Pauta Auxiliar 12

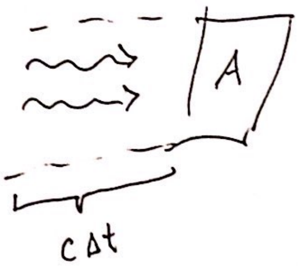
P1

$I \simeq 1300\, W/m^2$

Recordamos que para una onda EM, $I = \langle S \rangle = \frac{1}{2} c \varepsilon_0 E_0^2$

Por el otro lado tenemos una densidad de momentum asociada a las ondas. Promediado en el tiempo, esto es $\langle p \rangle = \frac{\varepsilon_0}{2c} E_0^2$

Durante un tiempo Δt (harto mayor que el periodo asociado a la onda), el "volumen de onda" que incide sobre el absorbedor será $A \cdot c \Delta t$



$\Rightarrow$ El momentum total que llega al absorbedor es $\frac{\varepsilon_0}{2\cancel{c}} E_0^2 \cdot A \cancel{c} \Delta t = \frac{E_0^2 A \Delta t}{2} \cdot \varepsilon_0$

$$F = \frac{\Delta p}{\Delta t} \Rightarrow F = \frac{E_0^2 A}{2} \cdot \varepsilon_0 \qquad P = \frac{F}{A} = \frac{E_0^2}{2} \cdot \varepsilon_0$$

Fuerza sobre el absorbedor, debida al cambio de momentum de la onda.

Presión

$$\left. \begin{array}{l} P = \frac{\varepsilon_0}{2} E_0^2 \\ I = \frac{\varepsilon_0 c E_0^2}{2} \end{array} \right\} I = cP \Rightarrow P = \frac{1300\, W/m^2}{3 \cdot 10^8\, m/s} \approx \boxed{4{,}3 \cdot 10^{-6}\, Pa}$$

Para un reflector perfecto, es el doble. ¿Por qué? Bueno, si la luz llega a un absorbedor, lo único que hace éste es "detenerla".

$p_i = -p \longrightarrow p_f = 0$

Absorbedor — Absorbedor

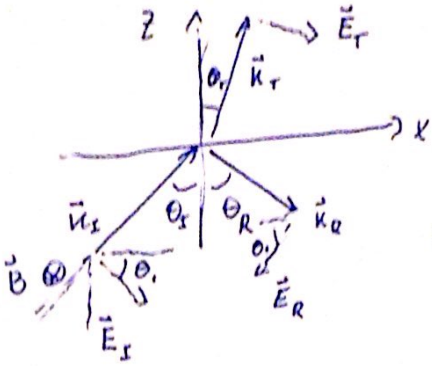
$\Delta p = p$

$p_i = -p \xrightarrow[\Delta p = 2p]{} p_f = p$

Reflector — Reflector

Para el reflector, el $\underline{\Delta p}$ de la onda es el doble, y la presión es proporcional a esto $\Rightarrow$ el reflector siente el doble de presión.

Parte 1 :



$\vec{B} = \frac{1}{v} \vec{k} \times \vec{E}$

$\vec{k} = \frac{\omega}{v} \hat{k} = n \frac{\omega}{c} \hat{k}$ $\quad |\vec{B}| = \frac{n}{c} |\vec{E}|$

Tipo "p" ($\vec{E}$ paralelo a plano de incidencia)

Cond. borde: distintas formas de decirlas

Forma "fancy" de decir condiciones de borde

1) $\varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp}$ $\qquad E_1^{\parallel} = E_2^{\parallel}$

$B_1^{\perp} = B_2^{\perp}$ $\qquad \frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel}$

2) $\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = 0$ $\qquad \hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$

$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$ $\qquad \hat{n} \times (\vec{H}_1 - \vec{H}_2) = 0$

Ahora, llegamos a Fresnel:

$\theta_I = \theta_R \equiv \theta_1$ por ley de reflexión ! , $\theta_T = \theta_2$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Snell)

