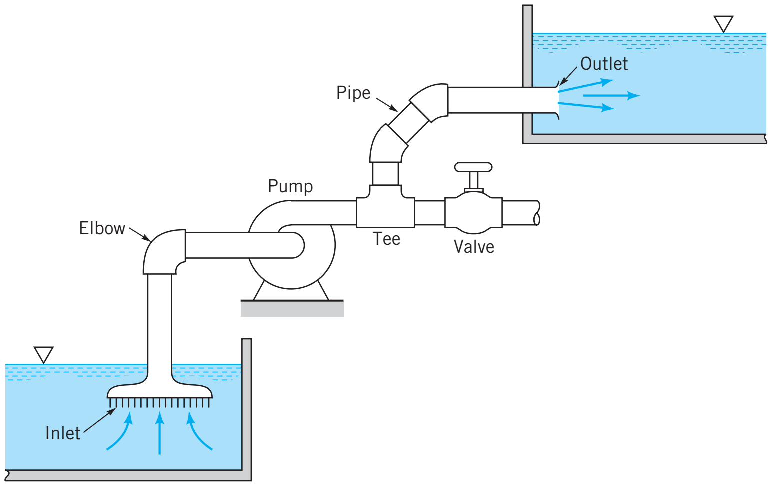
8 Viscous Flow in Pipes

In the previous chapters we have considered a variety of topics concerning the motion of fluids. The basic governing principles concerning mass, momentum, and energy were developed and applied, in conjunction with rather severe assumptions, to numerous flow situations. In this chapter we will apply the basic principles to a specific, important topic—the flow of viscous, incompressible fluids in pipes and ducts.

The transport of a fluid (liquid or gas) in a closed conduit (commonly called a *pipe* if it is of round cross section or a *duct* if it is not round) is extremely important in our daily operations. A brief consideration of the world around us will indicate that there is a wide variety of applications of pipe flow. Such applications range from the large, man-made Alaskan pipeline that carries crude oil almost 800 miles across Alaska, to the more complex (and certainly not less useful) natural systems of "pipes" that carry blood throughout our body and air into and out of our lungs. Other examples include the water pipes in our homes and the distribution system that delivers the water from the city well to the house. Numerous hoses and pipes carry hydraulic fluid or other fluids to various components of vehicles and machines. The air quality within our buildings is maintained at comfortable levels by the distribution of conditioned (heated, cooled, humidified/dehumidified) air through a maze of pipes and ducts. Although all of these systems are different, the fluid-mechanics principles governing the fluid motions are common. The purpose of this chapter is to understand the basic processes involved in such flows.

Pipe flow is very important in our daily operations.

Some of the basic components of a typical *pipe system* are shown in Fig. 8.1. They include the pipes themselves (perhaps of more than one diameter), the various fittings used to connect the individual pipes to form the desired system, the flowrate control devices (valves), and the pumps or turbines that add energy to or remove energy from the fluid. Even the most simple pipe systems are actually quite complex when they are viewed in terms of rigorous analytical considerations. We will use an "exact" analysis of the simplest pipe flow topics (such as laminar flow in long, straight, constant diameter pipes) and dimensional analysis considerations combined with experimental results for the other pipe flow topics. Such an approach is not unusual in fluid mechanics investigations. When "real world" effects are important (such as viscous effects in pipe flows), it is often difficult or "impossible" to use



■ FIGURE 8.1 **Typical pipe system components.**

only theoretical methods to obtain the desired results. A judicious combination of experimental data with theoretical considerations and dimensional analysis often provides the desired results. The flow in pipes discussed in this chapter is an example of such an analysis.

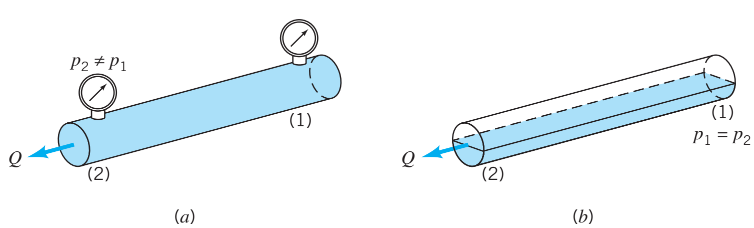
8.1 General Characteristics of Pipe Flow

Before we apply the various governing equations to pipe flow examples, we will discuss some of the basic concepts of pipe flow. With these ground rules established we can then proceed to formulate and solve various important flow problems.

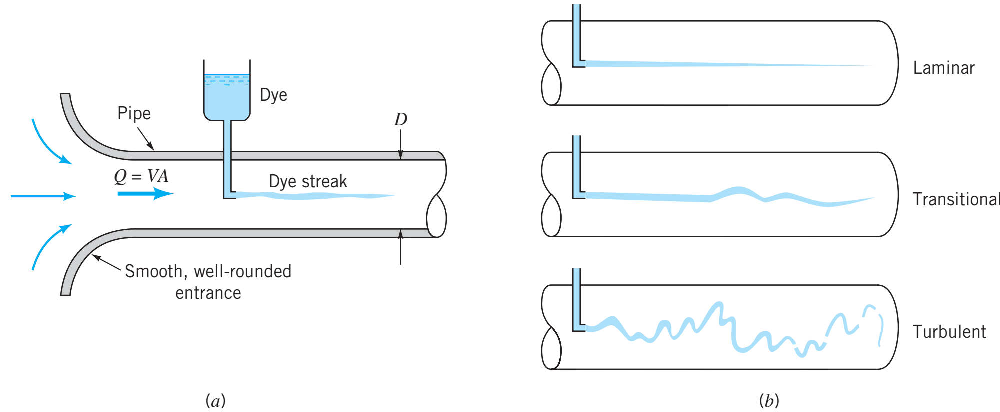
Although not all conduits used to transport fluid from one location to another are round in cross section, most of the common ones are. These include typical water pipes, hydraulic hoses, and other conduits that are designed to withstand a considerable pressure difference across their walls without undue distortion of their shape. Typical conduits of noncircular cross section include heating and air conditioning ducts that are often of rectangular cross section. Normally the pressure difference between the inside and outside of these ducts is relatively small. Most of the basic principles involved are independent of the cross-sectional shape, although the details of the flow may be dependent on it. Unless otherwise specified, we will assume that the conduit is round, although we will show how to account for other shapes.

The pipe is assumed to be completely full of the flowing fluid.

For all flows involved in this chapter, we assume that the pipe is completely filled with the fluid being transported as is shown in Fig. 8.2*a*. Thus, we will not consider a concrete pipe through which rainwater flows without completely filling the pipe, as is shown in Fig. 8.2*b*. Such flows, called open-channel flow, are treated in **Chapter 10**. The difference between open-channel flow and the pipe flow of this chapter is in the fundamental mechanism that drives the flow. For open-channel flow, gravity alone is the driving force—the wa-



■ FIGURE 8.2 **(*a*) Pipe flow. (*a*) Open-channel flow.**



■ **FIGURE 8.3** **(*a*) Experiment to illustrate type of flow. (*b*) Typical dye streaks.**

ter flows down a hill. For pipe flow, gravity may be important (the pipe need not be horizontal), but the main driving force is likely to be a pressure gradient along the pipe. If the pipe is not full, it is not possible to maintain this pressure difference, $p_1 - p_2$.

