

P3 $f(x) = e^{-x} \quad x \in [0, 2]$

$$f(x) = \begin{cases} 1 & \text{si } x \in [0, 1) \\ e^{-1} & \text{si } x \in [1, 2) \\ e^{-2} & \text{si } x = 2 \end{cases}$$

como queremos la serie en cosenos, la extendemos de forma par.

$$\tilde{f}(x) = \begin{cases} f(x) & \text{si } x \in [0, 2] \\ f(-x) & \text{si } x \in [-2, 0) \end{cases}$$

$$a_0 = \frac{1}{2} \int_{-2}^2 \tilde{f}(x) dx = \int_0^2 \tilde{f}(x) dx = \int_0^2 f(x) dx = \int_0^1 dx + \int_1^2 e^{-x} dx = 1 + e^{-1}$$

$\tilde{f}$ Par

$$a_k = \frac{1}{2} \int_{-2}^2 \tilde{f}(x) \cos\left(\frac{k\pi x}{2}\right) dx = \int_0^2 \tilde{f}(x) \cos\left(\frac{k\pi x}{2}\right) dx = \int_0^2 f(x) \cos\left(\frac{k\pi x}{2}\right) dx$$

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$$= \int_0^1 \cos\left(\frac{k\pi x}{2}\right) dx + e^{-1} \int_1^2 \cos\left(\frac{k\pi x}{2}\right) dx$$

$$= \frac{\sin\left(\frac{k\pi x}{2}\right)}{\frac{k\pi}{2}} \Big|_0^1 + e^{-1} \frac{\sin\left(\frac{k\pi x}{2}\right)}{\frac{k\pi}{2}} \Big|_1^2 = \frac{2}{k\pi} (1 - e^{-1}) \sin\left(\frac{k\pi}{2}\right)$$

$$\therefore f(x) = \frac{1+e^{-1}}{2} + \sum_{k=1}^{\infty} \frac{2}{k\pi} (1-e^{-1}) \sin\left(\frac{k\pi}{2}\right) \cos\left(\frac{k\pi x}{2}\right)$$