$E_{\text{abajo}}^{\parallel} = E_{\text{arriba}}^{\parallel} \Rightarrow E_I \cos\theta_1 - E_R \cos\theta_1 = E_T \cos\theta_2$

$\frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel} \Rightarrow \frac{1}{\mu_1}(B_I + B_R) = \frac{1}{\mu_2} B_T \Rightarrow \frac{1}{\mu_1} n_1 (E_I + E_R) = n_2 E_T \cdot \frac{1}{\mu_2}$ $\left(n = \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0 \varepsilon_0}} \right)$

$\Rightarrow \sqrt{\frac{\varepsilon_1}{\mu_1}} (E_I + E_R) = \sqrt{\frac{\varepsilon_2}{\mu_2}} E_T \Rightarrow Z_2 (E_I + E_R) = Z_1 E_T$

Queremos $r_p \equiv \left(\frac{E_R}{E_I}\right)_p = ?$ $\quad$ Arriba $\cdot Z_1$, abajo $\cdot \cos\theta_2$

$\Rightarrow Z_1 (E_I \cos\theta_1 - E_R \cos\theta_1) = E_T Z_1 \cos\theta_2$

$\cos\theta_2 Z_2 (E_I + E_R) = Z_1 E_T \cos\theta_2$

$\Rightarrow E_I (Z_1 \cos\theta_1 - Z_2 \cos\theta_2) = E_R (Z_2 \cos\theta_2 + Z_1 \cos\theta_1)$

$$\Rightarrow \boxed{r_p = \frac{Z_1 \cos\theta_1 - Z_2 \cos\theta_2}{Z_2 \cos\theta_2 + Z_1 \cos\theta_1}}$$

Además definimos $t_p \equiv \left(\frac{E_T}{E_I}\right)_p = ?$

$$E_I\cos\theta_1 - E_R\cos\theta_1 = E_T\cos\theta_2 \quad /\cdot Z_2$$

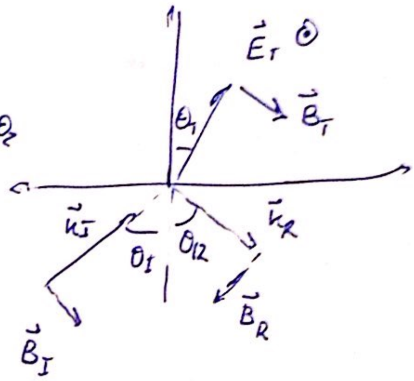
$$Z_2 E_I + Z_2 E_R = Z_1 E_T \quad /\cdot \cos\theta_1$$

$$E_I\cos\theta_1 Z_2 - E_R\cos\theta_1 Z_2 = E_T\cos\theta_2 Z_2$$

$$E_I\cos\theta_1 Z_2 + E_R\cos\theta_1 Z_2 = E_T\cos\theta_1 Z_1$$

$$2E_I\cos\theta_1 Z_2 = E_T(\cos\theta_1 Z_1 + \cos\theta_2 Z_2) \Rightarrow t_p = \frac{2\cancel{E_I}\cos\theta_1 Z_2}{Z_1\cos\theta_1 + Z_2\cos\theta_2}$$

Ahora caso $E \perp$ al plano, polarización tipo "s" senkrecht



$$E_1^{\parallel} = E_2^{\parallel} \Rightarrow E_I + E_R = E_T$$

$$\frac{1}{\mu_1}B_1^{\parallel} = \frac{1}{\mu_2}B_2^{\parallel} \Rightarrow \frac{1}{\mu_1}(B_I\cos\theta_1 - B_R\cos\theta_1) = \frac{1}{\mu_2}B_T\cos\theta_2$$

$$B = \frac{n}{c}E$$

$$\Rightarrow \frac{n_1}{\mu_1}(E_I\cos\theta_1 - E_R\cos\theta_1) = \frac{n_2}{\mu_2}E_R\cos\theta_2$$

$$\Rightarrow Z_2(E_I\cos\theta_1 - E_R\cos\theta_1) = Z_1 E_R\cos\theta_2$$

$$\rightsquigarrow r_s \equiv \left(\frac{E_R}{E_I}\right)_s = \frac{Z_2\cos\theta_1 - Z_1\cos\theta_2}{Z_2\cos\theta_1 + Z_1\cos\theta_2}$$

$$t_s \equiv \left(\frac{E_T}{E_I}\right)_s = \frac{2Z_2\cos\theta_1}{Z_2\cos\theta_1 + Z_1\cos\theta_2}$$

