$\longrightarrow$ considerando el lado izquierdo de la igualdad

$Av_1 = A\alpha v_2 = \alpha\lambda_2 v_2 = \lambda_2 v_1$

↳ lo cual nos dice que v_1 esta asociado a λ_1 y λ_2 ($\longrightarrow\!\!\leftarrow$ contradicción)

$\therefore$ $v_1 + v_2$ NO es vector propio

d) $A = A^2 \Longrightarrow 0$ y 1 son sus únicos valores propios

intuición $\longrightarrow$ $A^2 v = \lambda^2 v$, además sabemos que $A = A^2 \,/\cdot v$

$$\left.\begin{aligned} Av &= A^2 v \\ \lambda v &= \lambda^2 v \end{aligned}\right] \quad \begin{aligned} &A^2 v - Av = 0 \\ &\underbrace{(\lambda^2 - \lambda)}\, v = 0 \\ &\lambda^2 - \lambda = \lambda(\lambda - 1) = 0 \\ &\therefore \underline{\underline{\lambda = 1 \vee \lambda = 0}} \end{aligned}$$

P4) $\longrightarrow$ no tan difícil 4 a) y b) juntas.

$v \in \mathbb{R} \longrightarrow v_1 = v_1 + v_2$ es único

$\in V_1 \,|\oplus|\, \in V_2$

$$\left.\begin{aligned} P_1(v) &= v_1 \\ P_2(v) &= v_2 \end{aligned}\right] \therefore P_1(v_1) = P_1(v_1 + 0) = v_1 \cdot 1$$

v_1 es vector propio asociado a 1

$P_1(v_2) = P_1(0 + v_2) = 0 \cdot v_2$

v_2 es vector propio asociado a 0

como α_i son arbitrarios

$$v_1 = \sum_{i=1}^{n} \alpha_i a_i \;;\; P_1(v_1) = \sum_{i=1}^{n} \alpha_i P_1(a_i) = v_1 = \sum_{i=1}^{n} \alpha_i a_i \longrightarrow P_1(a_i) = a_i \;\forall i$$

$\Longrightarrow$ Para el caso de P_2, se debe realizar de la misma forma, pero cambiando el orden de los elementos

$$v_2 = \sum_{i=1}^{m} \beta_i b_i \;;\; P_1(v_2) = \sum_{i=1}^{m} \beta_i P_1(b_i) = 0 \Longrightarrow P_1(b_i) = 0 \;\forall i$$

Auxiliar # 12
(álgebra lineal)

$y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \quad w = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$

P1] $A = xy^t$

a) $w \in y^\perp \Longleftrightarrow w \in \mathbb{R}^n \wedge \boxed{\langle y, w \rangle = 0}$

$\langle y, w \rangle = \sum_{i=1}^{n} y_i w_i = 0 \longrightarrow y_k w_k + \sum_{i=1}^{n} y_i w_i = 0$

SPG. asumimos que $y_n \neq 0$

$\boxed{w_k = -\sum_{i=1}^{n} \frac{y_i}{y_k} w_i}$

$y = \sum_{i=1}^{n} y_i e_i \;;\; w = \sum_{i=1}^{n} w_i e_i$

$w = \sum_{\substack{i=1 \\ i \neq k}}^{n} w_i e_i + w_k y_k \Rightarrow w = \sum_{\substack{i=1 \\ i \neq k}}^{n} w_i e_i + \overbrace{\left(\sum_{\substack{i=1 \\ i \neq k}}^{n} \frac{y_i}{y_k} w_i \right)}^{w_k} e_k = \sum_{\substack{i=1 \\ i \neq k}}^{n} w_i \overbrace{\left(e_i - \frac{y_i}{y_k} e_k \right)}$

generador ∧ base $(y^\perp)$ $\left\{ e_i - \frac{y_i}{y_k} e_k \right\}_{\substack{i=1 \\ i \neq k}}^{n}$

