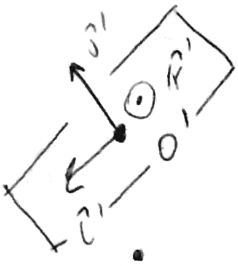
auxiliar #9

P1) Primero debemos definir nuestros sistema de referencia, y vectores unitarios

• O' rota con velocidad angular $\Omega\hat k$



• ahora debemos definir bien $\vec R$ y $\vec r'$, con sus derivadas, para usarlas después

$$\vec R = R\hat r, \quad \dot{\vec R} = R\Omega\hat\theta, \quad \ddot{\vec R} = -R\Omega^2\hat r$$

$$\vec r' = \left(-\frac{L}{2}+V_0 t\right)\hat i', \quad \dot{\vec r}' = V_0\hat i', \quad \ddot{\vec r}' = 0$$

• debemos ahora encontrar relaciones entre O y O', si queremos describir la respuesta en términos de O', entonces debemos describir O en términos de O'

$$\hat r = -\hat j', \quad \hat i' = -\hat\theta, \quad \hat k = \hat k'$$

• Describimos ahora las fuerzas ficticias

$$\vec F_{trans} = -m\frac{d^2\vec R}{dt^2} = mR\Omega^2\hat r = -mR\Omega^2\hat j'$$

$$\vec F_{cent} = -m\Omega\hat k\times\left(\Omega\hat k\times\left(-\frac{L}{2}+V_0 t\right)\hat i'\right),$$

$$= -m\Omega^2\left(V_0 t-\frac{L}{2}\right)\hat i'$$

$\hat k\times\hat i' = \hat k'\times\hat i'$
$= -\hat j'$
$\hat k\times(-\hat j') = \hat i'$

$$\vec F_{cor} = -2m\Omega\hat k\times V_0\hat i' = 2m\Omega V_0\hat j$$

$$\vec F_{az} = 0$$

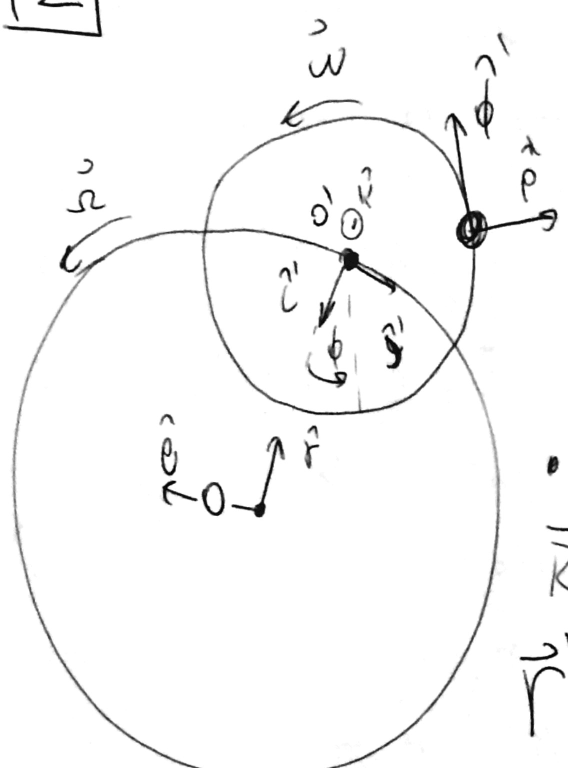
• escribimos la ecuación para la fuerza neta

$$\vec{F}_{neta} = m\vec{a}_{tot} = m\vec{a}' - F_{Ficticia}$$

$$= 0 + mR\Omega^2 \hat{j}' + m\Omega^2\left(V_0 t - \frac{L}{2}\right)\hat{i}' - 2m\Omega V_0 \hat{j}'$$

$$= m\Omega^2\left(V_0 t - \frac{L}{2}\right)\hat{i}' + \left(mR\Omega^2 - 2m\Omega V_0\right)\hat{j}'$$

