PAUTA AUXILIAR 2

P1] $\vec{H} = (y^2 z^3,\ 2xyz^3,\ 3xy^2z^2 + y)$

$$\int_C \vec{H}\cdot d\vec{r} = \int_{t_0}^{t_f} \vec{H}(\vec{r}(t))\cdot \frac{d\vec{r}}{dt}\,dt$$ donde $\vec{r}(t)$ es la parametrización de C

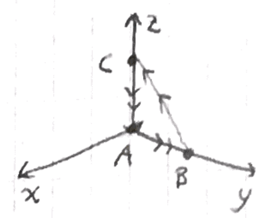
$$C = \overline{AB} + \overline{BC} + \overline{CA} \Rightarrow \int_C \vec{H}\cdot d\vec{r} = \int_{\overline{AB}} \vec{H}\cdot d\vec{r} + \int_{\overline{BC}} \vec{H}\cdot d\vec{r} + \int_{\overline{CA}} \vec{H}\cdot d\vec{r}$$

- Parametricemos $\overline{AB}$:

Para parametrizar un segmento entre dos puntos, se utiliza la siguiente parametrización:

$$\vec{r}(t) = \underset{\substack{\uparrow \\ \text{punto de partida}}}{A}\cdot(1-t) + \underset{\substack{\uparrow \\ \text{punto de llegada}}}{B}\cdot t = \begin{pmatrix}0\\0\\0\end{pmatrix}(1-t) + \begin{pmatrix}0\\1\\0\end{pmatrix}t = \begin{pmatrix}0\\t\\0\end{pmatrix} \quad t\in[0,1]$$

Notemos que cuando $t=0$ $\vec{r}(t) = A$ y $t=1 \Rightarrow \vec{r}(t) = B$



Entonces $\vec{r}(t) = (0, t, 0)$ $t\in[0,1]$

$$\frac{d\vec{r}}{dt} = (0,1,0)$$

$$\Rightarrow \int_{AB} \vec{H}\cdot d\vec{r} = \int_0^1 H(\vec{r}(t))\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}dt = \int_0^1 \begin{pmatrix}0\\0\\t\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}dt = 0 \,//$$

- Parametricemos $\overline{BC}$:

$$\vec{r}(t) = B(1-t) + Ct = \begin{pmatrix}0\\1\\0\end{pmatrix}(1-t) + \begin{pmatrix}0\\0\\1\end{pmatrix}t = \begin{pmatrix}0\\1-t\\t\end{pmatrix} \quad t\in[0,1]$$

$$\frac{d\vec{r}}{dt} = \begin{pmatrix}0\\-1\\1\end{pmatrix} \Rightarrow \int_{\overline{BC}} \vec{H}\cdot d\vec{r} = \int_0^1 \vec{H}(\vec{r}(t))\cdot\begin{pmatrix}0\\-1\\1\end{pmatrix}dt = \int_0^1 \begin{pmatrix}(1-t)^2t^3\\0\\1-t\end{pmatrix}\cdot\begin{pmatrix}0\\-1\\1\end{pmatrix}dt$$

$$= \int_0^1 1-t\ dt = \left(t - \frac{t^2}{2}\right)\Big|_0^1 = 1 - \frac{1}{2} = \frac{1}{2} \,//$$

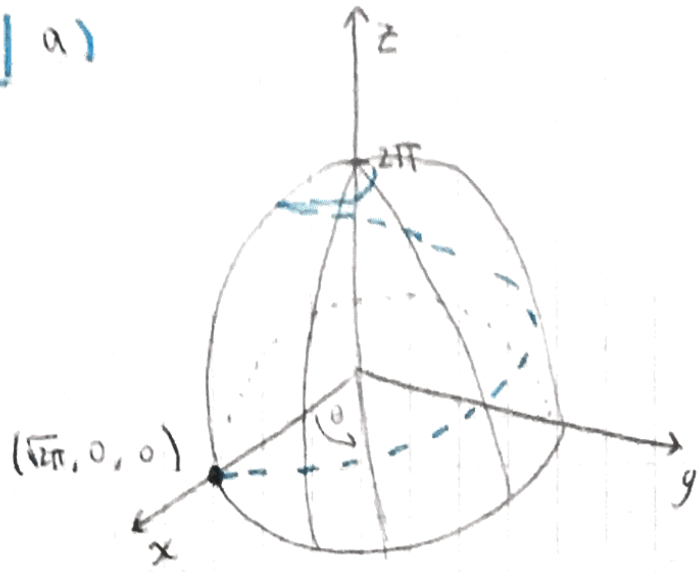
- Parametricemos $\overline{CA}$:

