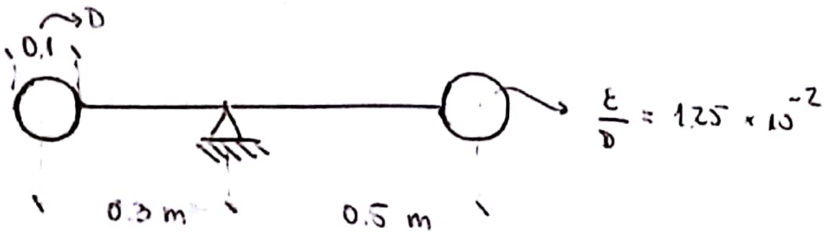
Auxiliar #10:

P1)



a) Consideremos las variables:

$\mathcal{D} \rightarrow$ Fuerza de arrastre

$\rho \rightarrow$ Densidad del fluido

$\mu \rightarrow$ Viscosidad

$U \rightarrow$ Velocidad

$D \rightarrow$ Diámetro de la esfera

$\varepsilon \rightarrow$ Rugosidad

$$\mathcal{D} = f(\rho; \mu; D, U; \varepsilon) \Rightarrow \text{Buckingham.}$$

1. $[\mathcal{D}] : MLT^{-2}$ $\quad [U] : LT^{-1}$ $\quad [\mu] : ML^{-1}T^{-1}$

   $[D] = [\varepsilon] : L$ $\quad [\rho] : ML^{-3}$

2. # de grupos = 6 var. − 3 un. = 3 grupos

3. Variables a repetir : ρ, U, D

4. $\Pi_1 = \mathcal{D} \cdot \rho^a U^b \cdot D^c$, $\quad \Pi_2 = \mu \rho^a U^b \cdot D^c$, $\quad \Pi_3 = \varepsilon \, \rho^a \cdot U^b \cdot D^c$.

5. $\Pi_1 : MLT^{-2} (ML^{-3})^a (LT^{-1})^b \cdot (L)^c$

$$\left.\begin{array}{ll} M] & 1 + a = 0 \\ L] & 1 - 3a + b + c = 0 \\ T] & -2 - b = 0 \end{array}\right\} \begin{array}{l} a = -1 \\ b = -2 \\ c = -2 \end{array} \qquad \Pi_1 = \frac{\mathcal{D}}{\rho U^2 D^2}$$

$$\Pi_2 : \mu \cdot \rho^a \cdot U^b \cdot D^c \Leftrightarrow \Pi_2 = \frac{\rho U D}{\mu} = Re.$$

$$\Pi_3 : L \cdot (ML^{-3})^a (LT^{-1})^b \cdot L^c \rightsquigarrow a = b = 0, \quad c = -1$$

$$\Pi_3 = \frac{\varepsilon}{D}$$

6. $\Pi_1 = \Phi(\Pi_2 ; \Pi_3)$

$$\Rightarrow \frac{D}{\rho U^2 D^2} = \Phi\left(\frac{\rho U D}{\mu} ; \frac{\varepsilon}{D}\right)$$ //

b) Giro en sentido de los punteros del reloj:

$M_{TOT} < 0 \iff 0.5\, \underset{\text{Arrastre esfera rugosa}}{D_1} - 0.3\, \underset{\text{Arrastre esfera suave}}{D_2} \leq 0$

$\Rightarrow 0.5 D_1 \leq 0.3 D_2$, $D_1 = \frac{1}{2}\rho C_{D_1} U_1^2 A_1$, $D_2 = \frac{1}{2}\rho C_{D_2} U_2^2 A_2$

Para este caso, $A_1 = A_2 \wedge U_1 = U_2$

$0.5 \cdot C_{D_1} \cancel{\frac{1}{2}\rho U^2 A} \leq 0.3\, C_{D_2} \cancel{\frac{1}{2}\rho U^2 A}$

$C_{D_1} \leq 0.6 \cdot C_{D_2}$

$\frac{C_{D_1}}{C_{D_2}} \leq 0.6 \longrightarrow$ según gráfico $\approx Re = 10^5$