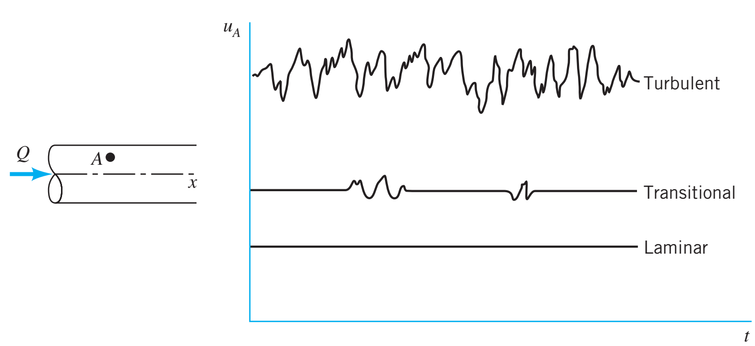
8.1.1 Laminar or Turbulent Flow

V8.1 Laminar/ turbulent pipe flow

A flow may be laminar, transitional, or turbulent.

The flow of a fluid in a pipe may be laminar flow or it may be turbulent flow. **Osborne Reynolds** (1842–1912), a British scientist and mathematician, was the first to distinguish the difference between these two classifications of flow by using a simple apparatus as shown in Fig. 8.3*a*. If water runs through a pipe of diameter D with an average velocity V, the following characteristics are observed by injecting neutrally buoyant dye as shown. For "small enough flowrates" the dye streak (a streakline) will remain as a well-defined line as it flows along, with only slight blurring due to molecular diffusion of the dye into the surrounding water. For a somewhat larger "intermediate flowrate" the dye streak fluctuates in time and space, and intermittent bursts of irregular behavior appear along the streak. On the other hand, for "large enough flowrates" the dye streak almost immediately becomes blurred and spreads across the entire pipe in a random fashion. These three characteristics, denoted as *laminar*, *transitional*, and *turbulent* flow, respectively, are illustrated in Fig. 8.3*b*.

The curves shown in Fig. 8.4 represent the x component of the velocity as a function of time at a point A in the flow. The random fluctuations of the turbulent flow (with the associated particle mixing) are what disperse the dye throughout the pipe and cause the blurred appearance illustrated in Fig. 8.3*b*. For laminar flow in a pipe there is only one component



■ **FIGURE 8.4** **Time dependence of fluid velocity at a point.**

of velocity, $\mathbf{V} = u\hat{\mathbf{i}}$. For turbulent flow the predominant component of velocity is also along the pipe, but it is unsteady (random) and accompanied by random components normal to the pipe axis, $\mathbf{V} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}$. Such motion in a typical flow occurs too fast for our eyes to follow. Slow motion pictures of the flow can more clearly reveal the irregular, random, turbulent nature of the flow.

Pipe flow characteristics are dependent on the value of the Reynolds number.

As was discussed in **Chapter 7**, we should not label dimensional quantities as being "large" or "small," such as "small enough flowrates" in the preceding paragraphs. Rather, the appropriate dimensionless quantity should be identified and the "small" or "large" character attached to it. A quantity is "large" or "small" only relative to a reference quantity. The ratio of those quantities results in a dimensionless quantity. For pipe flow the most important dimensionless parameter is the Reynolds number, Re—the ratio of the inertia to viscous effects in the flow. Hence, in the previous paragraph the term flowrate should be replaced by Reynolds number, $\text{Re} = \rho VD/\mu$, where V is the average velocity in the pipe. That is, the flow in a pipe is laminar, transitional, or turbulent provided the Reynolds number is "small enough," "intermediate," or "large enough." It is not only the fluid velocity that determines the character of the flow—its density, viscosity, and the pipe size are of equal importance. These parameters combine to produce the Reynolds number. The distinction between laminar and turbulent pipe flow and its dependence on an appropriate dimensionless quantity was first pointed out by Osborne Reynolds in 1883.

The Reynolds number ranges for which laminar, transitional, or turbulent pipe flows are obtained cannot be precisely given. The actual transition from laminar to turbulent flow may take place at various Reynolds numbers, depending on how much the flow is disturbed by vibrations of the pipe, roughness of the entrance region, and the like. For general engineering purposes (i.e., without undue precautions to eliminate such disturbances), the following values are appropriate: The flow in a round pipe is laminar if the Reynolds number is less than approximately 2100. The flow in a round pipe is turbulent if the Reynolds number is greater than approximately 4000. For Reynolds numbers between these two limits, the flow may switch between laminar and turbulent conditions in an apparently random fashion (transitional flow).

EXAMPLE 8.1

Water at a temperature of 50 °F flows through a pipe of diameter $D = 0.73$ in. (a) Determine the minimum time taken to fill a 12-oz glass (volume $= 0.0125\ \text{ft}^3$) with water if the flow in the pipe is to be laminar. (b) Determine the maximum time taken to fill the glass if the flow is to be turbulent. Repeat the calculations if the water temperature is 140 °F.

SOLUTION

(a) If the flow in the pipe is to remain laminar, the minimum time to fill the glass will occur if the Reynolds number is the maximum allowed for laminar flow, typically $\text{Re} = \rho VD/\mu = 2100$. Thus, $V = 2100\,\mu/\rho D$, where from Table B.1, $\rho = 1.94$ slugs/ft^3 and $\mu = 2.73 \times 10^{-5}\ \text{lb} \cdot \text{s/ft}^2$ at 50 °F, while $\rho = 1.91\ \text{slugs/ft}^3$ and $\mu = 0.974 \times 10^{-5}\ \text{lb} \cdot \text{s/ft}^2$ at 140 °F. Thus, the maximum average velocity for laminar flow in the pipe is

$$V = \frac{2100\mu}{\rho D} = \frac{2100(2.73 \times 10^{-5}\ \text{lb} \cdot \text{s/ft}^2)}{(1.94\ \text{slugs/ft}^3)(0.73/12\ \text{ft})} = 0.486\ \text{lb} \cdot \text{s/slug}$$
$$= 0.486\ \text{ft/s}$$

Similarly, $V = 0.176$ ft/s at 140 °F. With $\forall$ = volume of glass and $\forall = Qt$ we obtain

$$t = \frac{\forall}{Q} = \frac{\forall}{(\pi/4)D^2V} = \frac{4(0.0125 \text{ ft}^3)}{(\pi[0.73/12]^2\text{ft}^2)(0.486 \text{ ft/s})}$$

$$= 8.85 \text{ s at } T = 50 \text{ °F} \qquad \textbf{(Ans)}$$

Similarly, $t = 24.4$ s at 140 °F. To maintain laminar flow, the less viscous hot water requires a lower flowrate than the cold water.

(b) If the flow in the pipe is to be turbulent, the maximum time to fill the glass will occur if the Reynolds number is the minimum allowed for turbulent flow, Re = 4000. Thus, $V = 4000\mu/\rho D = 0.925$ ft/s and $t = 4.65$ s at 50 °F, while $V = 0.335$ ft/s and $t = 12.8$ s at 140 °F.