P2)



$$\vec{\Omega} = \left(\frac{2\pi}{T}\right)\hat{k}, \quad \vec{w} = \left(\frac{2\pi}{\cancel{\tau}}\right)\hat{k}$$

• O' rota con velocidad $\Omega\hat{k}$

• Describir $\vec{R}$ y $\vec{r}'$

$$\vec{R} = R\hat{r}, \quad \dot{\vec{R}} = R\Omega\hat{\theta}, \quad \ddot{\vec{R}} = -R\Omega^2\hat{r}$$

$$\vec{r}' = r\hat{\rho}', \quad \dot{\vec{r}}' = rw\hat{\phi}', \quad \ddot{\vec{r}}' = -rw^2\hat{\rho}'$$

• relaciones entre O y O'

$$\hat{i}' = -\hat{r}, \quad \hat{j}' = -\hat{\theta}, \quad \hat{k}' = \hat{k}$$

Si en $t=0$, $\phi = 0$

$$\hat{\rho}' = \hat{i}'\cos\phi + \hat{j}'\operatorname{sen}\phi = -\hat{r}\cos\phi - \hat{\theta}\operatorname{sen}\phi, \quad \phi = wt$$

• Fuerzas ficticias

$$\vec{F}_{Trans} = mR\Omega^2\hat{r} = -mR\Omega^2\hat{i}'$$

$$\vec{F}_{cent} = -m\Omega\hat{k}\times(\Omega\hat{k}\times r\hat{\rho}')$$
$$= -m\Omega^2 r\,\hat{k}\times(\hat{k}\times\hat{\rho}')$$
$$= -m\Omega^2 r\,\hat{k}\times\hat{\phi}'$$
$$= m\Omega^2 r\,\hat{\rho}'$$

$\vec F_{cor} = -2m\Omega\hat k \times r w\hat\phi' = 2m\Omega r w\hat\rho'$

$\vec F_{az} = 0$

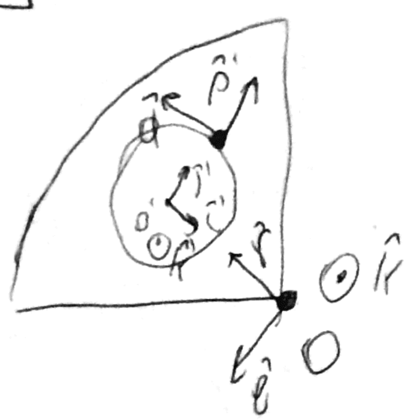
$\rightarrow \vec F_{neto} = m\vec a = m\vec a' - \vec F_{Ficticias}$

$= -mrw^2\hat\rho' - mR\Omega^2\hat r - m\Omega^2 r\hat\rho' - 2m\Omega r w\hat\rho'$

$= -mr(w^2+2w\Omega+\Omega^2)\hat\rho' - mR\Omega^2\hat r$

$= -mr(w+\Omega)^2\hat\rho' - mR\Omega^2\hat r$

P3



• O' rota con velocidad $\Omega\hat k$

$\vec R = 2R\hat r$

$\dot{\vec R} = 2R\Omega\hat\theta$

$\ddot{\vec R} = -2R\Omega^2\hat r$

$\vec r' = R\hat\rho'$, $\dot{\vec r}' = R\dot\phi\hat\phi'$, $\ddot{\vec r}' = -R\dot\phi^2\hat\rho' + R\ddot\phi\hat\phi'$

