Aux #10

$\boxed{P1}$ a) La longitud entre 0 y x de f es

$$\mathcal{L}_0^x(f) = \int_0^x \sqrt{1+(f'(t))^2}\,dt = x^2 + 2x - f(x)$$

derivando $\Rightarrow$ $\sqrt{1+(f'(x))^2} = 2x + 2 - f'(x)$

$\Rightarrow$ $1+\cancel{(f'(x))^2} = \left(2(x+1) - f'(x)\right)^2$

$= 4(x+1)^2 - 4(x+1)f'(x) + \cancel{(f'(x))^2}$

$\Rightarrow$ $1 = 4(x+1)^2 - 4(x+1)f'(x)$

$\Rightarrow$ $f'(x) = x + 1 - \dfrac{1}{4(x+1)}$

integramos $f(x) = \dfrac{x^2}{2} + x - \dfrac{1}{4}\ln(x+1) + C$

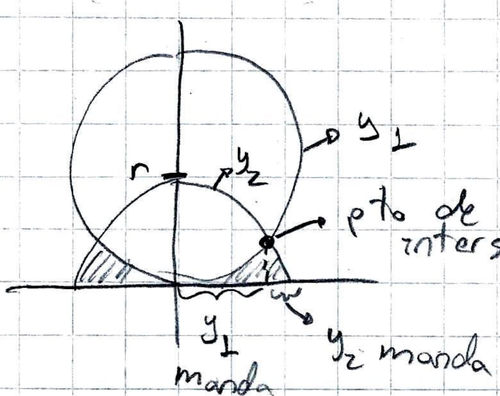
Ocupando que $f(0)=0$

$\Rightarrow f(0) = \dfrac{0^2}{2} + 0 - \dfrac{1}{4}\cancelto{0}{\ln(1)} + C = 0$

$\Rightarrow$ $C = 0$

$\therefore$ $f(x) = \dfrac{x^2}{2} + x - \dfrac{1}{4}\ln(x+1)$

b) Describamos las curvas analiticamente ya que al rotar la region pintada notamos que



$\Rightarrow$ y_1 := basta con notar la restricción $x^2+y^2-2ry \geq 0$

$\Rightarrow \quad x^2+y^2-2ry = 0 \quad / + r^2$

$\Rightarrow \quad x^2+y^2-2ry+r^2 = r^2$

$\Rightarrow \quad \boxed{x^2+(y-r)^2 = r^2}$ $\circledast$

Mientras que para y_2 basta con la otra restricción

$\Rightarrow \quad \boxed{x^2+y^2 = r^2}$

Obs: Usaremos la simetria del problema para ahorrar trabajo

$\Rightarrow$ de $\circledast$ despejamos y $\Rightarrow (y-r)^2 = r^2 - x^2$

$\Rightarrow \quad y_1 = -\sqrt{r^2-x^2} + r$

usaremos la parte de abajo de la circunferencia

mientras que $y_2 = \sqrt{r^2-x^2}$ $\rightarrow$ es un semicirculo

$y = \sqrt{r^2-x^2}$ es la unica opción

Ahora veamos donde se intersectan

$$\Rightarrow \quad y_2 = y_1 \Rightarrow \sqrt{r^2 - x^2} = -\sqrt{r^2-x^2} + r$$

$$\Rightarrow 2\sqrt{r^2-x^2} = r \qquad (\ )^2$$

$$\Rightarrow 4(r^2-x^2) = r^2$$

$$\Rightarrow 4r^2 - 4x^2 = r^2$$

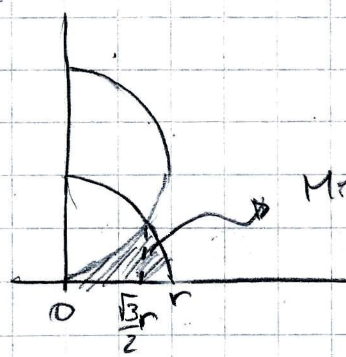
$$\Rightarrow 4x^2 = 3r^2$$

$$\Rightarrow x = \pm \frac{\sqrt{3}}{2} r$$

Ahora la simetría! ocuparemos el cuadrante 1

$\Rightarrow x = \frac{\sqrt{3}}{2} r$ por lo que nuestro estudio

Será en



Mitad de lo que piden

$\Rightarrow$ nuestro resultado lo multiplicaremos por 2

Luego el manto será

$$S_{ox} = 2\Big[\underbrace{2\pi \int_0^{\frac{\sqrt{3}}{2}r} y_1 \sqrt{1-(y_1')^2}\,dx}_{I_1} + \underbrace{2\pi \int_{\frac{\sqrt{3}}{2}r}^{r} y_2 \sqrt{1-(y_2)^2}\,dx}_{I_2}\Big]$$