$\dim(y^\perp) = n - 1$

b) $w \in y^\perp$ ¿$Aw = ?$ 0

$Aw = x(y^t w) = x \underbrace{\langle y, w \rangle}_{0} = 0 = 0 \cdot w$

0 es valor propio asociado a w

✓ ¿$\dim(W_0)$? (adelanto) $\longrightarrow \dim(y^\perp) = n-1$

si $w \in y^\perp \Rightarrow w \in W_0 \Rightarrow y^\perp \subseteq W_0$

$\dim(W_0) = n - 1$

c) A es diagonalizable $\Longleftrightarrow x \notin y^\perp$

(contraintuitivo a causa de que al ver la implicancia $\Rightarrow$] no hay datos suficientes para llegar a que $x \notin y^\perp$)

$\Leftarrow$] $x \notin y^\perp \Rightarrow$ A es diagonalizable

$A = xy^t \Rightarrow Ax = x \overbrace{\langle y, x \rangle}^{\lambda \neq 0 \text{ (ya que son } \parallel)} \Rightarrow Ax = \lambda x$ $x \notin y^\perp \longrightarrow$ valor propio

* al menos $\exists$ un vector extra de los que ya tenía.

$\dim(W_\lambda) \geq 1$

$n = \dim(\mathbb{R}^n) \geq \dim(W_\lambda) + \dim(W_0) \geq n - \cancel{1} + \cancel{1} = n$

$\therefore$ A es diagonalizable

$\Rightarrow$] A es diagonalizable $\Rightarrow x \notin y^\perp \Longleftrightarrow$ (contrarecíproca) $\boxed{x \in y^\perp \Rightarrow A \text{ NO es diagonalizable}}$

• $Ax = x \overbrace{\langle y, x \rangle}^{0} = 0$; tomemos $w \in \mathbb{R}^n$ tq $\boxed{w \notin y^\perp}$

(no me sirve x) ya lo tenemos

$Aw = x \overbrace{\langle y, w \rangle}^{\lambda}$ $\longleftarrow$ Buscamos $\vec{vp}$ fuera de los que tenemos

– si $x = \lambda w$ (contradicción)

$\langle y, w \rangle = \langle y, \lambda x \rangle = \lambda \langle y, x \rangle = 0$ { ya que $w \in y^\perp$

A al ser $\parallel$ a x, no puede ser paralelo a w.

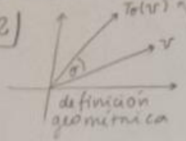
– si x no es paralelo a w

– si $Aw = \alpha w = \lambda x \longrightarrow \boxed{w = \frac{\lambda}{\alpha} x}$ (contradicción)

$\therefore w$ no puede ser vector propio $\Rightarrow$ hay $n-1$ vectores propio

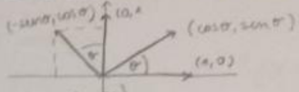
$\therefore$ A no es diagonalizable

P2)



a) $T_\theta\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cos\theta\\ \operatorname{sen}\theta\end{pmatrix}$

$\wedge\ T_\theta\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-\operatorname{sen}\theta\\ \cos\theta\end{pmatrix}$



$\Longrightarrow A_\theta = \begin{pmatrix}\cos\theta & -\operatorname{sen}\theta\\ \operatorname{sen}\theta & \cos\theta\end{pmatrix}$

b) $P(\lambda) = |A_\theta - \lambda I| = \begin{vmatrix}\cos\theta-\lambda & -\operatorname{sen}\theta\\ \operatorname{sen}\theta & \cos\theta-\lambda\end{vmatrix} = (\cos\theta-\lambda)^2 + \operatorname{sen}^2\theta$

$P(\lambda) = \underline{\cos^2\theta} - 2\lambda\cos\theta + \lambda^2 + \underline{\operatorname{sen}^2\theta}$

$= \lambda^2 - 2\lambda\cos\theta + 1 = 0 \longrightarrow \lambda = \dfrac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$

$* \cos^2\theta - 1 = -\operatorname{sen}^2\theta *$

$= \underline{\cos\theta \pm i\operatorname{sen}\theta}$

no se encuentran def en los $\mathbb{R}$

consideramos que estamos en $\mathbb{C}$

c) $L_\theta(v) = A_\theta\begin{pmatrix}z_1\\z_2\end{pmatrix} \longrightarrow \lambda_1 = \cos\theta + i\operatorname{sen}\theta$

