Aux #6

P1 Recordemos que

1) $\operatorname{Sen}^2(x) = \dfrac{1-\cos(2x)}{2}$ 2) $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$

a) $\int \operatorname{sen}^4(x)\,dx = \int (\operatorname{sen}^2(x))^2\,dx$

$$= \int \left(\frac{1-\cos(2x)}{4}\right)^2 dx$$

$$= \int \frac{1 - 2\cos(2x) + \cos^2(2x)}{4}\,dx$$

$$= \frac{1}{4}\int 1 - 2\cos(2x) + \frac{1+\cos(4x)}{2}\,dx$$

$$= \frac{1}{4}\left(\int 1\,dx - 2\int \cos(2x)\,dx + \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \cos(4x)\,dx\right)$$

$$= \frac{1}{4}\left(x - 2\frac{\operatorname{Sen}(2x)}{2} + \frac{1}{2}x + \frac{1}{2}\frac{\operatorname{Sen}(4x)}{4} + C\right)$$

$$= \frac{3x}{8} - \frac{\operatorname{Sen}(2x)}{4} + \frac{\operatorname{Sen}(4x)}{32} + K$$

Obs: $K = \frac{1}{4}C$ pero es irrelevante

b) $\int \cos^5(x)\,dx = \int \cos^4(x)\cos(x)\,dx$

$$= \int (1-\operatorname{sen}^2(x))^2 \cdot \cos(x)\,dx \qquad u = \operatorname{Sen}(x) \Rightarrow du = \cos(x)\,dx$$

$$= \int (1-u^2)^2\,du$$

$$= \int 1 - 2u^2 + u^4\,du$$

$$= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C$$

Deshago Cambio de Variable

$$= \operatorname{sen}(x) - \frac{2}{3}\operatorname{sen}^3(x) + \frac{\operatorname{sen}^5(x)}{5} + C$$

$\boxed{P2}$ a) $\int (x-1)\sqrt{x+4}\,dx$ Idea: pasar a $\int x^{\alpha} + x^{\beta} + x^{\gamma} \ldots$

hacemos $u = x+4$ $du = dx$

$\hookrightarrow x = u-4$

$$\Rightarrow \int (x-1)\sqrt{x+4}\,dx = \int ((u-4)-1)\sqrt{u}\,du$$

$$= \int (u-5)\sqrt{u}\,du$$

$$= \int u\sqrt{u} - 5\sqrt{u}\,du$$

$$= \int u^{3/2} - 5u^{1/2}\,du = \frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1} - 5\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{2}{5}u^{5/2} - \frac{10}{3}u^{3/2} + C$$

$u = x+4$

$$= \frac{2}{5}(x+4)^{5/2} - \frac{10}{3}(x+4)^{3/2} + C$$

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b) $\int \frac{dx}{x - x^{3/5}}$ Idea: hacer desaparecer $\sqrt[5]{\ }$

haciendo $x = u^5 \Rightarrow dx = 5u^4 du$

$$\Rightarrow \int \frac{dx}{x - x^{3/5}} = \int \frac{5u^4 du}{u^5 - u^3} = 5\int \frac{u^4}{u^3(u^2-1)} du$$

$$= 5\int \frac{\frac{2}{2}u}{u^2-1} du \qquad \begin{cases} t = u^2-1 \\ dt = 2u\,du \end{cases}$$

$$= \frac{5}{2}\int \frac{dt}{t}$$

$$= \frac{5}{2}\ln(t) + C$$

$x = u^5$

$x^{2/5} = u^2$

$$= \frac{5}{2}\ln(u^2-1) + C$$

$$= \frac{5}{2}\ln(x^{2/5} - 1) + C$$

c) $\int \frac{Sen(x)\cos(x)}{\sqrt{1+Sen(x)}}\,dx$ $\qquad u = 1+Sen(x)$

$du = \cos(x)\,dx$

$= \int \frac{(u-1)\,du}{\sqrt{u}}$

$= \int \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\,du$

$= \int u^{1/2} - u^{-1/2}\,du$

$= \frac{u^{1/2+1}}{\frac{1}{2}+1} - \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$