$$y_1 = -\sqrt{r^2-x^2} + r \quad , \quad y_2 = \sqrt{r^2-x^2}$$

$$y_1' = -\left(\frac{1}{\cancel{2}}(r^2-x^2)^{-\frac{1}{2}} \cdot -\cancel{2}x\right) \quad ; \quad y_2' = \frac{1}{\cancel{2}} \cdot (r^2-x^2)^{-\frac{1}{2}} \cdot -\cancel{2}x$$

$$= -\frac{x}{\sqrt{r^2-x^2}} \qquad\qquad = -\frac{x}{\sqrt{r^2-x^2}}$$

$$\Rightarrow I_1 = \int_0^{\frac{\sqrt{3}}{2}r} y_1 \sqrt{1-(y_1')^2}\, dx = \int_0^{\frac{\sqrt{3}}{2}r} \left(r-\sqrt{r^2-x^2}\right)\sqrt{1+\frac{x^2}{r^2-x^2}}\, dx$$

$$= \int_0^{\sqrt{3}/2\, r} \left(r-\sqrt{r^2-x^2}\right)\sqrt{\frac{r^2-\cancel{x^2}+\cancel{x^2}}{r^2-x^2}}\, dx$$

$$= \int_0^{\sqrt{3}/2\, r} \left(r-\sqrt{r^2-x^2}\right)\frac{r}{\sqrt{r^2-x^2}}\, dx$$

$$= \int_0^{\sqrt{3}/2\, r} \frac{r^2}{\sqrt{r^2-x^2}} - r\, dx$$

$$= r^2 \int_0^{\frac{\sqrt{3}}{2}r} \frac{dx}{\sqrt{r^2-x^2}} - r\int_0^{\sqrt{3}/2\, r} 1\, dx$$

$$= r^2 \operatorname{sen}^{-1}\left(\frac{x}{r}\right)\Big|_0^{\frac{\sqrt{3}}{2}r} - r\, x\Big|_0^{\frac{\sqrt{3}}{2}r}$$

$\operatorname{sen} 0 =$

$\operatorname{sen} x = \frac{\sqrt{3}}{2}$

$$= r^2\left(\frac{\pi}{3} - 0\right) - r\left(\frac{\sqrt{3}}{2}r - 0\right)$$

$$= r^2\frac{\pi}{3} - r^2\frac{\sqrt{3}}{2}$$

$$I_2 = \int_{\frac{\sqrt{3}}{2}r}^{r} \sqrt{r^2-x^2}\,\sqrt{1+\frac{x^2}{r^2-x^2}}\,dx$$

$$= \int_{\%}^{\%} \cancel{\sqrt{r^2-x^2}}\,\frac{r}{\cancel{\sqrt{r^2-x^2}}}\,dx$$

$$= \int_{\frac{\sqrt{3}}{2}r}^{r} r\,dx = r\left(r-\frac{\sqrt{3}}{2}r\right)$$

$$= r^2-\frac{\sqrt{3}}{2}r^2$$

$$\Rightarrow S_{ox} = 2\left[\,2\pi I_1 + 2\pi I_2\,\right]$$

$$= 4\pi\,(I_1+I_2)$$

$$= 4\pi\left(r^2\frac{\pi}{3} - r^2\frac{\sqrt{3}}{2} + r^2 - \frac{\sqrt{3}}{2}r^2\right)$$

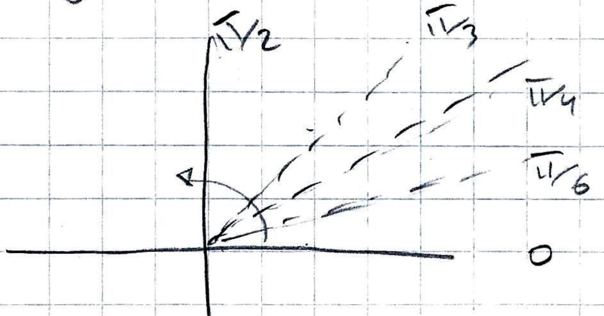
$$= 4\pi r^2\left(\frac{\pi}{3} + 1 - \sqrt{3}\right)$$