$\lambda_2 = \cos\theta - i\operatorname{sen}\theta$

⊙ $L_\theta(v) = A_\theta\begin{pmatrix}z_1\\z_2\end{pmatrix} - \lambda_1\begin{pmatrix}z_1\\z_2\end{pmatrix}$

$\begin{pmatrix}0\\0\end{pmatrix} = (A_\theta - \lambda_1 I)\begin{pmatrix}z_1\\z_2\end{pmatrix}$

$\begin{pmatrix}i\operatorname{sen}\theta & -\operatorname{sen}\theta\\ \operatorname{sen}\theta & -i\operatorname{sen}\theta\end{pmatrix}\begin{pmatrix}z_1\\z_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \Longrightarrow -\operatorname{sen}\theta(iz_1 + z_2) = 0$

$\operatorname{sen}\theta(z_1 - iz_2) = 0$

$\boxed{-iz_1 = z_2}$

$z_1 = iz_2 \ /\cdot i$

$iz_1 = -z_2$

$v_1 = \begin{pmatrix}z_1\\z_2\end{pmatrix} = z_1\begin{pmatrix}1\\-i\end{pmatrix}$

asociado a λ_1

$\boxed{\theta \neq n\pi}$

⊙ v_2 asociado a λ_2

$A_\theta v_2 = \lambda_2 v_2$

$\Rightarrow (A_\theta - \lambda_2 I)v_2 = 0$

$\begin{pmatrix}i\operatorname{sen}\theta & -\operatorname{sen}\theta\\ \operatorname{sen}\theta & i\operatorname{sen}\theta\end{pmatrix}\begin{pmatrix}z_1\\z_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$\operatorname{sen}\theta(iz_1 - z_2) = 0$

$\operatorname{sen}\theta(z_1 + iz_2)$

$\boxed{iz_1 = z_2}$

$z_1 = -iz_2 \ /\cdot i$

$\boxed{iz_1 = z_2}$

$v_2 = \begin{pmatrix}z_1\\z_2\end{pmatrix} = z_1\begin{pmatrix}1\\i\end{pmatrix}$

P3) a) $A, B \in M_{nn,\mathbb{R}}$ A inventible.

sabiendo : $|AB| = |A|\cdot|B|$

$P_{AB} = |AB - \lambda I||\hat{I}| = |AB - \lambda I||A\cdot A^{-1}| = |AB - \lambda I||A||A^{-1}| = |A^{-1}(AB - \lambda I)A|$

$= |(\underbrace{\cancel{A^{-1}}A}_{I}\cdot B\cdot A - \lambda\underbrace{\cancel{A^{-1}}A}_{I})| = |BA - \lambda I| = P_{AB}(\lambda)$

b) $\{v_i\}_{i=1}^n$ son los vectores propios para A, estan asociados a α_i

para B estan asociados a β_i

→ cuál es el vp de $A^3 + 2B$ asociado a v_i?

$(A^3 + 2B)v_i = \underline{\underline{A^3 v_i + 2Bv_i}} = \alpha_i^3 v_i + 2\beta_i v_i = (\alpha_i^3 + 2\beta_i)v_i$

$A^3 v_i = A^2\cdot Av_i = A^2\cdot\alpha_i v_i = \alpha_i\cdot A\cdot Av_i = \alpha_i^2\cdot Av_i = \alpha_i^3 v_i$

c) $\left.\begin{array}{l} Av_1 = \lambda_1 v_1 \\ Av_2 = \lambda_2 v_2 \end{array}\right|$ $\lambda_1 \neq \lambda_2$

$\Downarrow$

$\underbrace{\lambda_1 - \lambda \neq \lambda - \lambda_2}$

alguno de ellos es $\neq 0$

→ supongamos que $v_1 + v_2$ es $\vec{vp}$ (x contradicción)

$A(v_1 + v_2) = \lambda(v_1 + v_2)$

$= Av_1 + Av_2$

$= \lambda_1 v_1 + \lambda_2 v_2$

$\lambda_1 v_1 + \lambda_2 v_2 = \lambda(v_1 + v_2)$

$\Rightarrow (\lambda_1 - \lambda)v_1 = (\lambda - \lambda_2)v_2$

$\Rightarrow v_1 = \boxed{\dfrac{(\lambda - \lambda_2)}{(\lambda_1 - \lambda)}}\, v_2$

vemos que son //

a) Sea $\{u_i\}_{i=1}^{m}$ base de U $\quad P_U(x) = \sum_{i=1}^{m} \frac{\langle x, u_i \rangle u_i}{\|u_i\|^2}$

