P3)

$y'' + w^2 y = 0$, $y = \sum_0^\infty a_n x^{n+s}$

$= x^s (a_0 + a_1 x + \dots)$

$\Rightarrow \quad y' = \sum_0 a_n \cdot (n+s)\, x^{n+s-1}$

$y'' = \sum_0 a_n (n+s)(n+s-1)\, x^{n+s-2}$

$\Rightarrow \sum_0 a_n (n+s)(n+s-1)\, x^{n+s-2} + w^2 \sum a_n x^{n+s} = 0$

$\sum_2 a_n (n+s)(n+s-1)\, x^{n+s-2} + a_0 s(s-1) x^{s-2} + a_1 (s+1) s x^{s-1} + w^2 \sum_0 a_n x^{n+s}$

$= a_0 s(s-1) x^{s-2} + a_1 (s+1) s\, x^{s-1} + \sum_0 [a_{n+2} (n+s+2)(n+s+1) + w^2 a_n]\, x^{n+s} = 0$

$\Rightarrow$ Dado que se debe anular por ordenes de x^β

x^{s-2} / $a_0 s(s-1) = 0 \quad \rightarrow \quad s = 1$

$\rightarrow \quad s = 0$

x^{s-1} / $a_1 (s+1) s = 0 \quad \rightsquigarrow$ Si $s = 1 \Rightarrow a_1 = 0$

$\rightsquigarrow$ Si $s = 0 \Rightarrow a_1$ libre

x^{s+n} / $a_{n+2} (n+s+2)(n+s+1) + w^2 a_n = 0$

$$\Rightarrow a_{n+2} = -a_n \frac{w^2}{(n+s+2)(n+s+1)}$$

$\Rightarrow \underline{s=0)}$ $a_1 = a_3 = a_5 = \dots = 0$

$\hookrightarrow \quad a_{n+2} = a_n \dfrac{-w^2}{(n+2)(n+1)}$

$\therefore \quad a_2 = -\dfrac{w^2}{2!} a_0$

$a_4 = \dfrac{w^4}{4!} a_0 \qquad \rightarrow \quad a_{2n} = (-1)^n \dfrac{w^{2n}}{(2n)!} a_0$

$a_6 = \dfrac{-w^6}{6!} a_0$

$\Rightarrow \quad y = a_0 x^0 \left(1 - \dfrac{(wx)^2}{2!} + \dots \right)$

$= a_0 \cdot \cos(wx)$

$\rightarrow \underline{s=1)}$ $\quad a_{n+2} = a_n \dfrac{-w^2}{(n+3)(n+2)}$

$\therefore \quad a_2 = \dfrac{-w^2}{3!} a_0$

$a_4 = \dfrac{+w^4}{5!} a_0 \qquad \rightarrow \quad a_{2n} = (-1)^{2n} \dfrac{w^{2n}}{(2n+1)!}$

$a_6 = -\dfrac{w^6}{7!} a_0$

$\Rightarrow \quad y = \frac{w}{w} a_0 x^1 \left(1 - \dfrac{(wx)^2}{2!} + \dots \right)$

$\Rightarrow \quad \boxed{y = \dfrac{a_0}{w} \sin(wx)}$

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Resumen

EDO $\rightarrow$ $y'' + P(x)\, y' + Q(x)\, y = 0$ (1)

1) Pto ordinario: Si $P(x_0)$ y $Q(x_0)$ convergen (finitos) $\Rightarrow$ x_0 pto ordinario

2) Pto singular: Si $P(x_0)$ o $Q(x_0)$ diverge $\Rightarrow$ x_0 pto singular

2.1) Regular: Si x_0 es un pto singular pero y/o $\left\{ \begin{array}{l} \cdot)\ (x-x_0)\, P(x) \xrightarrow{x \to x_0} \text{converge} \\ \cdot)\ (x-x_0)^2\, Q(x) \xrightarrow{x \to x_0} \text{converge} \end{array} \right.$

2.2) Irregular: Si x_0 es un pto singular pero $\quad$ % $\rightarrow$ div

↳ $P(x_0)$ diverge mas rapido que $(x-x_0)$ $\quad$ % $\rightarrow$ div

3) Analisis en $x \to \infty$ $\rightsquigarrow$ $y(x) \Rightarrow w(z) = y(z^{-1})$

$x = \frac{1}{z} \Rightarrow z \to \infty$

↳ $y'(x) = \dfrac{dy(x)}{dx} = \dfrac{dy(z^{-1})}{dz} \cdot \dfrac{dz}{dx}$

$= \dfrac{dw(z)}{dz} \cdot \left(-\frac{1}{x^2}\right) = -z^2 w'$

↳ $y''(x) = \quad \ldots \quad = z^4 w'' + 2z^3 w'$

$\therefore$ (1) queda $\rightarrow$ $z^4 w'' + \left[2z^3 - z^2 P(z^{-1})\right] w' + Q(z^{-1})\, w = 0$

$\rightarrow$ $\tilde{P}(z) = \dfrac{2z - P(z^{-1})}{z^2}$ y $\tilde{Q}(z) = \dfrac{Q(z^{-1})}{z^4}$