P2 Para graficar en polares hay 2 formas

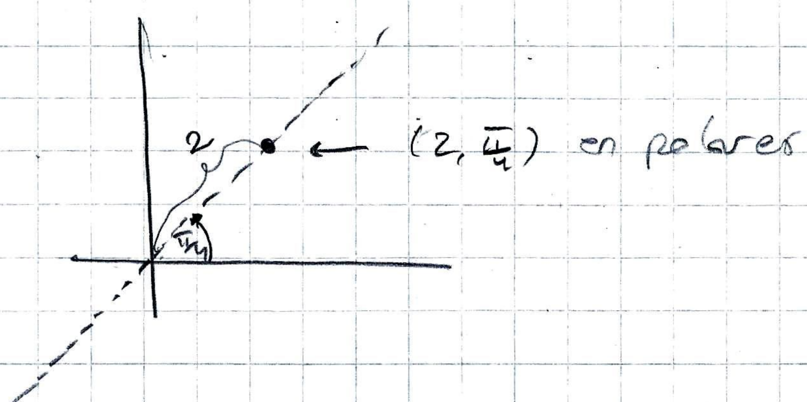
1) Tomar $r = f(\theta)$ y pasarlo a $y = y(x)$ y graficar en el plano normalmente

2) hacer una tabla $\begin{array}{c|c} r & \theta \\ \hline & \end{array}$ y dar valores de θ

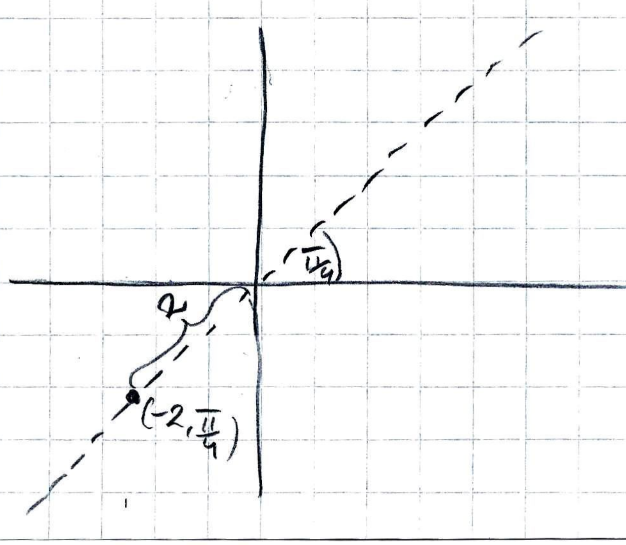
Siguiendo el sentido antihorario



por ejemplo poner el par $r = 2$ $\theta = \frac{\pi}{2}$



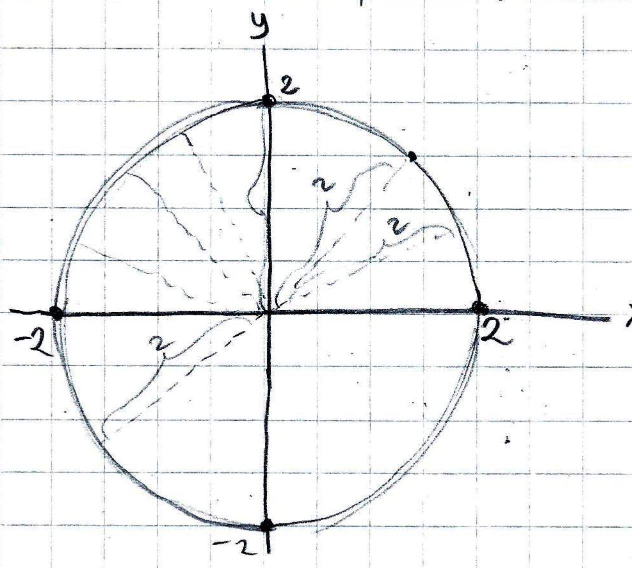
en caso de que $r = -2$ (negativo) se invierte el sentido



