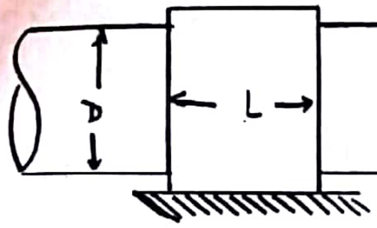
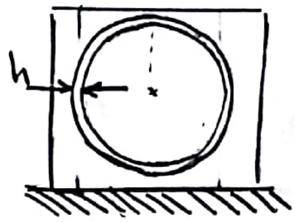
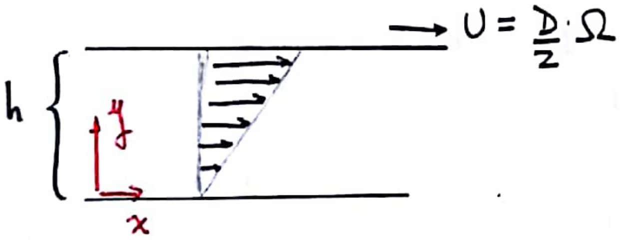
Auxiliar N°5:

Pregunta N°1:





Notemos que el eje rota con velocidad angular Ω, y $h << D$, entonces, dado que esa separación es mucho más pequeña que el radio de curvatura, el flujo del lubricante en el espacio, se puede modelar como un flujo de Couette, de modo que:



■ Ecuaciones que modelan el problema:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \vec{V}) = 0$$

Como el proceso es estacionario: $\partial / \partial t = 0$.

(*) supuestos del campo de velocidad:

→ El mov. es bidimensional $\Rightarrow v_z = \omega = 0$.

→ En general, se considerará que el flujo se mueve en el eje "x", así: $v_y = v = 0 \Rightarrow \vec{V} = u\,\hat{i}$

Luego, de la ec. de continuidad:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \vec{V}) \underset{\rho \text{ cte.}}{=} \rho\left(\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{0} + \cancel{\frac{\partial \omega}{\partial z}}^{0}\right) \Rightarrow \frac{\partial u}{\partial x} = 0$$

Entonces: $u = u(y)$.

Con las ecuaciones de Navier-stokes:

$\hat{i}$: $\rho\left(\cancel{\frac{\partial u}{\partial t}}^{0} + u\cancel{\frac{\partial u}{\partial x}}^{0} + \cancel{v}^{0}\frac{\partial u}{\partial y} + \cancel{w}^{0}\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{0} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$

$$-\frac{\partial p}{\partial x} + \rho \cancel{g_x}^{0} + \frac{\partial^2 u}{\partial y^2} = 0.$$

, supongamos que $\vec{g} = (0; g; 0)$.

$$\mu \cdot \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$$

$\hat{j}$: $\rho\left(\cancel{\frac{\partial v}{\partial t}}^{0} + u\cancel{\frac{\partial v}{\partial x}}^{0} + \cancel{v}^{0}\frac{\partial v}{\partial y} + w\cancel{\frac{\partial v}{\partial z}}^{0}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\cancel{\frac{\partial^2 v}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 v}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 v}{\partial z^2}}\right)^{0}$.

$$\rho g_y = \frac{\partial p}{\partial y}$$

$\hat{k}$: $\rho\left(\cancel{\frac{\partial w}{\partial t}}^{0} + u\cancel{\frac{\partial w}{\partial x}}^{0} + v\cancel{\frac{\partial w}{\partial y}}^{0} + w\cancel{\frac{\partial w}{\partial z}}^{0}\right) = -\frac{\partial p}{\partial z} + \rho \cancel{g_z}^{0} + \mu\left(\cancel{\frac{\partial^2 w}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 w}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 w}{\partial z^2}}^{0}\right)$

$$\frac{\partial p}{\partial z} = 0$$

Supongamos que no hay gradiente de presión en el eje 'x', entonces:

$$\frac{\partial^2 u}{\partial y^2} = 0 \Rightarrow u(y) = C_1 y + C_2$$

■ Condiciones de borde: $u(0) = 0 \Rightarrow C_2 = 0$

$$u(h) = \frac{D}{2}\Omega = C_1 \cdot h \Rightarrow C_1 = \frac{D\Omega}{2h}$$

$$\Rightarrow u(y) = \frac{D\Omega}{2h}\, y$$

eio, podemos calcular el esfuerzo de corte como:

$$\tau_w = \tau_{xy} = \mu \left(\frac{\partial u}{\partial y} \right) \Big|_{y=h} \hat{\imath} = \frac{\mu D \Omega}{2h} \hat{\imath}$$

Esfuerzo de corte que actúa en la capa superior

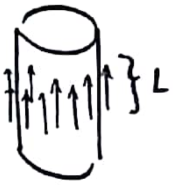
$$\Rightarrow \tau_w = \frac{\mu D \Omega}{2h}$$