$\Rightarrow Re = \frac{0.1\ U}{1.46 \times 10^{-5}\ m^2/s}$

↳ viscosidad cinemática del aire

$U_{min} = 14.6\ m/s$ //

P2] Sea $y^* = \dfrac{y}{h}$, $u^* = \dfrac{u}{U}$, $t^* = t \cdot \omega$

$$\Rightarrow \frac{\partial u}{\partial t} = \frac{\partial (U u^*)}{\partial t^*}\,\omega = U\omega \frac{\partial u^*}{\partial t^*}$$

$$\frac{\partial u}{\partial y} = \frac{\partial (U u^*)}{\partial (y^* h)} = \frac{U}{h}\frac{\partial u^*}{\partial y^*}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial (h y^*)}\left(\frac{U}{h}\frac{\partial u^*}{\partial y^*}\right) = \frac{U}{h^2}\cdot\frac{\partial^2 u^*}{\partial y^{*2}}$$

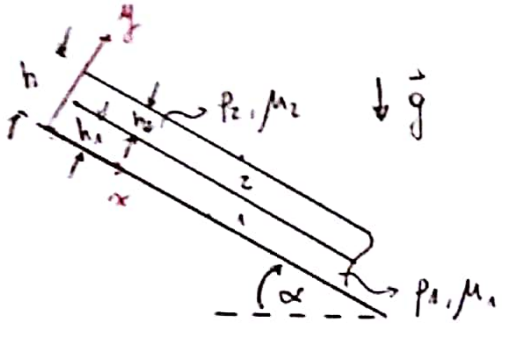
La ecuación diferencial original es:

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2}$$

$\Rightarrow$ Al adimensionalizar: $\rho \cancel{U} \omega \cdot \dfrac{\partial u^*}{\partial t^*} = \dfrac{\cancel{U}}{h^2}\dfrac{\partial^2 u^*}{\partial y^{*2}}\,\mu$

$$\left[\frac{\rho \omega h^2}{\mu}\right] \frac{\partial u^*}{\partial t^*} = \frac{\partial^2 u^*}{\partial y^{*2}}$$

P3]



Ec. de continuidad:

$$\frac{\partial \rho}{\partial t} + \rho \cdot \vec{\nabla} \cdot \vec{u} = 0$$

Navier stokes: (Bi-dimensional)

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Supuestos: $\rightarrow \partial/\partial t = 0$
$\rightarrow \vec{u} = u\hat{i}$
$\rightarrow \dfrac{\partial P}{\partial x} = 0$

Ec. de continuidad $\Rightarrow \dfrac{\partial u}{\partial x} = 0 \rightsquigarrow u = u(y)$

$$\Rightarrow 0 = \rho g \cdot \text{sen}(\alpha) + \mu \frac{\partial^2 u}{\partial y^2}$$

$$0 = -\frac{\partial P}{\partial y} - \rho g \cos(\alpha)$$

Condiciones de borde:

→ Campo de presión:

$$P_2(y=h) = P_{ATM} \quad , \quad P_1(y=h_1) = P_2(y=h_1)$$

→ Campo de velocidad:

$$\mu_2 \cdot \frac{\partial u_2}{\partial y}(y=h) = 0 \quad , \quad u_1(y=h_1) = u_2(y=h_1)$$

$$\mu_1 \frac{\partial u_1}{\partial y}(y=h_1) = \mu_2 \frac{\partial u_2}{\partial y}(y=h_2) \qquad u_1(y=0) = 0$$

Campo de presión: $\dfrac{\partial P}{\partial y} = -\rho g \cos(\alpha) \Rightarrow P(y) = -\rho g y \cos(\alpha) + A$

$$P_2(y=h) = P_{AT} = -\rho_2 g h \cos(\alpha) + A \Rightarrow A = \rho_2 g h \cos(\alpha) + P_{AT}$$

$$\Rightarrow P_2(y) = \rho_2 g \cos(\alpha)[h-y] + P_{AT}$$

$$P_1(y=h_1) = P_2(y=h_1) \Rightarrow \rho_1 g h_1 \cos(\alpha) + B = \rho_2 \cdot g [\underbrace{h-h_1}_{h_2}] \cos(\alpha) + P_{AT}$$

$$B = P_{AT} + [\rho_2 (\underbrace{h-h_1}_{h_2}) + \rho_1 h_1] g \cos(\alpha)$$

$$P_1(y) = P_{AT} + \rho_2 \cdot h_2 \cdot g \cdot \cos(\alpha) + \rho_1 g (h_1 - y) \cdot \cos(\alpha)$$