$= \frac{2}{3}u^{3/2} - 2u^{1/2} + C$

$= \frac{2}{3}(1+Sen(x))^{3/2} - 2(1+Sen(x))^{1/2} + C$

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d) $\int \frac{e^{3x}}{1+e^{2x}}\,dx$ $\qquad u = e^x \qquad du = e^x\,dx$

$= \int \frac{(e^x)^{2+1}}{1+(e^x)^2}\,dx = \int \frac{u^2\,du}{1+u^2}$

$= \int \frac{1+u^2-1}{1+u^2}\,du$

$= \int 1\,du - \int \frac{1}{1+u^2}\,du$

$= u - \arctan(u) + C$

$= e^x - \arctan(e^x) + C$

P3 a) $\int \frac{dx}{(a^2+x^2)^2}$

CV: $x = a\tan(v)$

$dx = a\sec^2(v)\,dv$

$$\int \frac{dx}{(a^2+x^2)^2} = \int \frac{a\sec^2(v)\,dv}{(a^2+a^2\tan^2(v))^2}$$

$$= \int \frac{a\sec^2(v)\,dv}{(a^2(1+\tan^2(v)))^2} \qquad 1+\tan^2(v) = \sec^2(v)$$

$$= \int \frac{a\sec^2(v)\,dv}{a^4\sec^4(v)}$$

$$= \frac{1}{a^3}\int \frac{1}{\sec^2(v)}\,dv$$

$$= \frac{1}{a^3}\int \cos^2(v)\,dv$$

$$= \frac{1}{a^3}\int \frac{1}{2}(\cos(2v)+1)\,dv$$

$$= \frac{1}{2a^3}\int \cos(2v)+1\,dv$$

$$= \frac{1}{2a^3}\left[\frac{1}{2}\operatorname{Sen}(2v)+v\right]+C$$

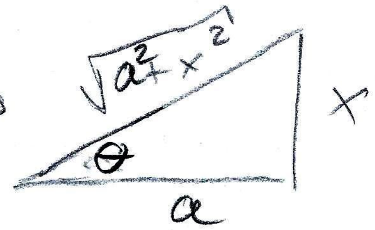
$v = \arctan(\frac{x}{a})$

$$= \frac{1}{2a^3}\left[\frac{1}{2}\operatorname{Sen}(2\arctan(\tfrac{x}{a}))+\arctan(\tfrac{x}{a})\right]+C$$

¿Que es esto?!!

$$\operatorname{Sen}(2\arctan(\tfrac{x}{a})) = 2\operatorname{Sen}(\arctan(\tfrac{x}{a}))\cos(\arctan(\tfrac{x}{a}))$$!!

Si $\boxed{\arctan(\frac{x}{a}) = \theta}$ $\Rightarrow$ $\tan(\theta) = \frac{x}{a} = \frac{\text{Cateto op}}{\text{Cateto ad}}$

$\rightarrow$ 

$\Rightarrow$ $\cos(\theta) = \frac{a}{\sqrt{a^2+x^2}}$

$\operatorname{Sen}(\theta) = \frac{x}{\sqrt{a^2+x^2}}$

$$\Rightarrow \quad \operatorname{sen}(2\theta) = 2\operatorname{sen}(\theta)\cos(\theta)$$

$$= 2\frac{x}{\sqrt{a^2+x^2}} \cdot \frac{a}{\sqrt{a^2+x^2}}$$

$$= \frac{2xa}{a^2+x^2}$$

$$\Rightarrow \int \frac{dx}{(a^2+x^2)^2} = \frac{1}{2a^3}\left[\frac{1}{\cancel{2}}\frac{\cancel{2}xa}{a^2+x^2} + \arctan\left(\frac{x}{a}\right)\right] + C$$

$$= \frac{1}{2a^3}\left[\frac{ax}{a^2+x^2} + \arctan\left(\frac{x}{a}\right)\right] + C$$