Matraca Fresnel

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

Parte 2: Teniendo r_s, r_p: (1er paso: $r_s, r_p \underbrace{(\mu, n)}_{\text{NO}}$ z!)

$$r_s = \frac{Z_2 \cos\theta_1 - Z_1 \cos\theta_2}{Z_2 \cos\theta_1 + Z_1 \cos\theta_2} \qquad r_p = \frac{Z_1 \cos\theta_1 - Z_2 \cos\theta_2}{Z_1 \cos\theta_1 + Z_2 \cos\theta_2}$$

$$\frac{Z_1}{Z_2} = \sqrt{\frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1}} = \frac{\mu_1 n_2}{\mu_2 n_1} \qquad \left(n = \frac{c}{v} = c\sqrt{\mu\varepsilon}\right)$$

$$\Rightarrow r_s = \frac{\cos\theta_1 - \frac{\mu_1 n_2}{\mu_2 n_1}\cos\theta_2}{\cos\theta_1 + \frac{\mu_1 n_2}{\mu_2 n_1}\cos\theta_2} = \frac{\mu_2 n_1 \cos\theta_1 - \mu_1 n_2 \cos\theta_2}{\mu_2 n_1 \cos\theta_1 + \mu_1 n_2 \cos\theta_2}$$

$$r_p = \frac{\mu_1 n_2 \cos\theta_1 - \mu_2 n_1 \cos\theta_2}{\mu_1 n_2 \cos\theta_1 + \mu_2 n_1 \cos\theta_2}$$

Ahora, si tenemos materiales no magnéticos, $\mu_1 = \mu_2 = \mu_0$

$$\Rightarrow r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \qquad n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow r_s = \frac{n_1 n_2}{n_1 n_2} \cdot \frac{\cos\theta_1 \sin\theta_2 - \cos\theta_2 \sin\theta_1}{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1}$$

$$\Rightarrow \boxed{r_s = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}}$$

Para la otra es parecido,

$\sin^2\theta_2 + \cos^2\theta_2$ $\qquad$ $\sin^2\theta_1 + \cos^2\theta_1$

$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{n_1 n_2}{n_1 n_2} \cdot \frac{\cos\theta_1 \sin\theta_1 - \cos\theta_2 \sin\theta_2}{\cos\theta_1 \sin\theta_1 + \cos\theta_2 \sin\theta_2}$$

$$= \frac{\cos\theta_1 \sin\theta_1 \sin^2\theta_2 + \cos\theta_1 \sin\theta_1 \cos^2\theta_2 - \cos\theta_2 \sin\theta_2 \sin^2\theta_1 - \cos\theta_2 \sin\theta_2 \cos^2\theta_1}{\cos\theta_1 \sin\theta_1 \sin^2\theta_2 + \cos\theta_1 \sin\theta_1 \cos^2\theta_2 + \cos\theta_2 \sin\theta_2 \sin^2\theta_1 + \cos\theta_2 \sin\theta_2 \cos^2\theta_1}$$

$$= \frac{\sin\theta_1 \cos\theta_2 (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + \sin\theta_2 \cos\theta_1 (\sin\theta_1 \sin\theta_2 - \cos\theta_1 \cos\theta_2)}{\sin\theta_1 \cos\theta_2 (\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2) + \sin\theta_2 \cos\theta_1 (\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2)}$$

$$= \frac{\sin(\theta_1 - \theta_2) \cdot \cos(\theta_1 + \theta_2)}{\sin(\theta_1 + \theta_2) \cdot \cos(\theta_1 - \theta_2)} = \boxed{\frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}}$$