$$\vec{r}(t) = C(1-t) + At = \begin{pmatrix}0\\0\\1\end{pmatrix}(1-t) + \begin{pmatrix}0\\0\\0\end{pmatrix}t = \begin{pmatrix}0\\0\\1-t\end{pmatrix} \quad t\in[0,1]$$

$$\frac{d\vec{r}}{dt} = \begin{pmatrix}0\\0\\-1\end{pmatrix} \Rightarrow \int_{\overline{CA}} \vec{H}\cdot d\vec{r} = \int_0^1 \vec{H}(\vec{r}(t))\cdot\begin{pmatrix}0\\0\\-1\end{pmatrix}dt = \int_0^1 \begin{pmatrix}0\\0\\0\end{pmatrix}\cdot\begin{pmatrix}0\\0\\-1\end{pmatrix}dt = 0 \,//$$

Luego $$\int_C \vec{H}\cdot d\vec{r} = \int_{\overline{AB}} \vec{H}\cdot d\vec{r} + \int_{\overline{BC}} \vec{H}\cdot d\vec{r} + \int_{\overline{CA}} \vec{H}\cdot d\vec{r} = 0 + \frac{1}{2} + 0 = \frac{1}{2} \,//$$

P2 | a)



$x^2+y^2+z=2\pi$

si $x=0, y=0 \Rightarrow z=2\pi \Rightarrow$ altura 2π

si $z=0 \Rightarrow x^2+y^2=2\pi \Rightarrow$ Base es círculo de radio $\sqrt{2\pi}$.

como hay un x^2+y^2, nos conviene parametrizar en coordenadas cilíndricas

$$\vec{r}(\rho,\theta,z)=\begin{pmatrix}\rho\cos\theta\\ \rho\,\mathrm{sen}\,\theta\\ z\end{pmatrix}$$

como es una curva, necesitamos que dependa de sólo 1 variable.

$$\underbrace{x^2+y^2}_{\rho^2}+z=2\pi \Rightarrow \rho=\sqrt{2\pi-z}$$

además $\dfrac{dz}{d\theta}=a \Rightarrow z(\theta)=a\theta+b$

inicialmente la altura es 0 → $z(0)=0 \Rightarrow b=0 \Rightarrow z(\theta)=\theta \Rightarrow z=\theta$

después de dar una vuelta está a altura 2π → $z(2\pi)=2\pi \Rightarrow a=1$

entonces $\vec{r}(\theta)=\begin{pmatrix}\sqrt{2\pi-\theta}\cos\theta\\ \sqrt{2\pi-\theta}\,\mathrm{sen}\,\theta\\ \theta\end{pmatrix}$ // $\theta\in[0,2\pi]$

b) $F(x,y,z)=\left(\dfrac{y^2}{2},\ y(x-z),\ -\dfrac{y^2}{2}\right)$. si $\exists f$ tal que $F=\nabla f$

$$\int_\Gamma \vec{F}\cdot d\vec{r}=f(\vec{r}(\theta_f))-f(\vec{r}(\theta_0))$$

Encontremos f: si $F=\nabla f$, entonces

$\dfrac{\partial f}{\partial x}=\dfrac{y^2}{2}$ (1) $\quad \dfrac{\partial f}{\partial y}=y(x-z)$ (2) $\quad \dfrac{\partial f}{\partial z}=-\dfrac{y^2}{2}$ (3)

$\dfrac{\partial f}{\partial x}=\dfrac{y^2}{2} \Rightarrow f=\dfrac{y^2}{2}x+C(y,z)$ (4) ← constante que no depende de x

como ya tenemos esta información, la usaremos

de (4), de (2)