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b) $$\int \frac{dx}{\sqrt{x^2+2x+2}} = \int \frac{dx}{\sqrt{(x+1)^2+1}}$$

$x+1 = \operatorname{senh}(v)$

$dx = \cosh(v)\,dv$

Se podía también usar $x+1 = \tan(u)$

$$= \int \frac{\cosh(v)\,dv}{\sqrt{\operatorname{senh}^2(v)+1}}$$

$$= \int \frac{\cosh(v)\,dv}{\cosh(v)}$$

$$= \int 1\,dv = v + C$$

$$= \operatorname{arcsenh}(x+1) + C$$

c) $\displaystyle\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx$ $\qquad x = a\,\text{sen}(y)$

$\qquad dx = a\cos(y)\,dy$

$$= \int \frac{\sqrt{a^2 - a^2\,\text{sen}^2(y)}\;a\cos(y)\,dy}{a^2\,\text{sen}^2(y)}$$

$$= \int \frac{\cancel{a^2}\sqrt{1-\text{sen}^2(y)}\;\cos(y)\,dy}{\cancel{a^2}\,\text{sen}^2(y)}$$

$$= \int \frac{\cos^2(y)}{\text{sen}^2(y)}\,dy$$

$$= \int \frac{1-\text{sen}^2(y)}{\text{sen}^2(y)}\,dy$$

$$= \int \frac{1}{\text{sen}^2(y)} - 1\,dy \qquad \left(\frac{\cos(x)}{\text{sen}(x)}\right)' = \frac{-\text{sen}^2(x)-\cos^2(x)}{\text{sen}^2(x)}$$

$$= -\text{cotan}(y) - y + C \qquad = \frac{-1}{\text{sen}^2(x)}$$

$$= -\text{cotan}\left(\text{arcsen}\left(\frac{x}{a}\right)\right) - \text{arcsen}\left(\frac{x}{a}\right) + C$$

Tarea hacer  Para calcular esto

P4 a) $\int \frac{1}{1+\operatorname{Sen}(x)+\cos(x)}\,dx$

$t = \tan\left(\frac{x}{2}\right) \Leftrightarrow \arctan(t) = \frac{x}{2}$

$dt = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$

$\frac{1}{1+t^2}\,dt = \frac{dx}{2}$

"iguales"

$\Rightarrow \cos^2\left(\frac{x}{2}\right) = \frac{1}{1+t^2}$

... seguir ...

$\operatorname{Sen}(x) = \frac{2t}{1+t^2}$ $\qquad dx = \frac{2dt}{1+t^2}$

$\cos(x) = \frac{1-t^2}{1+t^2}$

$$= \int \frac{\frac{2dt}{1+t^2}}{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$$

$$= \int \frac{\frac{2dt}{1+t^2}}{\frac{1+\cancel{t^2}+2t+1-\cancel{t^2}}{1+t^2}}$$

$$= \int \frac{\frac{2}{\cancel{1+t^2}}}{\frac{2+2t}{\cancel{1+t^2}}}\,dt$$

$$= \int \frac{1}{1+t}\,dt = \ln(1+t) + C$$

$$= \ln\left(1+\tan\left(\frac{x}{2}\right)\right) + C$$

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b) $\int \frac{\operatorname{Sen}(x)}{1+\operatorname{Sen}(x)}\,dx = I$

$t = \tan\left(\frac{x}{2}\right)$; $dx = \frac{2dt}{1+t^2}$

$\operatorname{Sen}(x) = \frac{2t}{1+t^2}$

$$= \int \frac{\frac{2t}{1+t^2}\cdot\frac{2dt}{1+t^2}}{1+\frac{2t}{1+t^2}}$$

$$= \int \frac{\frac{4t}{(1+t^2)^{\cancel{2}}}}{\frac{1+2t+t^2}{\cancel{(1+t^2)}}}\,dt \qquad (1+t)(1+t^2)$$