$\boxed{P_2}$ a) $r = 2$

r	θ
2	0
2	$\frac{\pi}{6}$
2	$\frac{\pi}{4}$
2	$\vdots$

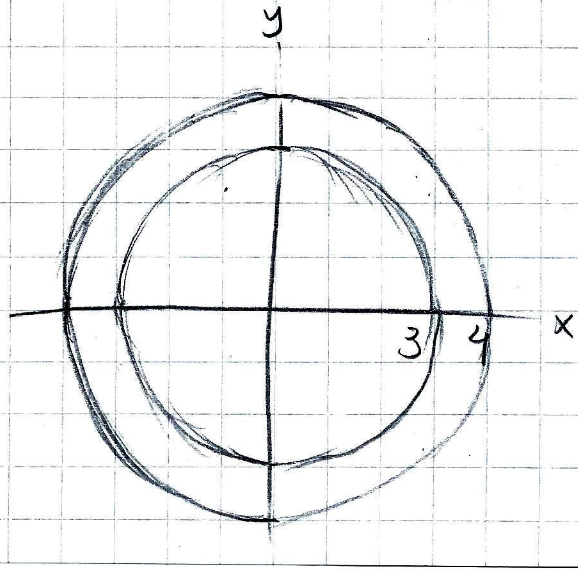
da lo mismo el θ siempre es 2. Esto forma una circunferencia



b) $r^2 - r - 12 = 0 \Rightarrow (r+3)(r-4) = 0$

$\Rightarrow r = -3$ y $r = 4$

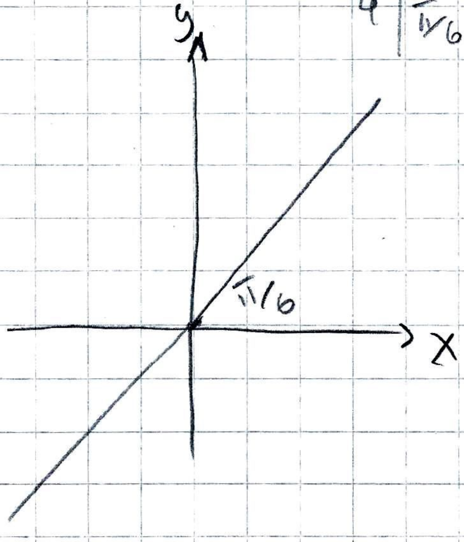
Dos circunferencias solo que $r = -3$ es en otro sentido



c) $\theta = \frac{\pi}{6}$

r	θ
-1	$\frac{\pi}{6}$
0	$\frac{\pi}{6}$
4	$\pi/6$

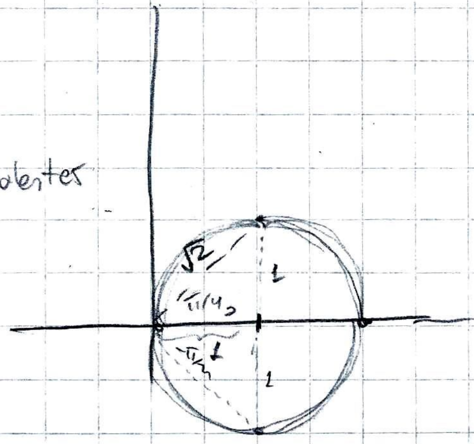
recta sobre la dirección $\theta = \frac{\pi}{6}$



f) $r = 2\cos(\theta)$

r	θ
2	0
$\sqrt{2}$	$\frac{\pi}{4}$
$\sqrt{2}$	$-\frac{\pi}{4}$
-2	π
0	$\pi/2$
1	$-\pi/3$

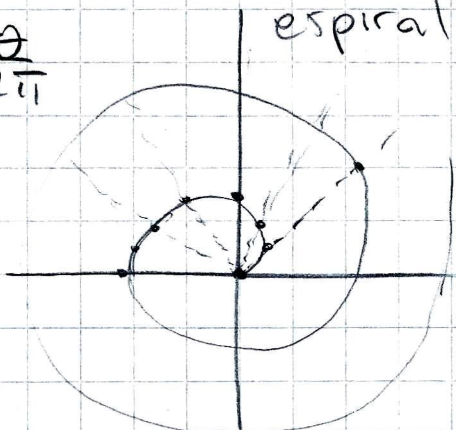
equivalentes



d) $r = \frac{\theta}{2\pi}$

r	θ
0	0
1	2π
2	4π
$\frac{1}{4}$	$\frac{\pi}{2}$
$\frac{1}{2}$	π

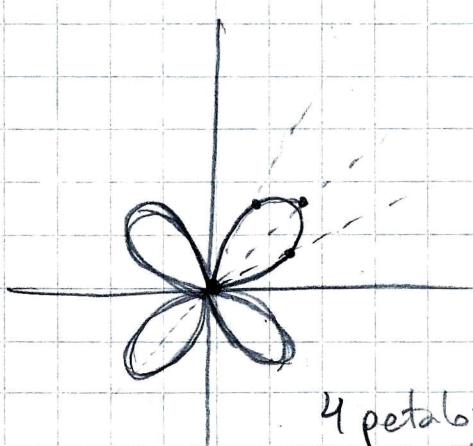
espiral



crece $\theta \Rightarrow$ crece r

g) $r = \operatorname{Sen}(2\theta)$

r	θ
0	0
$0.8 \approx \sqrt{3}/2$	$\pi/6$
1	$\pi/4$
$\sqrt{3}/2$	$\pi/3$
0	$\pi/2$
más valores	