$\Rightarrow \dfrac{\partial f}{\partial y}=yx+\dfrac{\partial C(y,z)}{\partial y}=yx-yz \Rightarrow \dfrac{\partial C(y,z)}{\partial y}=-yz$

$\Rightarrow C(y,z)=-\dfrac{y^2}{2}z+C(z) \Rightarrow f=\dfrac{y^2}{2}x-\dfrac{y^2}{2}z+C(z)$ (5)

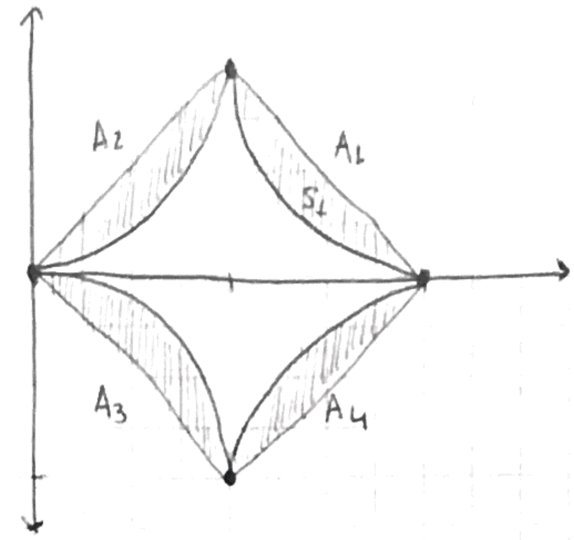
de (5), de (3)

$\Rightarrow \dfrac{\partial f}{\partial z}=-\dfrac{y^2}{2}+\dfrac{\partial C(z)}{\partial z}=-\dfrac{y^2}{2} \Rightarrow \dfrac{\partial C(z)}{\partial z}=0 \Rightarrow C(z)=C$

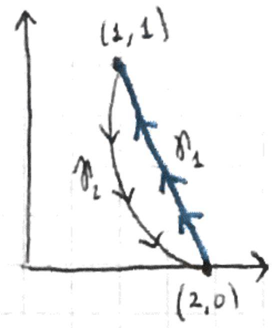
$\Rightarrow f=\dfrac{y^2}{2}x-\dfrac{y^2}{2}z+C$

$$\Rightarrow \int_\Gamma \vec{F}\cdot d\vec{r}=f(\vec{r}(2\pi))-f(\vec{r}(0))=f\begin{pmatrix}0\\0\\2\pi\end{pmatrix}-f\begin{pmatrix}\sqrt{2\pi}\\0\\0\end{pmatrix}=(0-0+C)-(0-0+C)=0 \quad //$$

P3



$A = A_1 + A_2 + A_3 + A_4 = 4 \cdot A_1 = 4 \cdot \iint_{S_1} dA = 4 \cdot \iint_{S_1} 1\, dA$



← en sentido anti-horario para usar Green.

usaremos Green pq' es más fácil parametrizar γ_1 y γ_2 que S_1.

TRUCO: encontrar $F = \begin{pmatrix} M \\ N \end{pmatrix}$ tal que $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 1$

como por ejemplo $\vec{F} = \begin{pmatrix} 0 \\ x \end{pmatrix}$

$$\Rightarrow A = 4 \iint_{S_1} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA \overset{\text{Green}}{=} 4 \int_{\gamma_1 + \gamma_2} \vec{F} \cdot d\vec{r} = 4 \int_{\gamma_1} \vec{F} \cdot d\vec{r} + 4 \int_{\gamma_2} \vec{F} \cdot d\vec{r}$$

- veamos $\int_{\gamma_1} \vec{F} \cdot d\vec{r} = \int_{t_0}^{t_f} \vec{F}(\vec{r}(t)) \cdot \frac{d\vec{r}}{dt}\, dt$

$$\vec{r}_{\gamma_1}(t) = \begin{pmatrix} 2 \\ 0 \end{pmatrix} (1-t) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} t \qquad t \in [0,1]$$

$$\vec{r}_{\gamma_1}(t) = \begin{pmatrix} 2(1-t) + t \\ t \end{pmatrix} = \begin{pmatrix} 2 - t \\ t \end{pmatrix} \qquad t \in [0,1]$$

$$\frac{d\vec{r}_{\gamma_1}}{dt} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \Rightarrow \int_{\gamma_1} \vec{F} \cdot d\vec{r} = \int_0^1 \vec{F}\begin{pmatrix} 2-t \\ t \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} dt$$

$$= \int_0^1 \begin{pmatrix} 0 \\ 2-t \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} dt = \int_0^1 2 - t = \left(2t - \frac{t^2}{2} \right) \Big|_0^1 = 2 - \frac{1}{2} = \frac{3}{2} \,//$$