• Relaciones O', O

$\hat i' = -\hat r'$, $\hat j' = -\hat r$, $\hat k' = \hat k$

• Fuerza ficticia

$\vec F_{trans} = -m(-2R\Omega^2\hat r) = 2Rmw^2\hat r$

$= -2Rm\Omega^2\hat i'$

$\vec F_{cent} = -m(\Omega\hat k)\times(\Omega\hat k\times R\hat\rho')$

$= m\Omega^2 R\hat\rho'$

• $\hat k\times\hat\rho' = \hat\phi'$

• $\hat k\times\hat\phi' = -\hat\rho$

$$\vec{F}_{cor} = -2m\,\Omega\,\hat{R} \times R\dot{\phi}\,\hat{\phi}'$$

$$= 2m\,\Omega R\dot{\phi}\,\hat{\rho}'$$

$$\vec{F}_{eZ} = 0$$

DCL

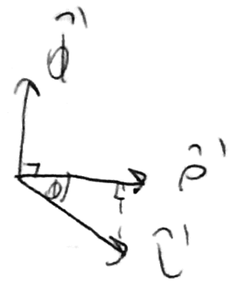
$$\vec{F}_{nete} = (N - mg)\hat{R} + N\hat{\rho}'$$

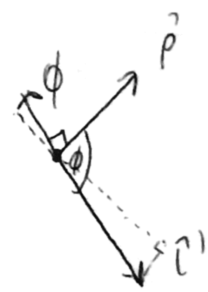
$$= \underbrace{m\left(R\ddot{\phi}\hat{\phi}' - R\dot{\phi}^2\hat{\rho}'\right)}_{m\vec{a}'} + \underbrace{2Rm\Omega^2\hat{\imath}'}_{-F_{Trans}}$$

$$\underbrace{-m\Omega^2 R\hat{\rho}'}_{-F_{cent}} \quad \underbrace{-2m\Omega R\dot{\phi}\hat{\rho}'}_{-F_{cor}}$$

~~4b~~ ~~N = 2mRΩ² Cos~~

$$\hat{\imath}' = -\hat{\phi}'\,\mathrm{Sen}\,\phi + \hat{\rho}'\,\mathrm{Cos}\,\phi$$





$$\therefore\ \hat{\phi}':\ mR\ddot{\phi} + 2mR\Omega^2(-\mathrm{Sen}\,\phi) = 0$$

$$\hat{\rho}':\ N = 2mR\Omega^2\,\mathrm{Cos}\,\phi - mR\dot{\phi}^2 - m\Omega^2 R - 2m\Omega R\dot{\phi}$$

$$\rightarrow \ddot{\phi} = 2\Omega\,\mathrm{Sen}\,\phi \Big/ \int_0^{\dot{\phi}} d\dot{\phi} \int_0^{\phi} d\phi$$

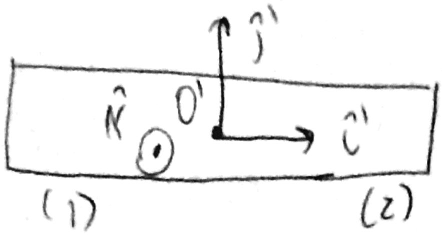
b)

$$\rightarrow \dot{\phi}^2 = 4\Omega^2(1 - \mathrm{Cos}\,\phi)$$

$N = 0 \rightarrow$ Separación

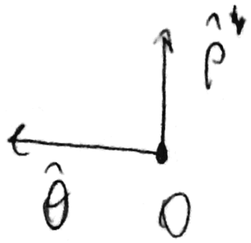
$$0 = 2mR\Omega^2\,\mathrm{Cos}\,\phi - mR\cdot 4\Omega^2(1 - \mathrm{Cos}\,\phi) - m\Omega^2 R - 2m\Omega R\cdot 2\Omega\sqrt{1 - \mathrm{Cos}\,\phi}$$

$$\rightarrow 6\,\mathrm{Cos}\,\phi_s - 4\sqrt{1 - \mathrm{Cos}\,\phi_s} - 5 = 0 \,//$$