4 petalos simetricos

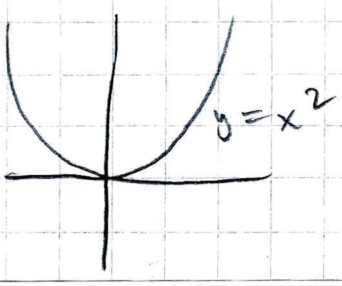
e) $r = \tan(\theta)\sec(\theta)$

$= \frac{\operatorname{sen}(\theta)}{\cos^2(\theta)}$

$\Rightarrow r\cos^2(\theta) = \operatorname{sen}(\theta)$

$\Rightarrow \underbrace{r^2\cos^2(\theta)}_{x^2} = \underbrace{r\operatorname{sen}(\theta)}_{y}$

$x^2 = y$

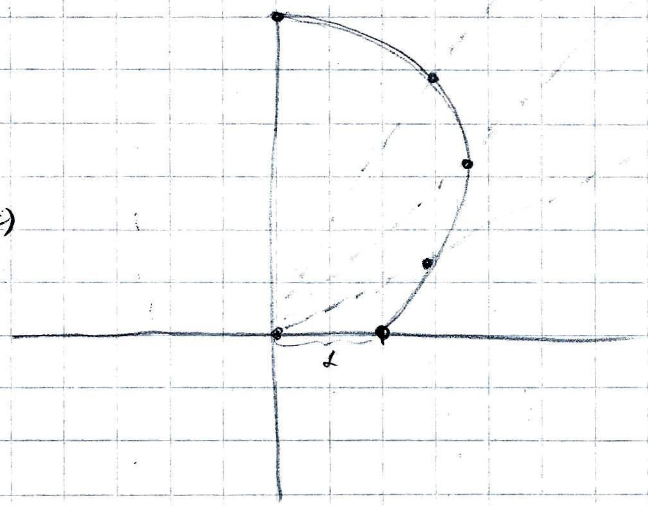


Tecnica 1)

b) $r = 1 + 2\operatorname{sen}(\theta)$

r	θ
1	0
2	$\pi/6$
$2{,}4 \approx 1+\sqrt{2}$	$\pi/4$
$2{,}8 \approx 1+\sqrt{3}$	$\pi/3$
3	$\pi/2$

$\Rightarrow$

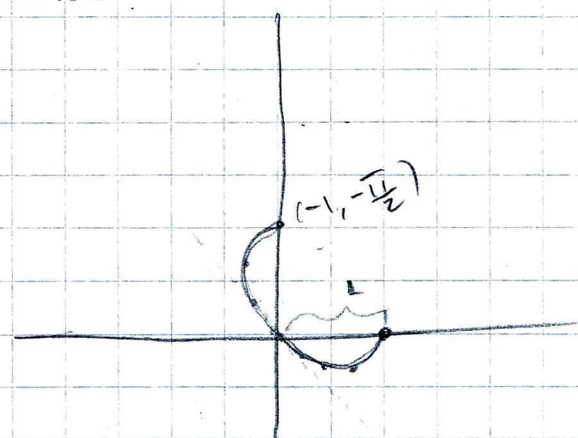


Angulos positivos

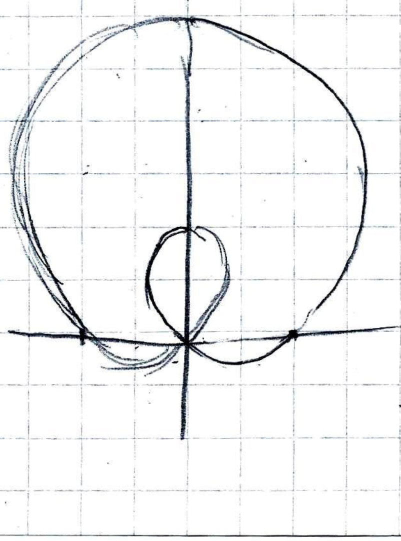
Angulos negativos

$\operatorname{sen}(\theta) \approx \theta$

r	θ
$1-\alpha$	$-\pi/a$
0	$-\pi/6$
$1-\sqrt{2}$	$-\pi/4$
$1-\sqrt{3}$	$-\pi/3$
-1	$-\pi/2$