- veamos $\int_{\gamma_2} \vec{F} \cdot d\vec{r} = \int_{t_0}^{t_f} \vec{F}(\vec{r}(t)) \cdot \frac{d\vec{r}}{dt}\, dt$

Tenemos que parametrizar γ_2, tenemos que encontrar $\vec{r}(t) = (x(t), y(t))$ tal que $(x-1)^{2/3} + y^{2/3} = 1$

imaginemos inicialmente que queremos que $x^{2/3} + y^{2/3} = 1$

una primera intuición sería $\vec{r}(t) = (\cos(t), \operatorname{sen}(t))$ pues $\cos^2(t) + \operatorname{sen}^2(t) = 1$, pero $\cos^{2/3}(t) + \operatorname{sen}^{2/3}(t) \neq 1$

Entonces podemos usar $\vec{r}(t) = (\cos^3(t), \operatorname{sen}^3(t))$ pues $(\cos^3(t))^{2/3} + (\operatorname{sen}^3(t))^{2/3} = 1$

Pero en realidad necesitamos que $(x-1)^{2/3} + y^{2/3} = 1$

Luego usamos $\vec{r}(t) = (\cos^3(t) + 1, \operatorname{sen}^3(t)) \qquad t \in [t_0, t_f]$

Falta encontrar t_0 y t_f. Sabemos que $\vec{r}(t_0) = (1,1) \Rightarrow \cos(t_0) = 0, \operatorname{sen}(t_0) = 1$
$\Rightarrow t_0 = \pi/2$

Por otro lado, $\vec{r}(t_f) = (2,0) \Rightarrow \cos(t_f) = 1, \operatorname{sen}(t_f) = 0 \Rightarrow t_f = 0$

$\frac{d\vec{r}}{dt} = (3\cos^2(t) \cdot -\operatorname{sen}(t), 3\operatorname{sen}^2(t)\cos(t))$

$$\Rightarrow \int_{\gamma_2} \vec{F} \cdot d\vec{r} = \int_{\pi/2}^{0} \vec{F}(\vec{r}(t)) \cdot d\vec{r}\, dt = - \int_0^{\pi/2} \begin{pmatrix} 0 \\ \cos^3(t) + 1 \end{pmatrix} \cdot \begin{pmatrix} -3\cos^2(t)\operatorname{sen}(t) \\ 3\operatorname{sen}^2(t)\cos(t) \end{pmatrix}$$

$$= -\underbrace{\int_0^{\pi/2} 3\cos^4(t)\,sen^2(t)\,dt}_{I} - \underbrace{\int_0^{\pi/2} 3\,sen^2(t)\cos(t)}_{=\left(sen^3(t)\right)\Big|_0^{\pi/2} = 1}$$

$$\Rightarrow A_1 = I - 1 \quad \Rightarrow \quad A = 4(I - 1)$$

- $sen(2t) = 2\,sen(t)\cos(t)$
- $\cos^2(t) = \frac{1+\cos(2t)}{2}$

veamos cuánto vale I

$$-\int_0^{\pi/2} 3\cos^4(t)\,sen^2(t)\,dt = -\frac{3}{4}\int_0^{\pi/2} (2\cos(t)\,sen(t))^2\cos^2(t)\,dt = -\frac{3}{4}\int_0^{\pi/2} sen^2(2t)\left(\frac{1+\cos(2t)}{2}\right)dt$$

$$= -\frac{3}{4}\left[\frac{1}{2}\int_0^{\pi/2} sen^2(2t)\,dt + \frac{1}{2}\int_0^{\frac{\pi}{2}} sen^2(2t)\cos(2t)\,dt\right]$$

$$= -\frac{3}{4}\left[\frac{1}{4}\int_0^{\frac{\pi}{2}} (1-\cos(4t))\,dt + \frac{1}{12}\int_0^{\pi/2}\frac{d}{dt}\left(sen^3(2t)\right)dt\right]$$

