\theta) + (\dot{x} + \cancel{\dot{s}\sin\theta} + s\dot{\theta}\cos\theta)(s\cos\theta)\right]$$

$$= m(s^2\dot{\theta} + \dot{x}s\cos\theta)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = m(\dot{x}\dot{s}\cos\theta - \dot{x}s\dot{\theta}\sin\theta - gs\sin\theta)$$

$$\rightarrow s\ddot{\theta} + 2\dot{s}\dot{\theta} + \ddot{x}\cos\theta + g\sin\theta = 0$$

$s\rfloor$

$$\frac{\partial \mathcal{L}}{\partial \dot{s}} = m\left((\dot{x} + \dot{s}\sin\theta + \cancel{s\dot{\theta}\cos\theta})\sin\theta + (\dot{s}\cos\theta - \cancel{s\dot{\theta}\sin\theta})\cos\theta \right)$$

$$= \cancel{m(\dot{x} + 2\dot{s}\cos\theta)\sin\theta}$$

$$= m(\dot{x}\sin\theta + \dot{s})$$

$$\frac{\partial \mathcal{L}}{\partial s} = m\Big((\dot{x} + \cancel{\dot{s}\sin\theta} + s\dot{\theta}\cos\theta)(\dot{\theta}\cos\theta)$$

$$+ (\cancel{\dot{s}\cos\theta} - s\dot{\theta}\sin\theta)(-\dot{\theta}\sin\theta)$$

$$+ mg\cos\theta + K(s-l)\Big)$$

$$= m\left(s\dot{\theta}^2 + \dot{x}\dot{\theta}\cos\theta + mg\cos\theta + K(s-l)\right)$$

$$\therefore\ m(\ddot{x}\sin\theta + \cancel{\dot{x}\dot{\theta}\cos\theta} + \ddot{s}) - m(s\dot{\theta}^2 + \cancel{\dot{x}\dot{\theta}\cos\theta}$$

$$+ mg\cos\theta + K(s-l)) = 0$$

$$\rightarrow m(\ddot{x}\sin\theta + \ddot{s} - s\dot{\theta}^2 - mg\cos\theta - K(s-l)) = 0$$