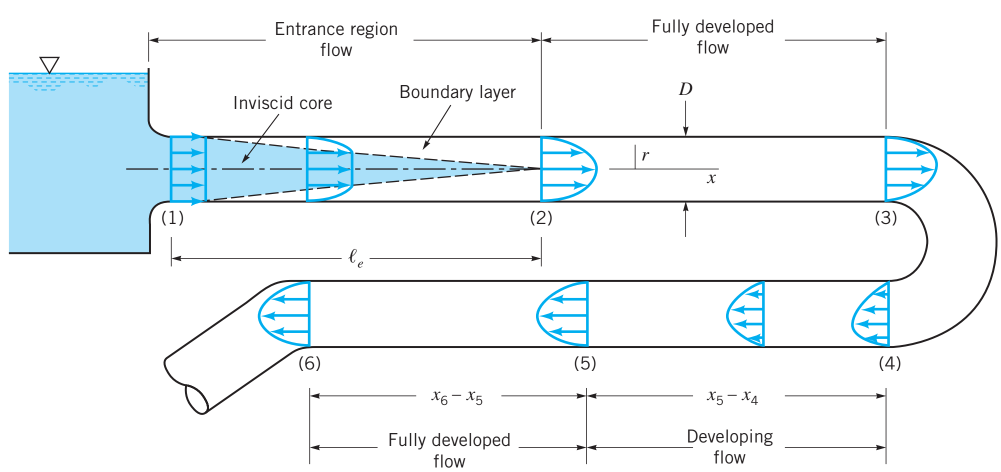
Note that because water is "not very viscous," the velocity must be "fairly small" to maintain laminar flow. In general, turbulent flows are encountered more often than laminar flows because of the relatively small viscosity of most common fluids (water, gasoline, air). If the flowing fluid had been honey with a kinematic viscosity ($\nu = \mu/\rho$) 3000 times greater than that of water, the above velocities would be increased by a factor of 3000 and the times reduced by the same factor. As we will see in the following sections, the pressure needed to force a very viscous fluid through a pipe at such a high velocity may be unreasonably large.

8.1.2 Entrance Region and Fully Developed Flow

Any fluid flowing in a pipe had to enter the pipe at some location. The region of flow near where the fluid enters the pipe is termed the *entrance region* and is illustrated in Fig. 8.5. It may be the first few feet of a pipe connected to a tank or the initial portion of a long run of a hot air duct coming from a furnace.

Flow in the entrance region of a pipe is quite complex.

As is shown in Fig. 8.5, the fluid typically enters the pipe with a nearly uniform velocity profile at section (1). As the fluid moves through the pipe, viscous effects cause it to stick to the pipe wall (the no-slip boundary condition). This is true whether the fluid is relatively inviscid air or a very viscous oil. Thus, a *boundary layer* in which viscous effects are important is produced along the pipe wall such that the initial velocity profile changes with distance along the pipe, x, until the fluid reaches the end of the entrance length, section (2), beyond which the velocity profile does not vary with x. The boundary layer has grown in



■ **FIGURE 8.5 Entrance region, developing flow, and fully developed flow in a pipe system.**

thickness to completely fill the pipe. Viscous effects are of considerable importance within the boundary layer. For fluid outside the boundary layer [within the *inviscid core* surrounding the centerline from (1) to (2)], viscous effects are negligible.

The shape of the velocity profile in the pipe depends on whether the flow is laminar or turbulent, as does the length of the entrance region, ℓ_e. As with many other properties of pipe flow, the dimensionless *entrance length*, ℓ_e/D, correlates quite well with the Reynolds number. Typical entrance lengths are given by

$$\frac{\ell_e}{D} = 0.06 \text{ Re for laminar flow} \tag{8.1}$$

The entrance length is a function of the Reynolds number.

and

$$\frac{\ell_e}{D} = 4.4 \, (\text{Re})^{1/6} \text{ for turbulent flow} \tag{8.2}$$

For very low Reynolds number flows the entrance length can be quite short ($\ell_e = 0.6D$ if $\text{Re} = 10$), whereas for large Reynolds number flows it may take a length equal to many pipe diameters before the end of the entrance region is reached ($\ell_e = 120D$ for $\text{Re} = 2000$). For many practical engineering problems, $10^4 < \text{Re} < 10^5$ so that $20D < \ell_e < 30D$.

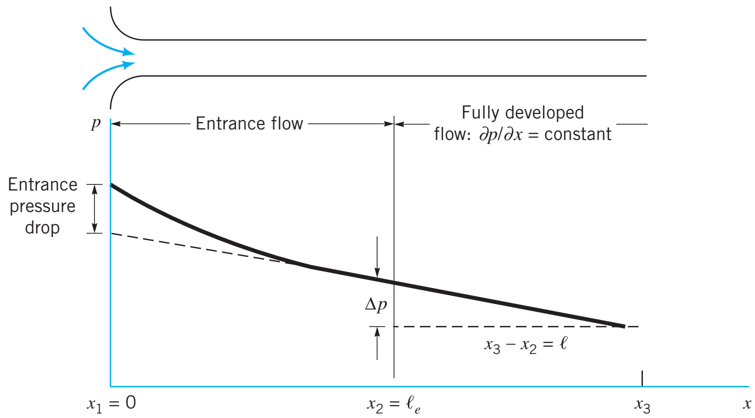
Calculation of the velocity profile and pressure distribution within the entrance region is quite complex. However, once the fluid reaches the end of the entrance region, section (2) of Fig. 8.5, the flow is simpler to describe because the velocity is a function of only the distance from the pipe centerline, r, and independent of x. This is true until the character of the pipe changes in some way, such as a change in diameter, or the fluid flows through a bend, valve, or some other component at section (3). The flow between (2) and (3) is termed *fully developed*. Beyond the interruption of the fully developed flow [at section (4)], the flow gradually begins its return to its fully developed character [section (5)] and continues with this profile until the next pipe system component is reached [section (6)]. In many cases the pipe is long enough so that there is a considerable length of fully developed flow compared with the developing flow length $[(x_3 - x_2) \gg \ell_e \text{ and } (x_6 - x_5) \gg (x_5 - x_4)]$. In other cases the distances between one component (bend, tee, valve, etc.) of the pipe system and the next component is so short that fully developed flow is never achieved.

8.1.3 Pressure and Shear Stress

Fully developed steady flow in a constant diameter pipe may be driven by gravity and/or pressure forces. For horizontal pipe flow, gravity has no effect except for a hydrostatic pressure variation across the pipe, γD, that is usually negligible. It is the pressure difference, $\Delta p = p_1 - p_2$, between one section of the horizontal pipe and another which forces the fluid through the pipe. Viscous effects provide the restraining force that exactly balances the pressure force, thereby allowing the fluid to flow through the pipe with no acceleration. If viscous effects were absent in such flows, the pressure would be constant throughout the pipe, except for the hydrostatic variation.

In non-fully developed flow regions, such as the entrance region of a pipe, the fluid accelerates or decelerates as it flows (the velocity profile changes from a uniform profile at the entrance of the pipe to its fully developed profile at the end of the entrance region). Thus, in the entrance region there is a balance between pressure, viscous, and inertia (acceleration) forces. The result is a pressure distribution along the horizontal pipe as shown in Fig. 8.6. The magnitude of the pressure gradient, $\partial p/\partial x$, is larger in the entrance region than in the fully developed region, where it is a constant, $\partial p/\partial x = -\Delta p/\ell < 0$.

The fact that there is a nonzero pressure gradient along the horizontal pipe is a result of viscous effects. As is discussed in Chapter 3, if the viscosity were zero, the pressure would not vary with x. The need for the pressure drop can be viewed from two different standpoints.