- $sen^2(2t) = \frac{1-\cos(4t)}{2}$
- $\frac{d\,(sen^3(2t))}{dt} = 6\,sen^2(2t)\cos(2t)$

$$= -\frac{3}{4}\left[\frac{1}{4}\left(\frac{\pi}{2} - \underbrace{\left(\frac{sen(2\pi) - sen(0)}{4}\right)}_{=0}\right) + \frac{1}{12}\underbrace{\left(sen^3(\pi) - sen^3(0)\right)}_{=0}\right] = -\frac{3\pi}{32}$$

$$\therefore\ A = 4\left(\frac{-3\pi}{32} - 1\right) = -\frac{3\pi}{8} - 4 \quad //$$

P4) $\vec{F}(x,y) = \underbrace{\dfrac{(x+y)}{x^2+y^2}}_{M}\,\hat{i} \quad \underbrace{-\dfrac{(x-y)}{x^2+y^2}}_{N}\,\hat{j}$

a) $$\frac{\partial N}{\partial x} = \frac{-1(x^2+y^2) - 2x(-x+y)}{(x^2+y^2)^2} = \frac{-x^2-y^2+2x^2-2xy}{(x^2+y^2)^2} = \frac{x^2-y^2-2xy}{(x^2+y^2)^2}$$

$$\frac{\partial M}{\partial y} = \frac{(x^2+y^2) - 2y(x+y)}{(x^2+y^2)^2} = \frac{x^2+y^2-2xy-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2-2xy}{(x^2+y^2)^2}$$

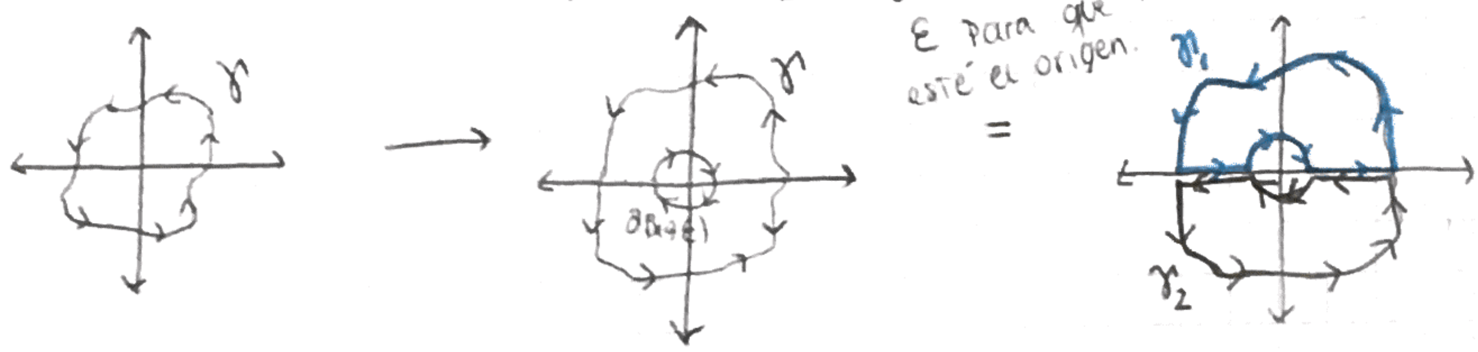
$$\Rightarrow \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0 \;//$$

b) caso 1: si γ no encierra el origen, se cumplen las hipótesis de Green

$$\Rightarrow I = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = 0 \quad \text{(por parte (a))}$$

caso 2: si γ encierra el origen no podemos aplicar Green directamente pues $(0,0)$ no está en el dominio de $\vec{F}$ entonces no podemos usar esa superficie tal y como está.

