P2) a) sea $u = v + w$

$$u_{xx} + u_{yy} = v_{xx} + v_{yy} + w_{xx} + w_{yy} = 0+0 = 0$$
$$u(0,y) = v(0,y) + w(0,y) = 0+0 = 0$$
$$u(1,y) = v(1,y) + w(1,y) = 1+0 = 1$$
$$u(x,0) = v(x,0) + w(x,0) = 0+0 = 0$$
$$u(x,1) = v(x,1) + w(x,1) = 0+1 = 1$$

$$u_{xx} + u_{yy} = 0$$
$$u(0,y) = 0 \quad u(1,y) = 1$$
$$u(x,0) = 0 \quad u(x,1) = 1$$

$\therefore$ $u = v + w$ es solución de (P).

b) (P_v)

$$v(x,y) = X(x)Y(y) \Rightarrow v_{xx} + v_{yy} = X''(x)Y(y) + X(x)Y''(y) = 0$$

$$\frac{X''(x)}{X(x)} = -\frac{Y''(y)}{Y(y)} = \lambda \Rightarrow \begin{cases} X'' - \lambda X = 0 \\ Y'' + \lambda Y = 0 \end{cases}$$ ← analizamos esta primero

- $Y'' + \lambda Y = 0$

(CB) $v(x,0) = 0 = X(x)Y(0) \Rightarrow Y(0) = 0$
$v(x,1) = 0 = X(x)Y(1) \Rightarrow Y(1) = 0$

caso 1 $\lambda = 0$

$Y'' = 0 \Rightarrow Y(y) = Ay + B$
$Y(0) = 0 \Rightarrow B = 0 \Rightarrow Y(y) = Ay$
$Y(1) = 0 \Rightarrow A = 0 \Rightarrow Y(y) = 0$ se descarta

caso 2 $\lambda < 0$

$Y(y) = A\cosh(\sqrt{-\lambda}y) + B\operatorname{senh}(\sqrt{-\lambda}y)$
$Y(0) = 0 = A \Rightarrow Y(y) = B\operatorname{senh}(\sqrt{-\lambda}y)$
$Y(1) = 0 = B\operatorname{senh}(\sqrt{-\lambda}) \to \sqrt{-\lambda} = 0 \to\leftarrow \lambda < 0$ se descarta

caso 3 $\lambda > 0$

$Y(y) = A\cos(\sqrt{\lambda}y) + B\operatorname{sen}(\sqrt{\lambda}y)$
$Y(0) = 0 = A \Rightarrow Y(y) = B\operatorname{sen}(\sqrt{\lambda}y)$
$Y(1) = 0 = B\operatorname{sen}(\sqrt{\lambda}) \Rightarrow \sqrt{\lambda} = k\pi \Rightarrow \lambda = (k\pi)^2 \quad k \in \mathbb{N}\setminus\{0\}$

$\therefore$ $Y_k = B_k \operatorname{sen}(k\pi y) \quad , \quad k \geq 1$

- $X'' - (k\pi)^2 X = 0 \Rightarrow X_k(x) = C_k \cosh(k\pi x) + D_k \operatorname{senh}(k\pi x)$

por principio de superposición

$$v(x,y) = \sum_{k\geq 1} X_k(x) Y_k(y) = \sum_{k \geq 1} (C_k \cosh(k\pi x) + D_k \operatorname{senh}(k\pi x)) B_k \operatorname{sen}(k\pi y)$$

$$= \sum_{k\geq 1} \left(\tilde{C}_k \cosh(k\pi x) + \tilde{D}_k \operatorname{senh}(k\pi x) \right) \operatorname{sen}(k\pi y)$$

$$v(0,y)=0=\sum_{k\geq 1}\tilde{C}_k \operatorname{sen}(k\pi y) \Rightarrow \tilde{C}_k=0 \quad \forall k$$

(pq la representación de Fourier es única)

$$\Rightarrow v(x,y)=\sum_{k\geq 1}\tilde{D}_k \operatorname{senh}(k\pi x)\operatorname{sen}(k\pi y)$$

$$v(1,y)=1=\underbrace{\sum_{k\geq 1}\overbrace{\tilde{D}_k \operatorname{senh}(k\pi)}^{b_k}\operatorname{sen}(k\pi y)}_{\text{serie de Fourier de } f(y)=1 \text{ en senos}} \Rightarrow \tilde{D}_k=\frac{b_k}{\operatorname{senh}(k\pi)}$$

Extendemos f de forma impar

$$\tilde{f}(y)=\begin{cases} f(y) & \text{si } y\in(0,1) \\ 0 & \text{si } y=0 \\ -f(-y) & \text{si } y\in(-1,0)\end{cases}$$

$$b_k=\underset{\uparrow\, L=1}{\frac{1}{1}}\int_{-1}^{1}\underbrace{\underset{\text{impar}}{\tilde{f}(y)}\,\underset{\text{impar}}{\operatorname{sen}(k\pi y)}}_{\text{par}}\,dy=2\int_0^1 f(y)\operatorname{sen}(k\pi y)\,dy$$

$$=2\int_0^1 \operatorname{sen}(k\pi y)\,dy=2\left[\frac{-\cos(k\pi y)}{k\pi}\right]\Big|_0^1$$

$$=2\left[\frac{-(-1)^k - -1}{k\pi}\right]=\frac{2}{k\pi}\left((-1)^{k+1}+1\right)$$

$$\therefore v(x,y)=\sum_{k\geq 1}\frac{2}{k\pi\operatorname{senh}(k\pi)}\left((-1)^{k+1}+1\right)\operatorname{senh}(k\pi x)\operatorname{sen}(k\pi y) \quad //$$

c) Igual que la parte (b) cambiando x por y

$$u(x,t)=v(x,t)+w(x,t) \quad //$$