■ **FIGURE 8.6** **Pressure distribution along a horizontal pipe.**

In terms of a force balance, the pressure force is needed to overcome the viscous forces generated. In terms of an energy balance, the work done by the pressure force is needed to overcome the viscous dissipation of energy throughout the fluid. If the pipe is not horizontal, the pressure gradient along it is due in part to the component of weight in that direction. As is discussed in **Section 8.2.1**, this contribution due to the weight either enhances or retards the flow, depending on whether the flow is downhill or uphill.

Laminar flow characteristics are different than those for turbulent flow.

The nature of the pipe flow is strongly dependent on whether the flow is laminar or turbulent. This is a direct consequence of the differences in the nature of the shear stress in laminar and turbulent flows. As is discussed in some detail in **Section 8.3.3**, the shear stress in laminar flow is a direct result of momentum transfer among the randomly moving molecules (a microscopic phenomenon). The shear stress in turbulent flow is largely a result of momentum transfer among the randomly moving, finite-sized bundles of fluid particles (a macroscopic phenomenon). The net result is that the physical properties of the shear stress are quite different for laminar flow than for turbulent flow.

8.2 Fully Developed Laminar Flow

As is indicated in the previous section, the flow in long, straight, constant diameter sections of a pipe becomes fully developed. That is, the velocity profile is the same at any cross section of the pipe. Although this is true whether the flow is laminar or turbulent, the details of the velocity profile (and other flow properties) are quite different for these two types of flow. As will be seen in the remainder of this chapter, knowledge of the velocity profile can lead directly to other useful information such as pressure drop, head loss, flowrate, and the like. Thus, we begin by developing the equation for the velocity profile in fully developed laminar flow. If the flow is not fully developed, a theoretical analysis becomes much more complex and is outside the scope of this text. If the flow is turbulent, a rigorous theoretical analysis is as yet not possible.

Although most flows are turbulent rather than laminar, and many pipes are not long enough to allow the attainment of fully developed flow, a theoretical treatment and full understanding of fully developed laminar flow is of considerable importance. First, it represents one of the few theoretical viscous analyses that can be carried out "exactly" (within the framework of quite general assumptions) without using other ad hoc assumptions or approximations. An understanding of the method of analysis and the results obtained provides a foundation from which to carry out more complicated analyses. Second, there are many practical situations involving the use of fully developed laminar pipe flow.

There are numerous ways to derive important results pertaining to fully developed laminar flow. Three alternatives include: (1) from $\mathbf{F} = m\mathbf{a}$ applied directly to a fluid element, (2) from the Navier–Stokes equations of motion, and (3) from dimensional analysis methods.

8.2.1 From $\mathbf{F} = m\mathbf{a}$ Applied Directly to a Fluid Element

We consider the fluid element at time t as is shown in Fig. 8.7. It is a circular cylinder of fluid of length ℓ and radius r centered on the axis of a horizontal pipe of diameter D. Because the velocity is not uniform across the pipe, the initially flat ends of the cylinder of fluid at time t become distorted at time $t + \delta t$ when the fluid element has moved to its new location along the pipe as shown in the figure. If the flow is fully developed and steady, the distortion on each end of the fluid element is the same, and no part of the fluid experiences any acceleration as it flows. The local acceleration is zero ($\partial \mathbf{V}/\partial t = 0$) because the flow is steady, and the convective acceleration is zero ($\mathbf{V} \cdot \nabla \mathbf{V} = u\, \partial u/\partial x\, \hat{\mathbf{i}} = 0$) because the flow is fully developed. Thus, every part of the fluid merely flows along its pathline parallel to the pipe walls with constant velocity, although neighboring particles have slightly different velocities. The velocity varies from one pathline to the next. This velocity variation, combined with the fluid viscosity, produces the shear stress.

Steady, fully developed pipe flow experiences no acceleration.

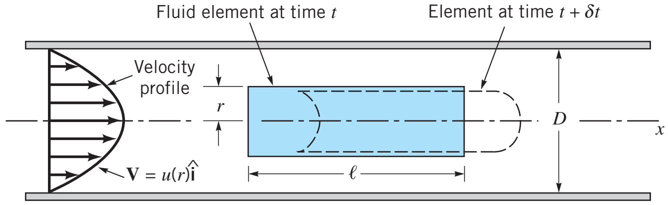
If gravitational effects are neglected, the pressure is constant across any vertical cross section of the pipe, although it varies along the pipe from one section to the next. Thus, if the pressure is $p = p_1$ at section (1), it is $p_2 = p_1 - \Delta p$ at section (2). We anticipate the fact that the pressure decreases in the direction of flow so that $\Delta p > 0$. A shear stress, τ, acts on the surface of the cylinder of fluid. This viscous stress is a function of the radius of the cylinder, $\tau = \tau(r)$.

As was done in fluid statics analysis (Chapter 2), we isolate the cylinder of fluid as is shown in Fig. 8.8 and apply Newton's second law, $F_x = ma_x$. In this case even though the fluid is moving, it is not accelerating, so that $a_x = 0$. Thus, fully developed horizontal pipe flow is merely a balance between pressure and viscous forces—the pressure difference acting on the end of the cylinder of area πr^2, and the shear stress acting on the lateral surface of the cylinder of area $2\pi r\ell$. This force balance can be written as

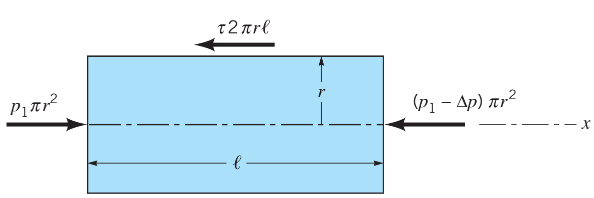
$$(p_1)\pi r^2 - (p_1 - \Delta p)\pi r^2 - (\tau)2\pi r\ell = 0$$

which can be simplified to give

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r} \tag{8.3}$$



■ **FIGURE 8.7** **Motion of a cylindrical fluid element within a pipe.**



■ **FIGURE 8.8** **Free-body diagram of a cylinder of fluid.**

Equation 8.3 represents the basic balance in forces needed to drive each fluid particle along the pipe with constant velocity. Since neither Δp nor ℓ are functions of the radial coordinate, r, it follows that $2\tau/r$ must also be independent of r. That is, $\tau = Cr$, where C is a constant. At $r = 0$ (the centerline of the pipe) there is no shear stress ($\tau = 0$). At $r = D/2$ (the pipe wall) the shear stress is a maximum, denoted τ_w, the *wall shear stress*. Hence, $C = 2\tau_w/D$ and the shear stress distribution throughout the pipe is a linear function of the radial coordinate

$$\tau = \frac{2\tau_w r}{D} \tag{8.4}$$

as is indicated in Fig. 8.9. The linear dependence of τ on r is a result of the pressure force being proportional to r^2 (the pressure acts on the end of the fluid cylinder; area $= \pi r^2$) and the shear force being proportional to r (the shear stress acts on the lateral sides of the cylinder; area $= 2\pi r\ell$). If the viscosity were zero there would be no shear stress, and the pressure would be constant throughout the horizontal pipe ($\Delta p = 0$). As is seen from Eqs. 8.3 and 8.4, the pressure drop and wall shear stress are related by

Basic pipe flow is governed by a balance between viscous and pressure forces.

$$\Delta p = \frac{4\ell\tau_w}{D} \tag{8.5}$$

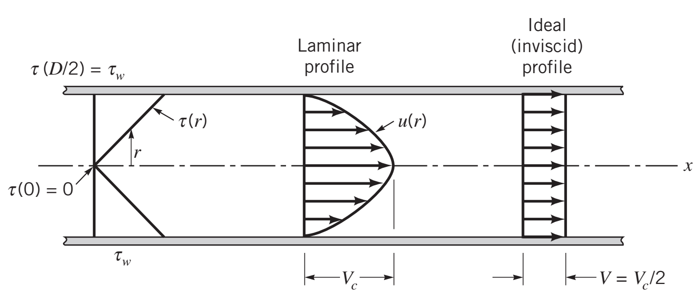
A small shear stress can produce a large pressure difference if the pipe is relatively long ($\ell/D \gg 1$).