Sea $\alpha, \beta \in \mathbb{K}$; $x, y \in \mathbb{R}^n$ arbitrarios

$\hookrightarrow$ PDQ- $P_U(\alpha x + \beta y) = \alpha P_U(x) + \beta P_U(y)$

$$\longrightarrow P_U(\alpha x + \beta y) = \sum_{i=1}^{m} \frac{\langle \alpha x + \beta y, u_i \rangle u_i}{\|u_i\|^2}$$

$$= \underbrace{\sum_{i=1}^{m} \frac{\langle \alpha x, u_i \rangle u_i}{\|u_i\|^2}}_{\alpha P_U(x)} + \underbrace{\sum_{i=1}^{m} \frac{\langle \beta y, u_i \rangle u_i}{\|u_i\|^2}}_{\beta P_U(y)}$$

Sea $P_U(x), P_U(y) \in R(V)$ arbitrario

$\hookrightarrow$ PDQ- $\underbrace{\alpha P_U(x) + \beta P_U(y)}_{= P_U(\alpha x + \beta y)} \in P_U(V)$

como V es e.v. $\longrightarrow \alpha x + \beta x \in V$

$$\Longrightarrow P_U(\alpha x + \beta y) \underset{\uparrow}{=} P_U(\alpha x + \beta y) \Rightarrow P_U(\alpha x + \beta y) \in P_U(V)$$

$\exists\, v = \alpha x + \beta y$ tq- $P_U(v) = P_U(\alpha x + \beta y)$

- Si $\{u_i\}_{i=1}^{n}$ es GENERADO

$\forall v \in V \ \exists\, \alpha_i$ tq- $v = \sum_{i=1}^{n} \alpha_i v_i$

$$P_U(V) \ni P_U(v) = P_U\left(\sum_{i=1}^{n} \alpha_i v_i\right) = \sum_{i=1}^{n} \alpha_i P_U(v_i)$$

$$\therefore P_U(V) = \langle \{P_U(v_i)\}_{i=1}^{n} \rangle$$

b) ii) $\left\{ \underset{u_1}{\overset{O_1}{\frac{1}{\sqrt{6}} \begin{pmatrix} -2 \\ 2 \\ 4 \\ 0 \end{pmatrix}}}, \underset{u_2}{\overset{O_2}{\frac{1}{\sqrt{30}} \begin{pmatrix} 2 \\ -4 \\ 3 \\ 1 \end{pmatrix}}} \right\}$

$$V = \left\langle \left\{ \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ -2 \\ 2 \end{pmatrix} \right\} \right\rangle$$

$P_U(v_1)$; $P_U(v_2)$

$$P_U(v_1) = \langle v_1, O_1 \rangle O_1 + \langle v_1, O_2 \rangle O_2$$

$$P_U(v_2) = \langle v_2, O_1 \rangle O_1 + \langle v_2, O_2 \rangle O_2$$

$$\therefore P_U(v_1) = \frac{-2}{2\sqrt{6}} \cdot \frac{1}{2\sqrt{6}} \begin{pmatrix} -2 \\ 2 \\ 4 \\ 0 \end{pmatrix} + \frac{(-7)}{\sqrt{30}} \cdot \frac{1}{\sqrt{30}} \begin{pmatrix} 2 \\ -4 \\ 3 \\ 1 \end{pmatrix}$$

desarrollando ...