La torsión se calcula como:

$$T = \int_A \tau \, r \, dA$$

r: $D/2$

dA: Área del manto del eje



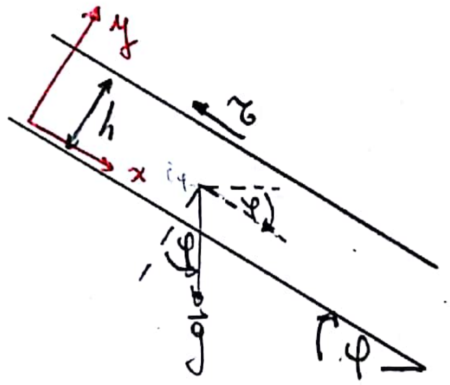
$$= \int_0^L \int_0^{\pi} \tau \frac{D}{2} (D \, d\theta) dz = \frac{\mu D \Omega}{2h} \cdot \frac{D}{2} \cdot D \cdot \pi \cdot L$$

$$\Rightarrow T = \frac{\pi \mu D^3 \Omega}{4h} L = 1.924 \times 10^{-2} \, [N \times m]$$

La potencia por otro lado es:

$$P = T \Omega = 7.255 \, [W]$$

Pregunta N° 2 :



Supuestos :

- Flujo estacionario
- Campo de velocidad sólo en la componente $\hat{\imath}$

$\downarrow \vec{g}$

Ecuación de continuidad: $\cancel{\dfrac{\partial \rho}{\partial t}}^{0} + \nabla \circ (\rho \vec{v}) = 0$

$$\frac{\partial u}{\partial x} = 0 \Rightarrow u = u(y). \rightsquigarrow \vec{v} = u(y)\,\hat{\imath}$$

$$\vec{g} = (g\,\text{sen}(\varphi); -g\cos(\varphi); 0)$$

Ecuación de Navier - stokes:

$$\hat{\imath}: \quad \rho\left(\cancel{\frac{\partial u}{\partial t}}^{0} + u\cancel{\frac{\partial u}{\partial x}}^{0} + \cancel{v}^{0}\frac{\partial u}{\partial y} + \cancel{w}^{0}\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{0} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}^{0}\right)$$

$$-\frac{\partial p}{\partial x} + \rho g\,\text{sen}(\varphi) + \mu\frac{\partial^2 u}{\partial y^2} = 0$$

$$\hat{\jmath}: \quad \rho\left(\cancel{\frac{\partial v}{\partial t}}^{0} + u\cancel{\frac{\partial v}{\partial x}}^{0} + \cancel{v}^{0}\frac{\partial v}{\partial y} + \cancel{w}\cancel{\frac{\partial v}{\partial z}}^{0}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\cancel{\frac{\partial^2 v}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 v}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 v}{\partial z^2}}^{0}\right)$$

$$\frac{\partial p}{\partial y} = -\rho g\cos(\varphi)$$

$$\hat{k}: \quad \rho\left(\cancel{\frac{\partial w}{\partial t}}^{0} + u\cancel{\frac{\partial w}{\partial x}}^{0} + v\cancel{\frac{\partial w}{\partial y}}^{0} + w\cancel{\frac{\partial w}{\partial z}}^{0}\right) = -\frac{\partial p}{\partial z} + \cancel{\rho g_z}^{0} + \mu\left(\cancel{\frac{\partial^2 w}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 w}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 w}{\partial z^2}}^{0}\right)$$

$$-\frac{\partial p}{\partial z} = 0$$

Además, para este caso: $\dfrac{\partial p}{\partial x} = 0$

...ando las condiciones de Borde:

$u(0)=0 \quad \wedge \quad \frac{\partial u}{\partial y}(y=h)\cdot\mu = -\tau$

$$\Rightarrow \quad \frac{\partial^2 u}{\partial y^2} = -\frac{\rho}{\mu} g\, \mathrm{sen}(\varphi)$$

$$\frac{\partial u}{\partial y} = -\frac{\rho}{\mu} g\, \mathrm{sen}(\varphi)\, y + C_1 \Big|_{y=h} = -\frac{\tau}{\mu}$$

$$-\frac{\rho g}{\mu}\mathrm{sen}(\varphi) h + C_1 = -\frac{\tau}{\mu}$$

$$C_1 = \frac{1}{\mu}\left[\rho g\, \mathrm{sen}(\varphi) h - \tau\right]$$

$$u(y) = -\frac{\rho g}{2\mu}\mathrm{sen}(\varphi) y^2 + \frac{1}{\mu}\left[\rho g\, \mathrm{sen}(\varphi) h - \tau\right] y + C_2.$$

$$u(y=0) = C_2 = 0$$

$$\Rightarrow u(y) = \frac{y}{\mu}\left(\rho g h\, \mathrm{sen}(\varphi) - \tau - \frac{\rho g y}{2}\mathrm{sen}(\varphi)\right)$$

$$\frac{Q}{W} = \int_0^h u(y)\,dy = \int_0^h \left[\frac{\rho g h}{\mu}\mathrm{sen}(\varphi) - \tau\right] y\, dy - \int_0^h \frac{\rho g y^2}{2\mu}\mathrm{sen}(\varphi)\, dy$$

$$= \left[\frac{\rho g h}{\mu}\mathrm{sen}(\varphi) - \tau\right]\frac{h^2}{2} - \frac{\rho g h^3}{6\mu}\mathrm{sen}(\varphi) \overset{!}{=} 0$$

$$\frac{3\cdot \rho g h}{3\cdot 2\mu}\mathrm{sen}(\varphi) - \tau - \frac{\rho g h}{6\mu}\mathrm{sen}(\varphi) = 0$$

$$\frac{\cancel{2}\rho g h}{3\cancel{6}\mu}\mathrm{sen}(\varphi) = \tau \rightarrow h = \frac{3\mu\tau}{\rho g\, \mathrm{sen}(\varphi)}$$