

Desigualdad final chapter 3

En el paper sale la siguiente desigualdad:

$$\int_{-T}^0 \int_{\Omega} P_1 w \frac{\partial w}{\partial t} dx dt \geq \frac{1}{2} \int_{\Omega} e^{2s\phi_{\lambda}(0)} |f|^2 a_0^2 dx - C s^2 \lambda^3 \int_{-T}^T \int_{\Omega} \phi_{\lambda}^3 |w|^2 dx dt. \quad (1)$$

Luego:

$$\frac{1}{2} \int_{\Omega} e^{2s\phi_{\lambda}(0)} |f|^2 a_0^2 dx \leq C s^2 \lambda^3 \int_{-T}^T \int_{\Omega} \phi_{\lambda}^3 |w|^2 dx dt + \int_{-T}^0 \int_{\Omega} P_1 w \frac{\partial w}{\partial t} dx dt. \quad (2)$$

Aplicando $2ab \leq a^2 + b^2$ a uno de los términos:

$$\int_{-T}^0 \int_{\Omega} P_1 w \frac{\partial w}{\partial t} dx dt = \int_{-T}^0 \int_{\Omega} (s^{-1/4} P_1 w) (s^{1/4} \frac{\partial w}{\partial t}) dx dt \leq C s^{-1/2} \int_{-T}^0 \int_{\Omega} (P_1 w)^2 dx dt + C s^{1/2} \int_{-T}^0 \int_{\Omega} (\frac{\partial w}{\partial t})^2 dx dt$$

Luego reemplazando en (2):

$$\frac{1}{2} \int_{\Omega} e^{2s\phi_{\lambda}(0)} |f|^2 a_0^2 dx \leq C s^2 \lambda^3 \int_{-T}^T \int_{\Omega} \phi_{\lambda}^3 |w|^2 dx dt + C s^{-1/2} \int_{-T}^0 \int_{\Omega} (P_1 w)^2 dx dt + C s^{1/2} \int_{-T}^0 \int_{\Omega} (\frac{\partial w}{\partial t})^2 dx dt. \quad (3)$$

Por otro lado, tenemos la otra ecuación del paper:

$$\begin{aligned} s^3 \lambda^3 \int_{-T}^T \int_{\Omega} \phi_{\lambda}^3 |w|^2 dx dt + \int_{-T}^0 \int_{\Omega} (P_1 w)^2 dx dt + s \lambda \int_{-T}^0 \int_{\Omega} (\frac{\partial w}{\partial t})^2 dx dt \\ \leq C \int_{\Omega} e^{2s\phi_{\lambda}(0)} |f|^2 dx + C s \lambda \int_{-T}^T \int_{\Gamma_0} \phi_{\lambda} e^{2s\phi_{\lambda}(0)} \left| \frac{\partial v}{\partial \nu} \right|^2 d\sigma dt \end{aligned}$$

Que al multiplicarla por $s^{-1/2}$ y ocupando (3) se concluye (para $\lambda > 1$):

$$\int_{\Omega} e^{2s\phi_{\lambda}(0)} |f|^2 a_0^2 dx \leq \frac{C}{\sqrt{s}} \int_{\Omega} e^{2s\phi_{\lambda}(0)} |f|^2 dx + C \sqrt{s} \lambda \int_{-T}^T \int_{\Gamma_0} \phi_{\lambda} e^{2s\phi_{\lambda}(0)} \left| \frac{\partial v}{\partial \nu} \right|^2 d\sigma dt \quad (4)$$