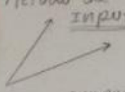
$$\frac{1}{90} \begin{pmatrix} -17 \\ 39 \\ -43 \\ -11 \end{pmatrix}$$

$$P_U(v_2) = \frac{1}{45} \begin{pmatrix} -27 \\ 69 \\ -48 \\ -21 \end{pmatrix}$$

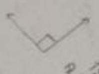
Auxiliar #13
(álgebra lineal)

Método de Gram-Smith

Input



OUTPUT



pensando en una computadora

INPUT

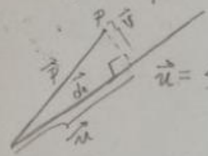
- 2 o más vectores

OUTPUT

- un conjunto de vectores "ortonormales" que genera el mismo espacio

→ perpendiculares entre sí

→ $\|x\| = 1$

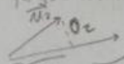


$$\vec{u} = \frac{\langle \vec{P}, \vec{d_1} \rangle}{\|\vec{d_1}\|^2} \vec{d_1}$$

para ortonormalizar

$\{u_i\}_{i=1}^{n}$

- (u_1) es "evidente" ya que es sólo él
- u_2



$$P_2 = \frac{\langle u_2, u_1 \rangle}{\|u_1\|^2} u_1$$

nos quedamos con

$O_2 = u_2 - P_2$

- u_3

$$P_3 = \frac{\langle u_3, u_1 \rangle}{\|u_1\|^2} u_1 + \frac{\langle u_3, O_2 \rangle}{\|O_2\|^2} O_2$$

$O_3 = u_3 - P_3$

... (así sucesivamente)

$\longrightarrow \{O_i\}_{i=1}^{n}$ → limitado x el espacio y no todos podrían ser ortogonales.

último paso: normalizar

$$\left\{ \frac{O_i}{\|O_i\|} \right\}_{i=1}^{n}$$

IDEA ÚTIL → Buscar base $\xrightarrow{\text{posteriormente}}$ ortonormalizar.

P3

a) Buscamos bases → vectores l.i

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 3 \\ 1 & 1 & 1 & 2 & 4 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 4 \end{pmatrix} \underset{\text{pivoteando}}{\leadsto} \begin{pmatrix} 1 & 0 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

↑ 4° → después en orden

$$\therefore \quad S = \left\langle \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\} \right\rangle$$

ahora ortonormalizamos

- $u_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = u_1$
- $u_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ $\quad P_2 = \dfrac{\langle u_2, u_1 \rangle}{\|u_1\|^2} u_1 = \dfrac{2}{3} u_1$ $\;\Big|\;$ $O_2 = u_2 - P_2 = \dfrac{1}{3}\begin{pmatrix} 0 \\ 3 \\ 0 \\ 3 \end{pmatrix} + \dfrac{1}{3}\begin{pmatrix} -2 \\ -2 \\ 0 \\ -2 \end{pmatrix} = \dfrac{1}{3}\begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$
- $u_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad P_3 = \underbrace{\dfrac{\langle u_3, u_1 \rangle}{\|u_1\|^2} u_1}_{u_1} + \underbrace{\dfrac{\langle u_3, O_2 \rangle}{\|O_2\|^2} O_2}_{0}$ $\;\Big|\;$ $O_3 = u_3 - P_3 = u_3 - u_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

normalizando $\quad O_1 = \dfrac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}; \; O_2 = \dfrac{1}{\sqrt{6}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}; \; O_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$$S = \left\langle \left\{ \frac{O_1}{\|O_1\|}; \frac{O_2}{\|O_2\|}; \frac{O_3}{\|O_3\|} \right\} \right\rangle \longrightarrow \left\langle \left\{ \overset{S_1}{O_1}, \overset{S_2}{O_2}, \overset{S_3}{O_3} \right\} \right\rangle$$

b) $\omega \in S^\perp \iff \forall y \in S,\ \langle \omega, y \rangle = 0$

$\langle \omega, \sum_{i=1}^{n} \alpha_i s_i \rangle = 0$

$\Rightarrow \langle \omega, \alpha_1 s_1 \rangle + \langle \omega, \alpha_2 s_2 \rangle + \langle \omega, \alpha_3 s_3 \rangle = 0$

$\Downarrow$

$\langle \omega, s_1 \rangle = 0$
$\langle \omega, s_2 \rangle = 0$
$\langle \omega, s_3 \rangle = 0$

$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix}$

$\left\{ \begin{array}{l} \omega_3 = 0 \\ -2\omega_1 + \omega_2 + \omega_4 = 0 \\ \omega_1 + \omega_2 + \omega_4 = 0 \end{array} \right.$ $\left| \begin{array}{l} \Rightarrow -3\omega_1 = 0 \\ \Downarrow \\ \omega_2 + \omega_4 = 0 \\ \omega_4 = -\omega_2 \end{array} \right.$