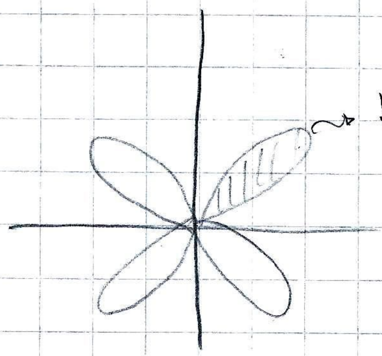


$\Rightarrow$



Si siguen que da asi

$\boxed{P3}$ a) por (b) Sabemos que $\rho(\theta) = \text{Sen}(2\theta)$



1er petalo

Simetrico!

Calculamos el area de un petalo y multiplicamos por 4

El primer petalo se forma $\theta = 0 \rightarrow \theta = \frac{\pi}{2}$

$$\Rightarrow A = 4 \cdot \frac{1}{2} \int_0^{\pi/2} (\text{Sen}(2\theta))^2 \, d\theta$$

$$\cos(2\theta) = \cos^2(\theta) - \text{sen}^2(\theta)$$
$$= 1 - 2\,\text{sen}^2(\theta)$$

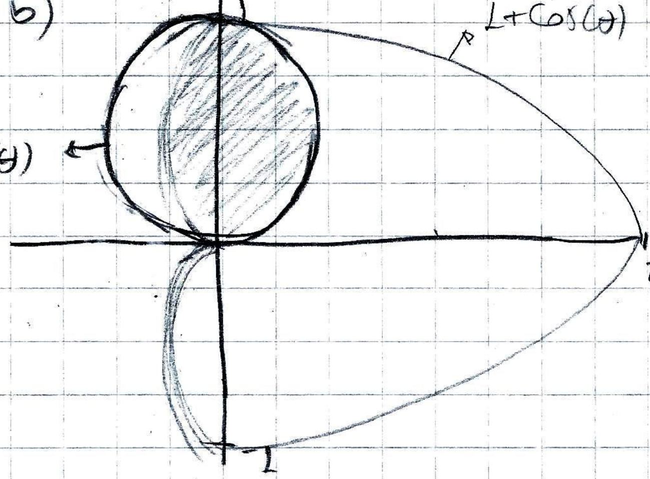
$$= 2 \int_0^{\pi/2} \left(\frac{1 - \cos(4\theta)}{2}\right) d\theta$$

$$= \left(\theta - \frac{1}{4}\,\text{sen}(4\theta)\right)\Big|_0^{\pi/2}$$

$$= \left(\frac{\pi}{2} - \frac{1}{4}\,\overset{0}{\text{Sen}(2\pi)} - 0\right)$$

$$= \frac{\pi}{2}$$

P3 b)



$r = \operatorname{Sen}(\theta)$

$\hookrightarrow$ circunferencia

radio $= \frac{1}{2}$ centro $(0, \frac{1}{2})$

area = [semicírculo] + [sección]

$\underbrace{\text{Semicirculo}}_{\frac{1}{2}\pi r^2}$

sección cardiode segundo cuadrante $\Big\}$ $\frac{\pi}{2} \to \pi$

$\Rightarrow$ Area $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{1}{2}\right)^2 = \frac{1}{2}\pi \frac{1}{4} = \frac{\pi}{8}$

y Area $= \frac{1}{2}\int_{\pi/2}^{\pi} (1+\cos(\theta))^2 d\theta \overset{\text{TAREA}}{=} \frac{3\pi}{8} - 1$

Area total $= \frac{3\pi}{8} + \frac{\pi}{8} - 1 = \frac{\pi}{2} - 1 > 0$

$\boxed{P4}$ notar que $A = \int_a^b f(x)dx$

$$\Rightarrow 2\pi \cdot A \cdot X_G = 2\pi \cancel{A} \cdot \frac{\int_a^b x f(x) dx}{\cancel{A}} = 2\pi \int_a^b x f(x) dx$$

$$= V_{oy}$$

Analogamente

$$\Rightarrow 2\pi A Y_G = 2\pi \cancel{A} \cdot \frac{\int_a^b \frac{f^2(x)}{2} dx}{\cancel{A}} = V_{ox}$$