• O' gira con velocidad $\omega_0 \hat{k}$

• Posiciones



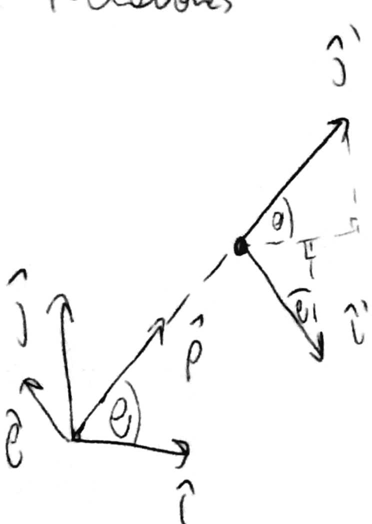
$$\vec{R} = R\hat{\rho}$$

$$\dot{\vec{R}} = R\omega_0 \hat{\theta}$$

$$\ddot{\vec{R}} = -R\omega_0^2 \hat{\rho}$$

$$\vec{r}_1 = x_1 \hat{i}' \quad , \quad \vec{r}_2 = x_2 \hat{i}'$$

• Relaciones



$$\hat{j}' = \hat{i}\cos\theta + \hat{j}\sin\theta$$

$$\hat{i}' = \hat{i}\sin\theta - \hat{j}\cos\theta$$

$$\hat{j}' = \hat{\rho} \quad , \quad \hat{i}' = -\hat{\theta}$$

• Fuerzas en (1) y (2)

$$\vec{F}^{(1)}_{Trans} = \vec{F}^{(2)}_{tras} = mR\omega_0^2 \hat{\rho}$$

$$\vec{F}^{(1)}_{Cent} = m\omega_0^2 x_1 \hat{i}' \quad , \quad \vec{F}^{(2)}_{cen} = m\omega_0^2 x_2 \hat{i}'$$

$$\vec{F}^{(1)}_{cor} = -2m\omega_0 \hat{k} \times \dot{x}_1 \hat{i}' = -2m\omega_0 \dot{x}_1 \hat{j}'$$

$$\vec{F}^{(2)}_{cor} = -2m\omega_0 \dot{x}_2 \hat{j}'$$

$\vec{F} = m\vec{a}' - \vec{F}_{F.C}$, para cada masa.

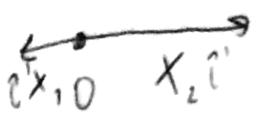
$$\vec{F}^{(1)} = m\ddot{x}_1\hat{i}' - mRW_0^2\hat{\rho} - mW_0^2 x_1\hat{i}' + 2mW_0\dot{x}_1\hat{j}'$$

Nos interesan la Fuerza en $\hat{i}$

$$-T = m\ddot{x}_1 - mW_0^2 x_1 \qquad |x_1| + |x_2| = L$$

De igual manera

$$T = m\ddot{x}_2 - mW_0^2 x_2$$



$$x_2 - x_1 = L \rightarrow \ddot{x}_2 = \ddot{x}_1$$

$x_2 - x_1 = L$

$\rightarrow x_1 = x_2 - L$

$$\rightarrow T = \frac{mW_0^2 L}{2}$$

Pues $x_1 < 0$

b) Cuerda se rompe $\rightarrow T = 0$

$$\rightarrow \cancel{m}\ddot{x}_1 = \cancel{m}W_0^2 x_1$$

$$\ddot{x}_2 = W_0 x_2$$

EDO, con solución conocida

$$\rightarrow x_1(t) = Ae^{w_0 t} + Be^{-w_0 t}$$

$$x_2(t) = Ce^{w_0 t} + De^{-w_0 t}$$

C.i.: $x_1(0) = -\frac{L}{2}$, $x_2(0) = \frac{L}{2}$, $\dot{x}_1(0) = \dot{x}_2(0) = 0$

Condiciones iniciales $\rightarrow A = B = \frac{L}{4}$, $C = D = \frac{L}{4}$

$$\therefore \quad x_1(t) = -\frac{L}{2}\cosh(W_0 t)$$

$$x_2(t) = +\frac{L}{2}\cosh(W_0 t)$$

$$\vec{r}_1'(t) = x_1\hat{i}', \quad \vec{r}_2'(t) = x_2\hat{i}'$$

$\rightarrow \vec{V}_3'(t) = \dot{x}_1 \hat{i}' = -\frac{L}{2} W_0 \operatorname{Senh}(W_0 t) \hat{i}'$ relativo