$$= \int \frac{4t}{(1+t)^2(1+t^2)}\,dt = 4\int \frac{t}{(1+t)^2(1+t^2)}\,dt$$

$\uparrow$

Fracciones Parciales

$$\frac{t}{(t+1)^2(t^2+1)} = \frac{A}{(t+1)} + \frac{B}{(t+1)^2} + \frac{Ct+D}{(t^2+1)}$$

$$= \frac{A(t+1)(t^2+1) + B(t^2+1) + (Ct+D)(t+1)^2}{(t+1)^2(t^2+1)}$$

$\Rightarrow \quad t = A(t+1)(t^2+1) + B(t^2+1) + (Ct+D)(t+1)^2$

$= A(t^3+t^2+t+1) + B(t^2+1) + (Ct+D)(t^2+2t+1)$

$= A(t^3+t^2+t+1) + B(t^2+1) + C(t^3+2t^2+t)$

$+ D(t^2+2t+1)$

$\Rightarrow (t^0) \quad 0 = A + B + D$

$(t^1) \quad 1 = A + C + 2D$

$(t^2) \quad 0 = A + B + 2C + D$

$(t^3) \quad 0 = A + C$

$(t^0) - (t^2) \Leftrightarrow \quad 0 = 2C \Rightarrow C = 0$

Si $C = 0$ por (t^3) . $A = 0$

Luego por (t^1) $D = \frac{1}{2}$ y por (t^0) $B = \frac{-1}{2}$

$$\Rightarrow \frac{t}{(t+1)^2(t^2+1)} = \frac{1}{2}\left(\frac{-1}{(t+1)^2} + \frac{1}{(t^2+1)}\right)$$

$$\Rightarrow I = 4\int \frac{t}{(1+t)^2(1+t^2)}\,dt = 4\int \frac{1}{2}\left(\frac{1}{t^2+1} - \frac{1}{(t+1)^2}\right)dt$$

$$= 2\left(\int \frac{dt}{t^2+1} - \int \frac{dt}{(t+1)^2}\right)$$

$$= 2\left(\arctan(t) + C_1 - \left(\frac{-1}{(t+1)}\right) + C_2\right)$$

$$= 2\arctan(t) + \frac{2}{t+1} + \overset{\nearrow C_1+C_2}{C}$$

$t = \tan\left(\frac{x}{2}\right)$

$$= 2\arctan\left(\tan\left(\frac{x}{2}\right)\right) + \frac{2}{\tan\left(\frac{x}{2}\right)+1} + C$$

$$= \cancel{2}\frac{x}{\cancel{2}} + \frac{2}{\tan\left(\frac{x}{2}\right)+1} + C$$

$$= \underbrace{x + \frac{2}{\tan\left(\frac{x}{2}\right)+1}}_{\int \frac{\operatorname{sen}(x)}{1+\operatorname{sen}(x)}\,dx} + C$$