Although we are discussing laminar flow, a closer consideration of the assumptions involved in the derivation of Eqs. 8.3, 8.4, and 8.5 reveals that these equations are valid for both laminar and turbulent flow. To carry the analysis further we must prescribe how the shear stress is related to the velocity. This is the critical step that separates the analysis of laminar from that of turbulent flow—from being able to solve for the laminar flow properties and not being able to solve for the turbulent flow properties without additional ad hoc assumptions. As is discussed in Section 8.3, the shear stress dependence for turbulent flow is very complex. However, for laminar flow of a Newtonian fluid, the shear stress is simply proportional to the velocity gradient, "$\tau = \mu\, du/dy$" (see Section 1.6). In the notation associated with our pipe flow, this becomes

$$\tau = -\mu \frac{du}{dr} \tag{8.6}$$

The negative sign is included to give $\tau > 0$ with $du/dr < 0$ (the velocity decreases from the pipe centerline to the pipe wall).

Equations 8.3 and 8.6 represent the two governing laws for fully developed laminar flow of a Newtonian fluid within a horizontal pipe. The one is Newton's second law of motion and the other is the definition of a Newtonian fluid. By combining these two equations we obtain



■ **FIGURE 8.9** **Shear stress distribution within the fluid in a pipe (laminar or turbulent flow) and typical velocity profiles.**

$$\frac{du}{dr} = -\left(\frac{\Delta p}{2\mu\ell}\right) r$$

which can be integrated to give the velocity profile as follows:

$$\int du = -\frac{\Delta p}{2\mu\ell} \int r\, dr$$

or

$$u = -\left(\frac{\Delta p}{4\mu\ell}\right) r^2 + C_1$$

where C_1 is a constant. Because the fluid is viscous it sticks to the pipe wall so that $u = 0$ at $r = D/2$. Thus, $C_1 = (\Delta p/16\mu\ell)D^2$. Hence, the velocity profile can be written as

$$u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right] \tag{8.7}$$

Under certain restrictions the velocity profile in a pipe is parabolic.

where $V_c = \Delta p D^2/(16\mu\ell)$ is the centerline velocity. An alternative expression can be written by using the relationship between the wall shear stress and the pressure gradient (Eqs. 8.5 and 8.7) to give

$$u(r) = \frac{\tau_w D}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where $R = D/2$ is the pipe radius.

This velocity profile, plotted in Fig. 8.9, is parabolic in the radial coordinate, r, has a maximum velocity, V_c, at the pipe centerline, and a minimum velocity (zero) at the pipe wall. The volume flowrate through the pipe can be obtained by integrating the velocity profile across the pipe. Since the flow is axisymmetric about the centerline, the velocity is constant on small area elements consisting of rings of radius r and thickness dr. Thus,

$$Q = \int u\, dA = \int_{r=0}^{r=R} u(r) 2\pi r\, dr = 2\pi\, V_c \int_0^R \left[1 - \left(\frac{r}{R}\right)^2\right] r\, dr$$

or

$$Q = \frac{\pi R^2 V_c}{2}$$

By definition, the average velocity is the flowrate divided by the cross-sectional area, $V = Q/A = Q/\pi R^2$, so that for this flow

$$V = \frac{\pi R^2 V_c}{2\pi R^2} = \frac{V_c}{2} = \frac{\Delta p D^2}{32\mu\ell} \tag{8.8}$$

and

$$Q = \frac{\pi D^4\, \Delta p}{128\mu\ell} \tag{8.9}$$

As is indicated in Eq. 8.8, the average velocity is one-half of the maximum velocity. In general, for velocity profiles of other shapes (such as for turbulent pipe flow), the average velocity is not merely the average of the maximum (V_c) and minimum (0) velocities as it is for the laminar parabolic profile. The two velocity profiles indicated in Fig. 8.9 provide the same flowrate—one is the fictitious ideal ($\mu = 0$) profile; the other is the actual laminar flow profile.

Poiseuille's law is valid for laminar flow only.

The above results confirm the following properties of laminar pipe flow. For a horizontal pipe the flowrate is (a) directly proportional to the pressure drop, (b) inversely proportional to the viscosity, (c) inversely proportional to the pipe length, and (d) proportional to the pipe diameter to the fourth power. With all other parameters fixed, an increase in diameter by a factor of 2 will increase the flowrate by a factor of 16—the flowrate is very strongly dependent on pipe size. A 2% error in diameter gives an 8% error in flowrate ($Q \sim D^4$ or $\delta Q \sim 4D^3\,\delta D$, so that $\delta Q/Q = 4\,\delta D/D$). This flow, the properties of which were first established experimentally by two independent workers, **G. Hagen** (1797–1884) in 1839 and **J. Poiseuille** (1799–1869) in 1840, is termed *Hagen–Poiseuille flow*. Equation 8.9 is commonly referred to as Poiseuille's law. Recall that all of these results are restricted to laminar flow (those with Reynolds numbers less than approximately 2100) in a horizontal pipe.

The adjustment necessary to account for nonhorizontal pipes, as shown in Fig. 8.10, can be easily included by replacing the pressure drop, Δp, by the combined effect of pressure and gravity, $\Delta p - \gamma \ell \sin\theta$, where θ is the angle between the pipe and the horizontal. (Note that $\theta > 0$ if the flow is uphill, while $\theta < 0$ if the flow is downhill.) This can be seen from the force balance in the x direction (along the pipe axis) on the cylinder of fluid shown in Fig. 8.10*b*. The method is exactly analogous to that used to obtain the Bernoulli equation (Eq. 3.6) when the streamline is not horizontal. The net force in the x direction is a combination of the pressure force in that direction, $\Delta p \pi r^2$, and the component of weight in that direction, $-\gamma \pi r^2 \ell \sin\theta$. The result is a slightly modified form of Eq. 8.3 given by

$$\frac{\Delta p - \gamma \ell \sin\theta}{\ell} = \frac{2\tau}{r} \tag{8.10}$$

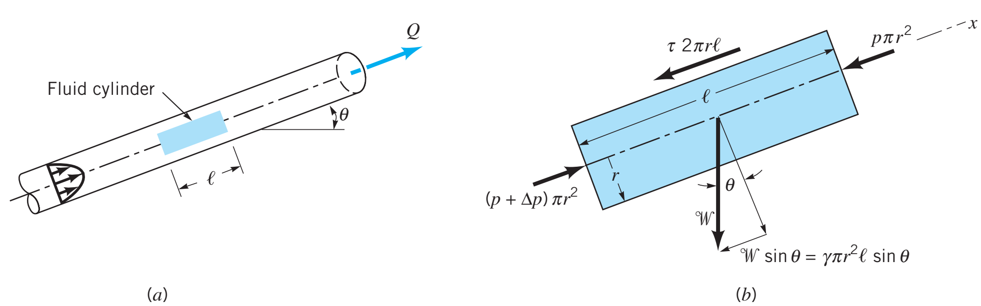
Thus, all of the results for the horizontal pipe are valid provided the pressure gradient is adjusted for the elevation term, that is, Δp is replaced by $\Delta p - \gamma \ell \sin\theta$ so that

$$V = \frac{(\Delta p - \gamma \ell \sin\theta)D^2}{32\mu\ell} \tag{8.11}$$

and

$$Q = \frac{\pi(\Delta p - \gamma \ell \sin\theta)D^4}{128\mu\ell} \tag{8.12}$$

It is seen that the driving force for pipe flow can be either a pressure drop in the flow direction, Δp, or the component of weight in the flow direction, $-\gamma \ell \sin\theta$. If the flow is downhill, gravity helps the flow (a smaller pressure drop is required; $\sin\theta < 0$). If the flow is uphill,