$\vec{V}_2'(t) = \frac{L}{2} W_0 \operatorname{Senh}(W_0 t) \hat{i}'$ al tubo

• Se escapan cuando $x_1(t^*) = -L$

$$\rightarrow -\frac{L}{2} \operatorname{Cosh}(W_0 t^*) = -L \rightarrow \operatorname{Cosh}(W_0 t^*) = 2$$

$$\operatorname{Cosh}(W_0 t^*)^2 - \operatorname{Sen}(W_0 t^*)^2 = 1$$

$$\rightarrow \operatorname{Senh}(W_0 t^*) = \sqrt{3}$$

$$\therefore \vec{V}_1'(t) = -\frac{L}{2}\sqrt{3} W_0 \hat{i}' \quad \vec{V}_1'(t) = \frac{\sqrt{3}}{2} L W_0 \hat{i}'$$

c) Recordemos que $\vec{V} = \vec{V}' + \dot{\vec{R}} + \vec{\Omega} \times \vec{r}'$

$$\cdot \dot{\vec{R}} = R W_0 \hat{\theta} = L W_0 \hat{\theta}$$

Cuando salen del tubo:

$$\cdot \vec{\Omega} \times \vec{r}_1' = W_0 \hat{k} \times (-L \hat{i}') = -W_0 L \hat{j}'$$

$$\cdot \vec{\Omega} \times \vec{r}_2' = W_0 \hat{k} \times (L \hat{i}') = W_0 L \hat{j}'$$

$$\therefore \vec{V}_1 = -\frac{\sqrt{3}}{2} L W_0 \hat{i}' + R W_0 \hat{\theta} - W_0 L \hat{j}'$$

$$= +\frac{\sqrt{3}}{2} L W_0 \hat{\theta} + L W_0 \hat{\theta} - W_0 L \hat{\rho}$$

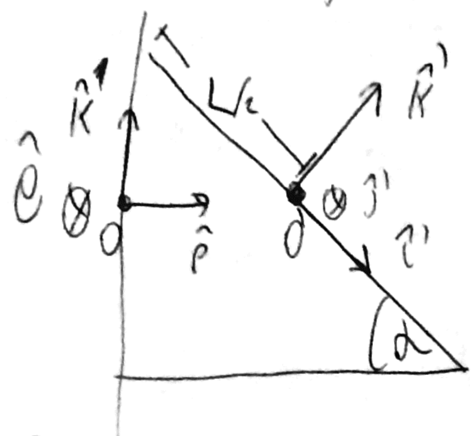
$$= L W_0 \left(\frac{\sqrt{3}}{2} + 1\right) \hat{\theta} - W_0 L \hat{\rho}$$

$$\vec{V}_2 = \frac{\sqrt{3}}{2} L W_0 \hat{i}' + L W_0 \hat{\theta} + W_0 L \hat{j}'$$

$$= L W_0 \left(-\frac{\sqrt{3}}{2} + 1\right) + W_0 L \hat{\rho}$$

P5)

• es mas facil describir un MAS cuando el origen esté en el punto de equilibrio



• O' rota de tal manera que siempre tiene la configuración de la figura, con velocidad $\Omega \hat{k}$

$$\vec{R} = \frac{L\cos\alpha}{2}\hat{\rho}, \quad \dot{\vec{R}} = \frac{L\cos\alpha\,\Omega}{2}\hat{\theta}, \quad \ddot{\vec{R}} = -\frac{L\cos\alpha\,\Omega^2}{2}\hat{\rho}$$

• el MAS tiene amplitud $L/2$

$$\vec{r}'(t) = \frac{L}{2}\cos\left(\frac{2\pi}{T}t + \phi\right)\hat{\imath}'$$ desfase