Entonces hacemos lo siguiente: sacamos un círculo de radio ε para que no esté el origen.



$$\Rightarrow \int_\gamma \vec{F}\cdot d\vec{r} + \int_{\partial B(0,\varepsilon)^-} \vec{F}\cdot d\vec{r} = \int_{\gamma_1} \vec{F}\cdot d\vec{r} + \int_{\gamma_2} \vec{F}\cdot d\vec{r}$$

pq' es en sentido horario

Pero γ_1 y γ_2 no encierran el origen, por lo que podemos usar Green

$$\Rightarrow \int_{\gamma_1} \vec{F}\cdot d\vec{r} + \int_{\gamma_2} \vec{F}\cdot d\vec{r} = 0 + 0 = 0$$

$$\Rightarrow \int_\gamma \vec{F}\cdot d\vec{r} - \int_{\partial B(0,\varepsilon)} \vec{F}\cdot d\vec{r} = 0 \quad \Rightarrow \quad \overbrace{\int_\gamma \vec{F}\cdot d\vec{r}}^{I} = \int_{\partial B(0,\varepsilon)} \vec{F}\cdot d\vec{r}$$

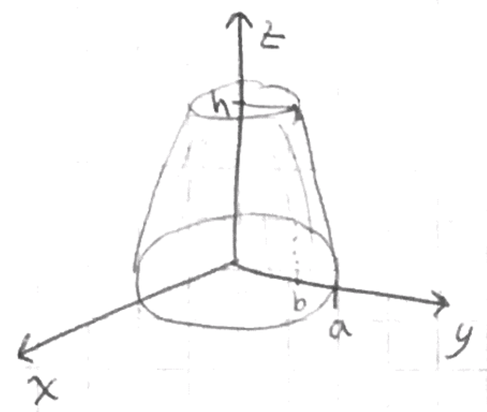
Luego basta integrar sobre $\partial B(0,\varepsilon)$

$$\vec{r}(t) = (\varepsilon\cos(t), \varepsilon\,\mathrm{sen}(t)) \quad t \in [0, 2\pi]$$
$$\frac{d\vec{r}}{dt} = (-\varepsilon\,\mathrm{sen}(t), \varepsilon\cos(t))$$

$$\int_{\partial B(0,\varepsilon)} \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \vec{F}\begin{pmatrix}\varepsilon\cos(t)\\ \varepsilon\,\mathrm{sen}(t)\end{pmatrix} \cdot \begin{pmatrix}-\varepsilon\,\mathrm{sen}(t)\\ \varepsilon\cos(t)\end{pmatrix} dt = \int_0^{2\pi} \begin{pmatrix}\dfrac{\varepsilon(\cos(t)+\mathrm{sen}(t))}{\varepsilon^2}\\ \dfrac{-\varepsilon(\cos(t)-\mathrm{sen}(t))}{\varepsilon^2}\end{pmatrix} \cdot \begin{pmatrix}-\varepsilon\,\mathrm{sen}(t)\\ \varepsilon\cos(t)\end{pmatrix} dt$$

$$= \int_0^{2\pi} -\cos(t)\,\mathrm{sen}(t) - \mathrm{sen}^2(t) - \cos^2(t) + \cancel{\cos(t)\,\mathrm{sen}(t)}\; dt = \int_0^{2\pi} -1 = -2\pi \;//$$

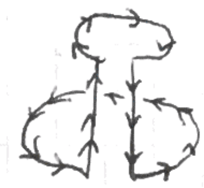
15 $\vec{F} = (y, z, x^2)$



Stokes

$$\iint_S \text{rot}(F) \cdot d\vec{s} = \oint_{\partial S} \vec{F} \cdot d\vec{r}$$

∂S



dirección tal que con regla de la mano derecha la normal apunte hacia afuera. en el manto

los segmentos rectos se anulan

Luego sólo debemos integrar sobre el círculo de radio b y el de radio a

$\vec{r}_b(t) = (b\cos(t), b\,\text{sen}(t), h) \quad t_0 = 2\pi, \ t_f = 0$ (pues va en sentido horario)