■ **FIGURE 8.10 Free-body diagram of a fluid cylinder for flow in a nonhorizontal pipe.**

gravity works against the flow (a larger pressure drop is required; $\sin\theta > 0$). Note that $\gamma\ell\sin\theta = \gamma\Delta z$ (where Δz is the change in elevation) is a hydrostatic type pressure term. If there is no flow, $V = 0$ and $\Delta p = \gamma\ell\sin\theta = \gamma\Delta z$, as expected for fluid statics.

EXAMPLE 8.2

An oil with a viscosity of $\mu = 0.40\ \text{N}\cdot\text{s/m}^2$ and density $\rho = 900\ \text{kg/m}^3$ flows in a pipe of diameter $D = 0.020$ m. (a) What pressure drop, $p_1 - p_2$, is needed to produce a flowrate of $Q = 2.0 \times 10^{-5}\ \text{m}^3/\text{s}$ if the pipe is horizontal with $x_1 = 0$ and $x_2 = 10$ m? (b) How steep a hill, θ, must the pipe be on if the oil is to flow through the pipe at the same rate as in part (a), but with $p_1 = p_2$? (c) For the conditions of part (b), if $p_1 = 200$ kPa, what is the pressure at section $x_3 = 5$ m, where x is measured along the pipe?

SOLUTION

(a) If the Reynolds number is less than 2100 the flow is laminar and the equations derived in this section are valid. Since the average velocity is $V = Q/A = (2.0 \times 10^{-5}\ \text{m}^3/\text{s})/[\pi(0.020)^2\text{m}^2/4] = 0.0637$ m/s, the Reynolds number is $\text{Re} = \rho VD/\mu = 2.87 < 2100$. Hence, the flow is laminar and from Eq. 8.9 with $\ell = x_2 - x_1 = 10$ m, the pressure drop is

$$\Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4}$$

$$= \frac{128(0.40\ \text{N}\cdot\text{s/m}^2)(10.0\ \text{m})(2.0 \times 10^{-5}\ \text{m}^3/\text{s})}{\pi(0.020\ \text{m})^4}$$

or

$$\Delta p = 20{,}400\ \text{N/m}^2 = 20.4\ \text{kPa}$$ **(Ans)**

(b) If the pipe is on a hill of angle θ such that $\Delta p = p_1 - p_2 = 0$, Eq. 8.12 gives

$$\sin\theta = -\frac{128\mu Q}{\pi\rho g D^4} \quad (1)$$

or

$$\sin\theta = \frac{-128(0.40\ \text{N}\cdot\text{s/m}^2)(2.0 \times 10^{-5}\ \text{m}^3/\text{s})}{\pi(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.020\ \text{m})^4}$$

Thus, $\theta = -13.34°$. **(Ans)**

This checks with the previous horizontal result as is seen from the fact that a change in elevation of $\Delta z = \ell\sin\theta = (10\ \text{m})\sin(-13.34°) = -2.31$ m is equivalent to a pressure change of $\Delta p = \rho g\,\Delta z = (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.31\ \text{m}) = 20{,}400\ \text{N/m}^2$, which is equivalent to that needed for the horizontal pipe. For the horizontal pipe it is the work done by the pressure forces that overcomes the viscous dissipation. For the zero-pressure-drop pipe on the hill, it is the change in potential energy of the fluid "falling" down the hill that is converted to the energy lost by viscous dissipation. Note that if it is desired to increase the flowrate to $Q = 1.0 \times 10^{-4}\ \text{m}^3/\text{s}$ with $p_1 = p_2$, the value of θ given by Eq. 1 is $\sin\theta = -1.15$. Since the sine of an angle cannot be greater than 1, this flow would not be possible. The weight of the fluid would not be large enough to offset the viscous force generated for the flowrate desired. A larger diameter pipe would be needed.

(c) With $p_1 = p_2$ the length of the pipe, ℓ, does not appear in the flowrate equation (Eq. 1). This is a statement of the fact that for such cases the pressure is constant all along the pipe (provided the pipe lies on a hill of constant slope). This can be seen by substituting the values of Q and θ from case (b) into Eq. 8.12 and noting that $\Delta p = 0$ for any ℓ. For example, $\Delta p = p_1 - p_3 = 0$ if $\ell = x_3 - x_1 = 5$ m. Thus, $p_1 = p_2 = p_3$ so that

$$p_3 = 200 \text{ kPa} \qquad \textbf{(Ans)}$$

Note that if the fluid were gasoline ($\mu = 3.1 \times 10^{-4}\ \text{N} \cdot \text{s/m}^2$ and $\rho = 680\ \text{kg/m}^3$), the Reynolds number would be Re $= 2790$, the flow would probably not be laminar, and a use of Eqs. 8.9 and 8.12 would give incorrect results. Also note from Eq. 1 that the kinematic viscosity, $\nu = \mu/\rho$, is the important viscous parameter. This is a statement of the fact that with constant pressure along the pipe, it is the ratio of the viscous force ($\sim\mu$) to the weight force ($\sim\gamma = \rho g$) that determines the value of θ.

8.2.2 From the Navier–Stokes Equations

Poiseuille's law can be obtained from the Navier–Stokes equations.

In the previous section we obtained results for fully developed laminar pipe flow by applying Newton's second law and the assumption of a Newtonian fluid to a specific portion of the fluid—a cylinder of fluid centered on the axis of a long, round pipe. When this governing law and assumptions are applied to a general fluid flow (not restricted to pipe flow), the result is the Navier–Stokes equations as discussed in **Chapter 6**. In **Section 6.9.3** these equations were solved for the specific geometry of fully developed laminar flow in a round pipe. The results are the same as those given in Eq. 8.7.

We will not repeat the detailed steps used to obtain the laminar pipe flow from the Navier–Stokes equations (see Section 6.9.3) but will indicate how the various assumptions used and steps applied in the derivation correlate with the analysis used in the previous section.