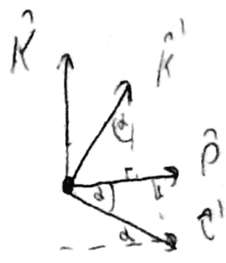
$$\vec{r}'(0) = \frac{L}{2}\hat{\imath}' \rightarrow \frac{L}{2}\hat{\imath}' = \frac{L}{2}\cos\phi\,\hat{\imath}' \rightarrow \cos\phi = 1$$
$$\rightarrow \phi = 0$$

$$\therefore \vec{r}'(t) = \frac{L}{2}\cos\left(\frac{2\pi}{T}t\right)\hat{\imath}'$$

$$\dot{\vec{r}}'(t) = -\frac{L}{2}\left(\frac{2\pi}{T}\right)\sin\left(\frac{2\pi}{T}t\right)$$

$$\ddot{\vec{r}}'(t) = -\frac{L}{2}\left(\frac{2\pi}{T}\right)^2\cos\left(\frac{2\pi}{T}t\right)$$

• Relaciones entre O y O'



$$\hat{k}' = \hat{k}\cos\alpha + \hat{\rho}\sin\alpha$$
$$\hat{\imath}' = \hat{\rho}\cos\alpha - \sin\alpha\,\hat{k}$$
$$\hat{\jmath}' = \hat{\theta}$$

Usamos $\vec{V}_{\text{Inercial}} = \vec{V}'_{\text{No inercial}} + \dot{\vec{R}} + \vec{\Omega} \times \vec{r}'_{\text{No inercial}}$

$$\vec{\Omega} \times \vec{r}' = \Omega \hat{k} \times \frac{L}{2} \cos\left(\frac{2\pi}{\cancel{T}} t\right) \hat{\imath} = \frac{L}{2} \Omega \cos\left(\frac{2\pi}{T} t\right) \hat{k} \times \hat{\imath}'$$

$$\hat{k} \times \hat{\imath}' = \hat{k} \times (\hat{\rho} \cos\alpha - \operatorname{Sen}\alpha\, \hat{k}) = \hat{e} \cos\alpha$$

$$\therefore \vec{V} = -\frac{L}{2}\left(\frac{2\pi}{T}\right) \operatorname{Sen}\left(\frac{2\pi}{T} t\right) \underbrace{(\hat{\rho} \cos\alpha - \operatorname{Sen}\alpha\, \hat{k})}_{\hat{\imath}'}$$

$$+ \frac{L \cos\alpha}{2} \Omega \hat{e} + \frac{L \cos\alpha}{2} \Omega \cos\left(\frac{2\pi}{T} t\right) \hat{e}$$

en $t = T/4$;

$$\vec{V}(T/4) = \frac{L \cos\alpha}{2} \Omega \hat{e} - \frac{L}{2}\left(\frac{2\pi}{T}\right)(\hat{\rho} \cos\alpha - \operatorname{Sen}\alpha\, \hat{k})$$

$$\vec{a} = \dot{\vec{V}} = -\frac{L}{2}\left(\frac{2\pi}{T}\right)\left[\left(\frac{2\pi}{T}\right) \cos\left(\frac{2\pi}{T} t\right)(\hat{\rho} \cos\alpha - \operatorname{Sen}\alpha\, \hat{k})\right.$$

$$\left. + \operatorname{Sen}\left(\frac{2\pi}{T} t\right) \cos\alpha \cdot \Omega \hat{e}\right]$$

$$+ \frac{L \cos\alpha}{2} \Omega^2 (-\hat{\rho}) + \frac{L \cos\alpha}{2} \Omega \left[-\left(\frac{2\pi}{T}\right) \operatorname{Sen}\left(\frac{2\pi}{T} t\right) \hat{e} - \cos\left(\frac{2\pi}{T} t\right) \hat{\rho} \Omega\right]$$

$$\vec{a}(t = T/4) = +\frac{L}{2}\left(\frac{2\pi}{T}\right) \cdot \cos\alpha\, \Omega\, \hat{e}$$

$$-\frac{L \cos\alpha}{2} \Omega^2 \hat{\rho} - \frac{L \cos\alpha}{2} \Omega \left(\frac{2\pi}{T}\right) \cancel{\operatorname{Sen}\left(\frac{2\pi}{T} t\right)} \hat{e}$$

$$= -\frac{L \cos\alpha}{2} \Omega^2 \hat{\rho}$$