$\boxed{P5}$ a) $\int x^2 e^x\,dx = I$

$u = x^2 \Rightarrow du = 2x\,dx$

$dv = e^x dx \Rightarrow v = e^x$

$uv - \int v\,du$

$I = x^2e^x - \int e^x\, 2x\,dx$

$= x^2e^x - 2\underbrace{\int xe^x dx}_{J}$

$= x^2e^x - 2(xe^x - x) + K$

Idea: e^x derivando o integrando nunca morirá

x^2 muere despues de 3 derivadas

$\downarrow$

$u = x^2$

$dv = e^x dx$

$J = \int xe^x dx$

$= xe^x - \int e^x dx$

$= xe^x - e^x + C$

misma idea

$u = x \Rightarrow du = dx$

$dv = e^x dx \Rightarrow v = e^x$

b) $\int \arctan(x)\,dx$

$u = \arctan(x) \Rightarrow du = \frac{dx}{1+x^2}$

$dv = dx \Rightarrow v = x$

$= x\arctan(x) - \underbrace{\int \frac{x}{1+x^2}dx}_{J}$

$\Rightarrow$

$J = \int \frac{x}{1+x^2}dx$

Cambio de variable

$t = 1+x^2 \Rightarrow dt = 2t\,dt$

$= \frac{1}{2}\int \frac{1}{t}dt$

$= \frac{1}{2}\ln(t) + C = \frac{1}{2}\ln(1+x^2) + C$

$\Rightarrow \int \arctan(x)\,dx = x\arctan(x) - \frac{1}{2}\ln(1+x^2) + C$

c) $\int \cos(\ln(x))\,dx = I$

CAMBIO DE VARIABLE

$y = \ln(x) \Leftrightarrow x = e^y$

$\Rightarrow dx = e^y\,dy$

$= \int \cos(y)\, e^y\, dy$

Nombremos $J = \int \cos(y)\, e^y\, dy$

$\Rightarrow J = \int \cos(y) e^y dy$

$= \cos(y) e^y + \int \text{sen}(y) e^y dy$

Por Partes

$u = \cos(y) \rightarrow du = -\text{sen}(y)dy$

$dv = e^y dy \rightarrow v = e^y$

De Nuevo por Partes

$u = \text{sen}(y) \rightarrow du = \cos(y)\,dy$

$dv = e^y dy \rightarrow v = e^y$

$\Downarrow$

$= \cos(y)e^y + \text{sen}(y)e^y - \underbrace{\int e^y \cos(y)\,dy}_{J}$ ¡¡O!!

$\Rightarrow J = \cos(y)\,e^y + \text{sen}(y)\,e^y - J$

$\Rightarrow 2J = e^y(\text{sen}(y) + \cos(y))$

$\Rightarrow J = \frac{e^y}{2}(\text{sen}(y) + \cos(y)) + C$

No olvidar

Luego $y = \ln(x)$

$\Rightarrow \int \cos(\ln(x))dx = \frac{e^{\ln(x)}}{2}(\text{sen}(\ln(x)) + \cos(\ln(x))) + C$

$= \frac{x}{2}(\text{sen}(\ln(x)) + \cos(\ln(x))) + C$

$\boxed{P6}$ a) $\int \frac{dx}{1-x^2} = \int \frac{1}{(1+x)(1-x)}$

F.P. $\frac{1}{(1+x)(1-x)} = \frac{A}{1+x} + \frac{B}{1-x}$

$\Rightarrow \quad 1 = A(1-x) + B(1+x)$

•) Si $x = 1$

$\Rightarrow \quad 1 = B \cdot (2) \Rightarrow \quad B = \frac{1}{2}$

•) Si Ahora $x = -1$

$\Rightarrow \quad 1 = A \cdot (2) \Rightarrow \quad A = \frac{1}{2}$

$\Rightarrow \frac{1}{(1+x)(1-x)} = \frac{1}{2}\left(\frac{1}{1+x} + \frac{1}{1-x} \right)$

$\Rightarrow \int \frac{dx}{1-x^2} = \frac{1}{2} \int \frac{1}{1+x} + \frac{1}{1-x} \, dx$

$= \frac{1}{2}\left(\ln(1+x) - \ln(1-x) \right) + C$

b) $\int \frac{4x^3-3x^2+3}{(x-1)^2(x^2+1)}\,dx$

Fracciones parciales

$$\frac{4x^3-3x^2+3}{(x-1)^2(x^2+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$$

$$\Rightarrow 4x^3-3x^2+3 = A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x^2-2x+1)$$

$$= A(x^3+x-x^2-1) + B(x^2+1) + C(x^3-2x^2+x) + D(x^2-2x+1)$$

(x^3) $4 = A + C$

(x^2) $-3 = -A + B - 2C + D$

(x^1) $0 = A + C - 2D$

(x^0) $3 = -A + B + D$

$(x^3)-(x^1) \Leftrightarrow$ $4 = 2D$ $\Rightarrow D = 2$

$(x^2)-(x^0) \Leftrightarrow$ $-6 = -2C$ $\Rightarrow C = 3$

$C = 3$ en (x^3) $\Rightarrow A = 1$

en (x^0) $A = 1$ y $D = 2$ $\Rightarrow B = 2$

$$\Rightarrow \int \frac{4x^3-3x^2+3}{(x-1)^2(x^2+1)}\,dx = \int \frac{1}{(x-1)} + \frac{2}{(x-1)^2} + \frac{3x+2}{x^2+1}\,dx$$