$\dfrac{d\vec{r}_b}{dt} = (-b\,\text{sen}(t), b\cos(t), 0)$

$$\int_{\partial B(0,b)} \vec{F} \cdot d\vec{r} = \int_{2\pi}^{0} \begin{pmatrix} b\,\text{sen}(t) \\ h \\ b^2\cos^2(t) \end{pmatrix} \cdot \begin{pmatrix} -b\,\text{sen}(t) \\ b\cos(t) \\ 0 \end{pmatrix} = -\int_0^{2\pi} -b^2\text{sen}^2(t) + hb\cos(t) \, dt$$

$$= b^2 \int_0^{2\pi} \text{sen}^2(t) - hb \underbrace{\int_0^{2\pi} \cos(t)}_{\text{sen}(t)\big|_0^{2\pi} = 0} = \frac{b^2}{2} \int_0^{2\pi} 1 - \cos(2t) \, dt$$

$$= \frac{b^2}{2} \cdot 2\pi - \frac{b^2}{2} \cdot \underbrace{\frac{\text{sen}(2t)}{2}\bigg|_0^{2\pi}}_{=0} = b^2\pi$$

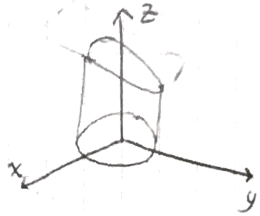
notemos que no depende de la altura, luego análogamente

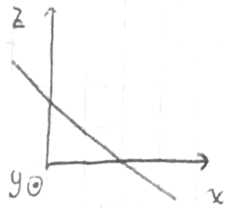
$$\int_{\partial B(0,a)} \vec{F} \cdot d\vec{r} = -a^2\pi$$

pq' se recorre en sentido opuesto

$$\therefore \iint_S \text{rot}(F) \, d\vec{s} = \pi(b^2 - a^2) \ //$$

P6) a) $F = \begin{pmatrix} x-z \\ z-y \\ x-y \end{pmatrix}$





$x^2+y^2=1 \Rightarrow \rho^2 = 1 \Rightarrow \rho = 1$

$x+z=1 \Rightarrow z = 1-x = 1-\rho\cos\theta = 1-\cos\theta$

$\vec{r}(\theta) = \begin{pmatrix} \cos(\theta) \\ \operatorname{sen}(\theta) \\ 1-\cos(\theta) \end{pmatrix} \Rightarrow \frac{d\vec{r}}{d\theta} = \begin{pmatrix} -\operatorname{sen}\theta \\ \cos\theta \\ \operatorname{sen}\theta \end{pmatrix}$

$\theta \in [0, 2\pi]$

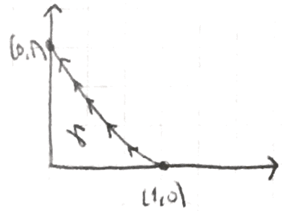
$$\int_\Gamma \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \begin{pmatrix} \cos\theta-(1-\cos\theta) \\ 1-\cos\theta-\operatorname{sen}\theta \\ \cos\theta-\operatorname{sen}\theta \end{pmatrix} \cdot \begin{pmatrix} -\operatorname{sen}\theta \\ \cos\theta \\ \operatorname{sen}\theta \end{pmatrix} d\theta = \int_0^{2\pi} \begin{pmatrix} 2\cos\theta-1 \\ 1-\cos\theta-\operatorname{sen}\theta \\ \cos\theta-\operatorname{sen}\theta \end{pmatrix} \cdot \begin{pmatrix} -\operatorname{sen}\theta \\ \cos\theta \\ \operatorname{sen}\theta \end{pmatrix} d\theta$$

$$= \int_0^{2\pi} -2\cos\theta\operatorname{sen}\theta + \operatorname{sen}\theta + \cos\theta - \cos^2\theta \cancel{-\cos\theta\operatorname{sen}\theta} \cancel{+\cos\theta\operatorname{sen}\theta} - \operatorname{sen}^2\theta \, d\theta$$

$$= \int_0^{2\pi} -\operatorname{sen}(2\theta) + \operatorname{sen}\theta + \cos\theta - 1 \, d\theta$$

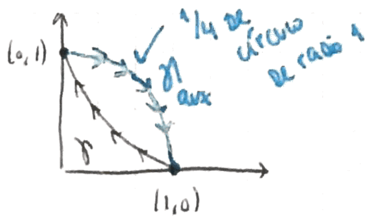
$$= \left[\frac{\cos(2\theta)}{2} - \cos\theta + \operatorname{sen}\theta - \theta\right]\Bigg|_0^{2\pi} = \left(\cancel{\tfrac{1}{2}} - \cancel{1} + 0 - 2\pi\right) - \left(\cancel{\tfrac{1}{2}} - \cancel{1} + 0 - 0\right) = -2\pi \;//$$

b)



no podemos usar Green porque no es una curva cerrada

¡Idea!: cerrarla



$\gamma_{aux}: \vec{r}(t) = \begin{pmatrix} \cos(t) \\ \operatorname{sen}(t) \end{pmatrix}$

