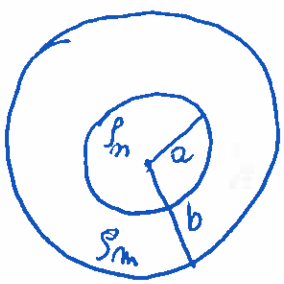
PAUTA AUX 1

P1)



$g_0 = g(r=b) \wedge \rho_m$

$\Rightarrow g_0 = \frac{GM}{b^2}$ * / por definición

Pero M:

$$M = 4\pi \left[\int_0^a r^2 \rho_n \, dr + \int_a^b r^2 \rho_m \, dr \right]$$

$$= 4\pi \left[\rho_n \frac{a^3}{3} + \frac{\rho_m}{3} (b^3 - a^3) \right] \quad ** $$

Entonces ** en *

$$g_0 b^2 = G 4\pi \left[\rho_n \frac{a^3}{3} + \frac{\rho_m}{3} (b^3 - a^3) \right]$$

Despejo ρ_n

$$\rho_n = \left\{ \frac{g_0 b^2 \cdot 3}{G 4\pi} - \rho_m (b^3 - a^3) \right\} \frac{1}{a^3}$$

$\Rightarrow \rho_m \cong 10,65\ [gr/cm^3]$

ii) GRAVEDAD INTERNA:

$0 < r < a$

$$\vec{g}(r) = \frac{GM}{r^2}\hat{r}$$

$$M(r) = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_m\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \frac{4\pi}{3}\rho_m r^3$$

$$\Rightarrow g(r) = \frac{G\rho_m}{\cancel{r^2}}\frac{4\pi}{3}\cdot r^{\cancel{3}} = \frac{G\rho_m 4\pi}{3}\cdot r$$

$a < r < b$

$$M = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho(r)\, r^2 \sin\theta \, dr\, d\theta\, d\phi$$

r: Genérico $\forall r$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho_n r^2 \sin\theta \, dr\, d\theta\, d\phi + \int_0^{2\pi}\int_0^{\pi}\int_a^{r} \rho_m r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \frac{4}{3}\pi a^3 \rho_n + 4\pi \rho_m \left[\frac{r^3}{3} \Big|_a^r \right]$$

$$\Rightarrow g = \frac{4\pi G}{3}\left[a^3 \rho_n + \rho_m (r^3 - a^3) \right] \cdot \frac{1}{r^2}$$

P2)

a) $g_0 = \frac{GM}{R^2} \Rightarrow M = \frac{g_0 R^2}{G}$

DENSIDAD

$M = \rho_p \cdot \frac{4\pi R^3}{3} \Rightarrow \rho_p = \frac{3}{4} \frac{M}{\pi R^3}$

$\Rightarrow M = 5{,}96 \cdot 10^{24}\,kg \quad \wedge \quad \rho_p = 5{,}5\,g/cm^3$

b) Lineal $\rho(r=0) = \rho_c \quad \wedge \quad \rho(r=R) = \rho_0$

$$\rho(r) = \rho_0 + \frac{\rho_c - \rho_0}{R}(R - r)$$

$$M = \int_0^R \int_0^\pi \int_0^{2\pi} \rho(r)\, r^2 \sin\theta \, d\phi \, d\theta \, dr$$

$$= 4\pi \int_0^R r^2 \left(\rho_0 + \frac{\rho_c - \rho_0}{R}(R - r) \right) dr$$

$$= 4\pi R^3 \left[\frac{\rho_c}{3} - \frac{(\rho_c - \rho_0)}{4} \right]$$

$$M = 4\pi R^3 \left[\frac{\rho_c}{12} + \frac{\rho_0}{4} \right]$$

c) $M_a = M_b$

$$\frac{4\pi R^3 \rho_p}{3} = 4\pi R^3 \left[\frac{\rho_c}{12} + \frac{\rho_0}{4} \right]$$

$$\rho_p = \frac{\rho_c}{4} + \frac{3}{4}\rho_0 \rightarrow \rho_c = 4\rho_p - 3\rho_0$$

$$\left.\begin{array}{l} \rho_p = 5{,}5 \ g/cm^3 \\ \rho_0 = 3 \ gr/cm^3 \end{array}\right\} \rho_c = 13 \ gr/cm^3$$