$$= \int \underbrace{\frac{1}{(x-1)}}_{I_1} + \underbrace{\frac{2}{(x-1)^2}}_{I_2} + \underbrace{\frac{3x}{x^2+1}}_{I_3} + \underbrace{\frac{2}{x^2+1}}_{I_4}\,dx$$

$$I_1 = \int \frac{1}{x-1}\,dx = \ln(x-1) + C_1$$

$$I_2 = \int \frac{2}{(x-1)^2}\,dx = \frac{-2}{(x-1)} + C_2$$

$$I_3 = \int \frac{3x}{x^2+1}\,dx = 3\int \frac{x}{x^2+1}\,dx = \frac{3}{2}\ln(x^2+1) + C_3$$

$$I_4 = \int \frac{2}{x^2+1}\,dx = 2\int \frac{1}{x^2+1}\,dx = 2\arctan(x) + C_4$$

$$\Rightarrow \int \frac{4x^3-3x^2+3}{(x-1)^2(x^2+1)}\,dx = \ln(x-1) - \frac{2}{x-1} + \frac{3}{2}\ln(x^2+1)$$

$$+ 2\arctan(x)$$

$$+ C$$

$\boxed{P7}$ $I_{p,q} = \int x^p (1+x)^q\,dx$

Por partes $\quad u = (1+x)^q \Rightarrow du = q(1+x)^{q-1}dx$

$\quad dv = x^p dx \Rightarrow v = \frac{x^{p+1}}{p+1}$

$$\Rightarrow I_{p,q} = \frac{x^{p+1}(1+x)^q}{p+1} - \int \frac{x^{p+1}}{p+1}\, q(1+x)^{q-1}\,dx$$

$$= \frac{x^{p+1}(1+x)^q}{p+1} - \frac{q}{p+1}\underbrace{\int x^{p+1}(1+x)^{q-1}\,dx}_{I_{p+1,\,q-1}}$$

$$\Rightarrow I_{p,q} = \frac{x^{p+1}(1+x)^q}{p+1} - \frac{q}{p+1} I_{p+1,\,q-1} \quad /\cdot\ p+1$$

$$\Rightarrow (p+1) I_{p,q} = x^{p+1}(1+x)^{q-1} - q\, I_{p+1,q-1}$$

b) 1) $J_n = \int x^n \operatorname{sen}(x)\,dx$

Partes $\quad u = x^n \Rightarrow du = n x^{n-1} dx$

$\quad dv = \operatorname{sen}(x) dx \Rightarrow v = -\cos(x)$

$$\Rightarrow J_n = -\cos(x) x^n + n \int \cos(x)\, x^{n-1} dx$$

Partes $\quad u = x^{n-1} \to du = n-1\, x^{n-2}$

$\quad dv = \cos(x) \to v = \operatorname{sen}(x)$

$$= -\cos(x) x^n + n\left(\operatorname{sen}(x) x^{n-1} - (n-1)\underbrace{\int x^{n-2}\operatorname{sen}(x) dx}_{J_{n-2}}\right)$$

$$= -x^n \cos(x) + n x^{n-1}\operatorname{sen}(x) - (n-1) J_{n-2}$$

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2) $K_n = \int \cos^n(x)\,dx$

$$= \int \cos^{n-1}(x) \cos(x)\, dx$$

Partes

- $u = \cos^{n-1}(x)$
  $\to du = (n-1)\cos^{n-2}(x) \cdot \operatorname{sen}(x)\, dx$
- $dv = \cos(x) dx$
  $\to v = \operatorname{sen}(x) dx$

$$= \cos^{n-1}(x)\operatorname{sen}(x) + (n-1)\int \cos^{n-2}(x)\operatorname{sen}^2(x) dx$$

$$= \cos^{n-1}(x)\operatorname{sen}(x) + (n-1)\int \cos^{n-2}(x)(1-\cos^2(x)) dx$$