General motion of an incompressible Newtonian fluid is governed by the continuity equation (conservation of mass, Eq. 6.31) and the momentum equation (Eq. 6.127), which are rewritten here for convenience:

$$\nabla \cdot \mathbf{V} = 0 \tag{8.13}$$

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{\nabla p}{\rho} + \mathbf{g} + \nu \nabla^2 \mathbf{V} \tag{8.14}$$

For steady, fully developed flow in a pipe, the velocity contains only an axial component, which is a function of only the radial coordinate $[\mathbf{V} = u(r)\hat{\mathbf{i}}]$. For such conditions, the left-hand side of the Eq. 8.14 is zero. This is equivalent to saying that the fluid experiences no acceleration as it flows along. The same constraint was used in the previous section when considering $\mathbf{F} = m\mathbf{a}$ for the fluid cylinder. Thus, with $\mathbf{g} = -g\hat{\mathbf{k}}$ the Navier–Stokes equations become

$$\nabla \cdot \mathbf{V} = 0$$

$$\nabla p + \rho g \hat{\mathbf{k}} = \mu \nabla^2 \mathbf{V} \tag{8.15}$$

The flow is governed by a balance of pressure, weight, and viscous forces in the flow direction, similar to that shown in Fig. 8.10 and Eq. 8.10. If the flow were not fully developed (as in an entrance region, for example), it would not be possible to simplify the Navier–Stokes equations to that form given in Eq. 8.15 (the nonlinear term $\mathbf{V} \cdot \nabla \mathbf{V}$ would not be zero), and the solution would be very difficult to obtain.

Because of the assumption that $\mathbf{V} = u(r)\hat{\mathbf{i}}$, the continuity equation, Eq. 8.13, is automatically satisfied. This conservation of mass condition was also automatically satisfied by the incompressible flow assumption in the derivation in the previous section. The fluid flows across one section of the pipe at the same rate that it flows across any other section (see Fig. 8.8).

The governing differential equations can be simplified by appropriate assumptions.

When it is written in terms of polar coordinates (as was done in Section 6.9.3), the component of Eq. 8.15 along the pipe becomes

$$\frac{\partial p}{\partial x} + \rho g \sin\theta = \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{8.16}$$

Since the flow is fully developed, $u = u(r)$ and the right-hand side is a function of, at most, only r. The left-hand side is a function of, at most, only x. It was shown that this leads to the condition that the pressure gradient in the x direction is a constant—$\partial p/\partial x = -\Delta p/\ell$. The same condition was used in the derivation of the previous section (Eq. 8.3).

It is seen from Eq. 8.16 that the effect of a nonhorizontal pipe enters into the Navier–Stokes equations in the same manner as was discussed in the previous section. The pressure gradient in the flow direction is coupled with the effect of the weight in that direction to produce an effective pressure gradient of $-\Delta p/\ell + \rho g \sin\theta$.

The velocity profile is obtained by integration of Eq. 8.16. Since it is a second-order equation, two boundary conditions are needed—(1) the fluid sticks to the pipe wall (as was also done in Eq. 8.7) and (2) either of the equivalent forms that the velocity remains finite throughout the flow (in particular $u < \infty$ at $r = 0$), or because of symmetry, $\partial u/\partial r = 0$ at $r = 0$. In the derivation of the previous section, only one boundary condition (the no-slip condition at the wall) was needed because the equation integrated was a first-order equation. The other condition ($\partial u/\partial r = 0$ at $r = 0$) was automatically built into the analysis because of the fact that $\tau = -\mu\, du/dr$ and $\tau = 2\tau_w r/D = 0$ at $r = 0$.

The results obtained by either applying $\mathbf{F} = m\mathbf{a}$ to a fluid cylinder (**Section 8.2.1**) or solving the Navier–Stokes equations (**Section 6.9.3**) are exactly the same. Similarly, the basic assumptions regarding the flow structure are the same. This should not be surprising because the two methods are based on the same principle—Newton's second law. One is restricted to fully developed laminar pipe flow from the beginning (the drawing of the free-body diagram), and the other starts with the general governing equations (the Navier–Stokes equations) with the appropriate restrictions concerning fully developed laminar flow applied as the solution process progresses.

8.2.3 From Dimensional Analysis

Although fully developed laminar pipe flow is simple enough to allow the rather straightforward solutions discussed in the previous two sections, it may be worthwhile to consider this flow from a dimensional analysis standpoint. Thus, we assume that the pressure drop in the horizontal pipe, Δp, is a function of the average velocity of the fluid in the pipe, V, the length of the pipe, ℓ, the pipe diameter, D, and the viscosity of the fluid, μ. We have not included the density or the specific weight of the fluid as parameters because for such flows they are not important parameters. There is neither mass (density) times acceleration nor a component of weight (specific weight times volume) in the flow direction involved. Thus,

$$\Delta p = F(V, \ell, D, \mu)$$

There are five variables that can be described in terms of three reference dimensions (M, L, T). According to the results of dimensional analysis (**Chapter 7**), this flow can be described in terms of $k - r = 5 - 3 = 2$ dimensionless groups. One such representation is

$$\frac{D\,\Delta p}{\mu V} = \phi\left(\frac{\ell}{D}\right) \tag{8.17}$$

where $\phi(\ell/D)$ is an unknown function of the length to diameter ratio of the pipe.

Although this is as far as dimensional analysis can take us, it seems reasonable to impose a further assumption that the pressure drop is directly proportional to the pipe length. That is, it takes twice the pressure drop to force fluid through a pipe if its length is doubled. The only way that this can be true is if $\phi(\ell/D) = C\ell/D$, where C is a constant. Thus, Eq. 8.17 becomes

$$\frac{D\,\Delta p}{\mu V} = \frac{C\ell}{D}$$

which can be rewritten as

$$\frac{\Delta p}{\ell} = \frac{C\mu\, V}{D^2}$$

or

$$Q = AV = \frac{(\pi/4C)\,\Delta p D^4}{\mu\ell} \tag{8.18}$$

The basic functional dependence for laminar pipe flow given by Eq. 8.18 is the same as that obtained by the analysis of the two previous sections. The value of C must be determined by theory (as done in the previous two sections) or experiment. For a round pipe, $C = 32$. For ducts of other cross-sectional shapes, the value of C is different (see **Section 8.4.3**).

Dimensional analysis can be used to put pipe flow parameters into dimensionless form.

It is usually advantageous to describe a process in terms of dimensionless quantities. To this end we rewrite the pressure drop equation for laminar horizontal pipe flow, Eq. 8.8, as $\Delta p = 32\mu\ell V/D^2$ and divide both sides by the dynamic pressure, $\rho V^2/2$, to obtain the dimensionless form as

$$\frac{\Delta p}{\frac{1}{2}\,\rho V^2} = \frac{(32\mu\ell V/D^2)}{\frac{1}{2}\,\rho V^2} = 64\left(\frac{\mu}{\rho V D}\right)\left(\frac{\ell}{D}\right) = \frac{64}{\text{Re}}\left(\frac{\ell}{D}\right)$$

This is often written as

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$$

where the dimensionless quantity

$$f = \Delta p(D/\ell)/(\rho V^2/2)$$

is termed the *friction factor*, or sometimes the *Darcy friction factor* [**H. P. G. Darcy** (1803–1858)]. (This parameter should not be confused with the less-used Fanning friction factor, which is defined to be $f/4$. In this text we will use only the Darcy friction factor.) Thus, the friction factor for laminar fully developed pipe flow is simply

$$f = \frac{64}{\text{Re}} \tag{8.19}$$

By substituting the pressure drop in terms of the wall shear stress (Eq. 8.5), we obtain an alternate expression for the friction factor as a dimensionless wall shear stress

$$f = \frac{8\tau_w}{\rho V^2} \tag{8.20}$$

Knowledge of the friction factor will allow us to obtain a variety of information regarding pipe flow. For turbulent flow the dependence of the friction factor on the Reynolds number is much more complex than that given by Eq. 8.19 for laminar flow. This is discussed in detail in Section 8.4.