Campo de velocidad

$$\frac{\partial^2 u}{\partial y^2} = -\frac{\rho g}{\mu} \cdot sen(\alpha) \Rightarrow u(y) = -\frac{\rho g}{2\mu} y^2 \cdot sen(\alpha) + C y + D$$

$$\left.\begin{array}{l} u_1(y) = -\frac{\rho_1 g}{2\mu_1} \cdot sen(\alpha)\, y^2 + C_1 \cdot y + D_1 \\ u_1(y=0) = 0 = D_1 \Rightarrow D_1 = 0 \end{array}\right\} u_1(y) = -\frac{\rho_1 g}{2\mu_1} \cdot sen(\alpha) y^2 + C_1 y$$

$$u_2(y) = -\frac{\rho_2 \cdot g}{2\mu_2} \, sen(\alpha)\, y^2 + C_2 \cdot y + D_2$$

$$\mu_2 \cdot \frac{\partial u_2}{\partial y}(h) = -\rho_2 g \, sen(\alpha)\, h + \mu_2 C_2 = 0 \Rightarrow C_2 = \frac{\rho_2 g}{\mu_2} sen(\alpha) h.$$

$$\Rightarrow u_2(y) = \frac{\rho_2 \cdot g\, sen(\alpha)}{\mu_2} \cdot h^2 \left[\left(\frac{y}{h}\right) - \frac{y^2}{2h^2}\right] + D_2$$

$$u_1(y=h_1) = -\frac{\rho_1 \cdot g}{2\mu_1} \, sen(\alpha) \cdot h_1^2 + C_1 \cdot h_1 = \frac{\rho_2 \cdot g}{\mu_2} \cdot sen(\alpha)\, h^2 \left[\frac{h_1}{h} - \frac{h_1^2}{h^2}\right] + D_2$$

$$\mu_1 \frac{\partial u_1}{\partial y}(h_1) = -\rho_1 \cdot g \cdot sen(\alpha) \cdot h_1 + \mu_1 C_1 = \rho_2 \cdot g\, sen(\alpha) h^2 \left[\frac{1}{h} - \frac{h_1}{h^2}\right]$$

$$C_1 = \frac{\rho_2 \cdot g}{\mu_1} \, sen(\alpha) h_2 + \frac{\rho_1 g}{\mu_1} \, sen(\alpha) h_1$$

$$\Rightarrow -\frac{\rho_1 g}{2\mu_1} \cdot sen(\alpha) \cdot h_1^2 + \frac{\rho_2 \cdot g}{\mu_1} \cdot sen(\alpha) h_2 \cdot h_1 + \frac{\rho_1 \cdot g \times 2}{\mu_1 \times 2} \cdot sen(\alpha) \cdot h_1^2$$

$$= \frac{\rho_2 \cdot g}{\mu_2} \cdot sen(\alpha) \cdot h h_1 - \frac{\rho_2 \cdot g}{\mu_2} \cdot sen(\alpha) \cdot h_1^2 + D_2$$

$$D_2 = \frac{\rho_1 g}{2\mu_1} \cdot sen(\alpha_1) \cdot h_1^2 + \frac{\rho_2 g}{\mu_1} \cdot sen(\alpha)\, h_1 \cdot h_2 - \frac{\rho_2 g}{\mu_2} \cdot sen(\alpha)\, h h_1$$

$$+ \frac{\rho_2 g}{\mu_2} \cdot sen(\alpha) \cdot h_1^2$$

$$\therefore\; u_1(y) = -\frac{\rho_1 g}{2\mu_1}\, sen(\alpha)\, y^2 + \left[\frac{\rho_2\, g\, h_2}{\mu_1} + \frac{\rho_1\, g\, h_1}{\mu_1}\right] sen(\alpha)\, y$$

$$u_2(y) = \frac{\rho_2 \cdot g \cdot sen(\alpha)}{\mu_2} h^2 \left[\frac{y}{h} - \frac{y^2}{2h^2}\right] + \frac{\rho_1\, g\, sen(\alpha)\, h_1^2}{2\mu_1}$$

$$\frac{\rho_2 \cdot g}{\mu_1}\, sen(\alpha_1) - \frac{\rho_2 \cdot g}{\mu_2}\, sen(\alpha)\, h \cdot h_1 + \frac{\rho_2 g}{\mu_2}\, sen(\alpha)\, h_1^2$$

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