La integral es la misma que la P4 en la parte (b) pero entre $\pi/2$ y 0 en vez de 0 y 2π

$\Rightarrow \int_{\gamma_{aux}} F\cdot dr = \frac{\pi}{2}$

por Green ✓

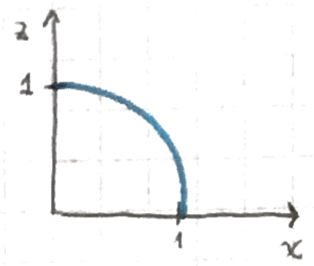
$\Rightarrow \int_{\gamma+\gamma_{aux}} F\cdot dr = 0 \Rightarrow \int_\gamma F\cdot d\vec{r} = -\int_{\gamma_{aux}} F\cdot dr$

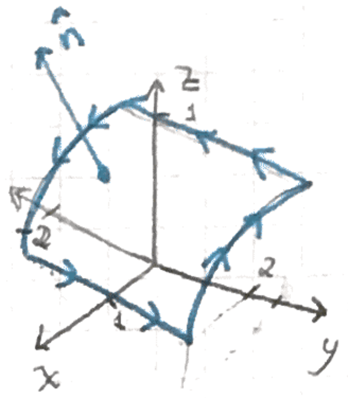
$\Rightarrow \int_\gamma \vec{F}\cdot d\vec{r} = -\pi/2 \;//$

c) $z = 1 - x^2 \quad 0 \le x \le 1 \quad -2 \le y \le 2 \quad ; \quad \vec{F} = (xy, yz, xz)$

$$\oint_{\partial S} \vec{F} \cdot d\vec{r} \underset{\text{Stokes}}{=} \iint_S \operatorname{rot}(\vec{F}) \cdot d\vec{A}$$

Bosquejo de S :





$$\operatorname{rot}(F) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & xz \end{vmatrix} = \hat{\imath}(0-y) - \hat{\jmath}(z-0) + \hat{k}(0-x) = (-y, -z, -x)$$

Parametricemos S : $\vec{r}(x,y,z) = (x, y, z) = \vec{r}(x,y) = (x, y, 1-x^2)$ $\quad x \in [0,1]$, $y \in [-2,2]$

$$\frac{\partial \vec{r}}{\partial x} = (1, 0, -2x) \; ; \; \frac{\partial \vec{r}}{\partial y} = (0, 1, 0) \Rightarrow \frac{\partial \vec{r}}{\partial x} \times \frac{\partial \vec{r}}{\partial y} = (2x, 0, 1)$$

$$\operatorname{rot}(F(\vec{r}(x,y))) = (-y, -(1-x^2), -x) = (-y, x^2-1, -x)$$

$$\iint_S \operatorname{rot}(F) \cdot dA = \int_{-2}^{2}\int_0^1 \operatorname{rot}(F(\vec{r}(x,y))) \cdot \frac{\partial \vec{r}}{\partial x} \times \frac{\partial r}{\partial y} \, dx\, dy = \int_{-2}^{2}\int_0^1 \begin{pmatrix} -y \\ x^2-1 \\ -x \end{pmatrix} \cdot \begin{pmatrix} 2x \\ 0 \\ 1 \end{pmatrix} dx\, dy$$

$$= \int_{-2}^{2}\int_0^1 -2xy - x \, dx\, dy = -2\int_{-2}^{2} y \left[\frac{x^2}{2} \right]\Big|_0^1 dy - \int_{-2}^{2} \left(\frac{x^2}{2} \right)\Big|_0^1 dy$$

$$= -\cancel{\left(\frac{y^2}{2} \right)\Big|_{-2}^{2}}^{\,0} - (2 - -2) \cdot \frac{1}{2} = -2 \; //$$