$\Longrightarrow \omega = \omega_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$

$\Downarrow$

$S^\perp = \left\langle \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \right\} \right\rangle$

P1 a) $(U+V)^\perp = U^\perp \cap W^\perp$

(doble inclusión)

$\omega \in (U+V)^\perp \iff \forall y \in U+V$ (arbitrario) $\langle \omega, y \rangle = 0$

$y = \underset{\in U}{u} + \underset{\in V}{v}$ $\quad \langle \omega, u+v \rangle = 0$

$\iff \langle \omega, u \rangle + \langle \omega, v \rangle = 0$

$\iff \forall u \in U, v \in V$

$\langle \omega, u \rangle = 0$ y $\langle \omega, v \rangle = 0$

$\iff \omega \in U^\perp$ y $\omega \in V^\perp$

$\iff \omega \in U^\perp \cap V^\perp$

b) $(U^\perp)^\perp = U$

ya que son complementarios

(computan todo el espacio)

$\cdot\ \omega \in (U^\perp)^\perp \iff \forall y \in U^\perp;\ \langle \omega, y \rangle = 0$ $\qquad \cdot\ y \in U^\perp \Rightarrow \forall y \in U^\perp\ \langle y, u \rangle = 0$

$U^\perp \oplus U = \mathbb{R}^n \left\{ \begin{array}{l} \cdot\ \omega \in U^\perp \iff \langle \omega, \omega \rangle = 0 \Rightarrow \omega = 0 \Rightarrow \omega \in U \Rightarrow \omega \in U \\ \cdot\ \omega \in U \Rightarrow \omega \in U \end{array} \right.$

$(U^\perp)^\perp \subseteq U$

c) $(U \cap V)^\perp = U^\perp + V^\perp$; tomando $F = U^\perp$; $E = V^\perp$

por (a) $\longrightarrow (F^\perp + E^\perp)^\perp = F \cap E \ /(\ )^\perp$

$= [(F^\perp + E^\perp)^\perp]^\perp = (F \cap E)^\perp$

por (b) $\longrightarrow F^\perp + E^\perp = (F \cap E)^\perp$

d) $U \oplus V = \mathbb{R}^n \iff \begin{array}{l} U + V = \mathbb{R}^n \\ U \cap V = \{0\} \end{array} \iff \begin{array}{l} (U+V)^\perp = \mathbb{R}^{n\perp} = \{0\} \\ (U \cap V)^\perp = \{0\}^\perp = \mathbb{R}^n \end{array}$

$\iff U^\perp \cap V^\perp = \{0\}$
$\iff U^\perp + V^\perp = \mathbb{R}^n$ $\iff U^\perp \oplus V^\perp = \mathbb{R}^n$

P2 b) i) $M = \begin{pmatrix} -2 & 1 & 5 \\ 2 & -3 & -7 \\ 4 & 5 & -3 \\ 0 & 1 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} -2 & 1 & 5 \\ 0 & -2 & -2 \\ 0 & \cancel{7} & \cancel{7} \\ 0 & \cancel{1} & \cancel{1} \end{pmatrix}$

$O_1 = \begin{pmatrix} -2 \\ 2 \\ 4 \\ 0 \end{pmatrix}$

$O_2 = u_2 - p_2 = u_2 - \frac{\langle u_2, O_1 \rangle}{\| O_1 \|^2} O_1$

$O_2 = \begin{pmatrix} 1 \\ -3 \\ 5 \\ 1 \end{pmatrix} - \cancel{\frac{12}{24}}^{1/2} \begin{pmatrix} -2 \\ 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+1 \\ -3-1 \\ 5-2 \\ 1+0 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 3 \\ 1 \end{pmatrix}$

$\therefore$

$U = \left\langle \left\{ \frac{1}{2\sqrt{6}} \begin{pmatrix} -2 \\ 2 \\ 4 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{30}} \begin{pmatrix} 2 \\ -4 \\ 3 \\ 1 \end{pmatrix} \right\} \right\rangle$