$$= \cos^{n-1}(x)\operatorname{sen}(x) + (n-1)\left(\int \cos^{n-2}(x) - \int \cos^n(x) dx\right)$$

$$= \cos^{n-1}(x)\operatorname{sen}(x) + (n-1)(K_{n-2} - K_n)$$

$$= \cos^{n-1}(x)\operatorname{sen}(x) + (n-1)K_{n-2} - (n-1)K_n$$

Luego

$$K_n = \cos^{n-1}(x)\,\mathrm{Sen}(x) + (n-1)K_{n-2} - (n-1)K_n$$

$$\Rightarrow K_n + (n-1)K_n = \cos^{n-1}(x)\,\mathrm{Sen}(x) + (n-1)K_{n-2}$$

$$\Rightarrow n\cdot K_n = \cos^{n-1}(x)\,\mathrm{sen}(x) - (n-1)K_{n-2}$$

$$\Rightarrow K_n = \frac{1}{n}\cdot\cos^{n-1}(x)\,\mathrm{Sen}(x) - \frac{(n-1)}{n}K_{n-2}$$

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c) $$J_{m,n} = \int \cos^m(x)\,\mathrm{Sen}^n(x)\,dx$$

$$= \int \mathrm{Sen}^{n-1}(x)\cos^m(x)\,\mathrm{Sen}(x)\,dx$$

Partes
$$\boxed{\begin{array}{l} u = \mathrm{Sen}^{n-1}(x) \Rightarrow du = n-1\,\mathrm{Sen}^{n-2}(x)\cos(x)dx \\ dv = \cos^m(x)\,\mathrm{Sen}(x)dx \Rightarrow V = -\dfrac{\cos^{m+1}(x)}{m+1} \end{array}}$$

$$\int \cos^m(x)\,\mathrm{sen}(x)dx = \int -u^m\,du = -\frac{u^{m+1}}{m+1}$$

C.V: $u = \cos(x)$ $\quad du = -\mathrm{sen}(x)dx$

$$= -\frac{\cos^{m+1}(x)}{m+1}\mathrm{Sen}^{n-1}(x) + (n-1)\int \frac{\cos^{m+1}(x)}{m+1}\mathrm{Sen}^{n-2}(x)\cos(x)dx$$

$$= \underbrace{-\frac{\cos^{m+1}(x)\,\mathrm{sen}^{n-1}(x)}{m+1}}_{} + \frac{n-1}{m+1}\int \cos^{m+2}(x)\,\mathrm{Sen}^{n-2}(x)dx$$

$$= \% + \frac{n-1}{m+1}\int \cos^m(x)\,(1+\mathrm{Sen}^2(x))\,\mathrm{Sen}^{n-2}(x)dx$$

$$= \% + \frac{n-1}{m+1}\left(\int \cos^m(x)\,\mathrm{Sen}^{n-2}(x) + \int \cos^m(x)\,\mathrm{Sen}^n(x)dx\right.$$

$$= \% + \frac{n-1}{m+1}\left(J_{m,n-2} + J_{m,n}\right)$$

$$J_{m,n} = -\frac{\cos^{m+1}(x)\,\text{sen}^{n-1}(x)}{m+1} + \frac{n-1}{m+1} J_{m,n-2} + \frac{n-1}{m+1} J_{m,n}$$

$$\Rightarrow J_{m,n}\left(1 - \frac{n-1}{m+1}\right) = \frac{-\cos^{m+1}(x)\,\text{sen}^{n-1}(x)}{m+1} + \frac{n-1}{m+1} J_{m,n-2}$$

d) TAREA ¡¡ Sorry mor: demasiada matraca

$$I = \frac{e^{ax}\,\text{sen}(bx)}{a} - \frac{b}{a} J$$

$$J = \frac{e^{ax}\cos(bx)}{a} + \frac{b}{a} I$$

$$\Rightarrow \quad aI + bJ = e^{ax}\,\text{sen}(bx)$$

$$-bI + aJ = e^{ax}\cos(bx)$$

Despejar I, J