8.2.4 Energy Considerations

In the previous three sections we derived the basic laminar flow results from application of $\mathbf{F} = m\mathbf{a}$ or dimensional analysis considerations. It is equally important to understand the implications of energy considerations of such flows. To this end we consider the energy equation for incompressible, steady flow between two locations as is given in **Eq. 5.89**

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L \tag{8.21}$$

Recall that the kinetic energy coefficients, α_1 and α_2, compensate for the fact that the velocity profile across the pipe is not uniform. For uniform velocity profiles $\alpha = 1$, whereas for any nonuniform profile, $\alpha > 1$. The head loss term, h_L, accounts for any energy loss associated with the flow. This loss is a direct consequence of the viscous dissipation that occurs throughout the fluid in the pipe. For the ideal (inviscid) cases discussed in previous chapters, $\alpha_1 = \alpha_2 = 1$, $h_L = 0$, and the energy equation reduces to the familiar Bernoulli equation discussed in Chapter 3 (**Eq. 3.7**).

Even though the velocity profile in viscous pipe flow is not uniform, for fully developed flow it does not change from section (1) to section (2) so that $\alpha_1 = \alpha_2$. Thus, the kinetic energy is the same at any section ($\alpha_1 V_1^2/2 = \alpha_2 V_2^2/2$) and the energy equation becomes

$$\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = h_L \tag{8.22}$$

The energy dissipated by the viscous forces within the fluid is supplied by the excess work done by the pressure and gravity forces.

A comparison of Eqs. 8.22 and 8.10 shows that the head loss is given by

$$h_L = \frac{2\tau\ell}{\gamma r}$$

(recall $p_1 = p_2 + \Delta p$ and $z_2 - z_1 = \ell \sin\theta$), which, by use of Eq. 8.4, can be rewritten in the form

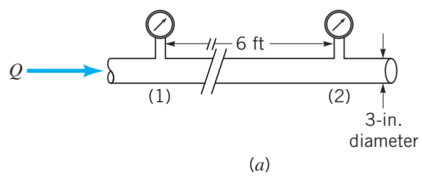
The head loss in a pipe is a result of the viscous shear stress on the wall.

$$h_L = \frac{4\ell\tau_w}{\gamma D} \tag{8.23}$$

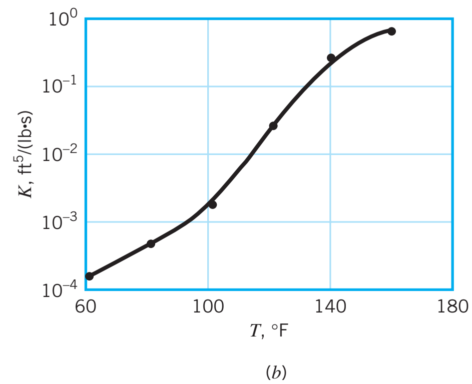
It is the shear stress at the wall (which is directly related to the viscosity and the shear stress throughout the fluid) that is responsible for the head loss. A closer consideration of the assumptions involved in the derivation of Eq. 8.23 will show that it is valid for both laminar and turbulent flow.

EXAMPLE 8.3

The flowrate, Q, of corn syrup through the horizontal pipe shown in Fig. E8.3 is to be monitored by measuring the pressure difference between sections (1) and (2). It is proposed that $Q = K\,\Delta p$, where the calibration constant, K, is a function of temperature, T, because of the temperature variation of the syrup's viscosity and density. These variations are given in Table E8.3. (a) Plot $K(T)$ versus T for 60 °F $\leq T \leq$ 160 °F. (b) Determine the wall shear stress and the pressure drop, $\Delta p = p_1 - p_2$, for $Q = 0.5\ \text{ft}^3/\text{s}$ and $T = 100\ °\text{F}$. (c) For the conditions of part (b), determine the net pressure force, $(\pi D^2/4)\ \Delta p$, and the net shear force, $\pi D \ell \tau_w$, on the fluid within the pipe between the sections (1) and (2).



(a)



(b)

■ FIGURE E8.3

■ TABLE E8-3

T (°F)	ρ (slugs/ft^3)	μ (lb · s/ft^2)
60	2.07	4.0×10^{-2}
80	2.06	1.9×10^{-2}
100	2.05	3.8×10^{-3}
120	2.04	4.4×10^{-4}
140	2.03	9.2×10^{-5}
160	2.02	2.3×10^{-5}

SOLUTION

(a) If the flow is laminar it follows from Eq. 8.9 that

$$Q = \frac{\pi D^4\ \Delta p}{128 \mu \ell} = \frac{\pi(\frac{3}{12}\ \text{ft})^4\ \Delta p}{128 \mu (6\ \text{ft})}$$

or

$$Q = K\ \Delta p = \frac{1.60 \times 10^{-5}}{\mu} \Delta p \qquad \textbf{(1)}$$

where the units on Q, Δp, and μ are ft^3/s, lb/ft^2, and lb · s/ft^2, respectively. Thus

$$K = \frac{1.60 \times 10^{-5}}{\mu} \qquad \textbf{(Ans)}$$

where the units of K are ft^5/lb · s. By using values of the viscosity from Table E8.3, the calibration curve shown in Fig. E8.3*b* is obtained. This result is valid only if the flow is laminar. As shown in Section 8.5, for turbulent flow the flowrate is not linearly related to the pressure drop so it would not be possible to have $Q = K\ \Delta p$. Note also that the value of K is independent of the syrup density (ρ was not used in the calculations) since laminar pipe flow is governed by pressure and viscous effects; inertia is not important.

(b) For $T = 100\ °\text{F}$, the viscosity is $\mu = 3.8 \times 10^{-3}$ lb · s/ft^2 so that with a flowrate of $Q = 0.5\ \text{ft}^3/\text{s}$ the pressure drop (according to Eq. 8.9) is

$$\Delta p = \frac{128\mu\ell Q}{\pi D^4} = \frac{128(3.8 \times 10^{-3}\ \text{lb} \cdot \text{s/ft}^2)(6\ \text{ft})(0.5\ \text{ft}^3\text{/s})}{\pi(\frac{3}{12}\ \text{ft})^4}$$

$$= 119\ \text{lb/ft}^2 \qquad \textbf{(Ans)}$$

provided the flow is laminar. For this case

$$V = \frac{Q}{A} = \frac{0.5\ \text{ft}^3\text{/s}}{\frac{\pi}{4}(\frac{3}{12}\ \text{ft})^2} = 10.2\ \text{ft/s}$$

so that

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(2.05\ \text{slugs/ft}^3)(10.2\ \text{ft/s})(\frac{3}{12}\ \text{ft})}{(3.8 \times 10^{-3}\ \text{lb} \cdot \text{s/ft}^2)}$$

$$= 1380 < 2100$$

Hence, the flow is laminar. From Eq. 8.5 the wall shear stress is

$$\tau_w = \frac{\Delta p D}{4\ell} = \frac{(119\ \text{lb/ft}^2)(\frac{3}{12}\ \text{ft})}{4(6\ \text{ft})} = 1.24\ \text{lb/ft}^2 \qquad \textbf{(Ans)}$$

(c) For the conditions of part (b), the net pressure force, F_p, on the fluid within the pipe between sections (1) and (2) is

$$F_p = \frac{\pi}{4} D^2\, \Delta p = \frac{\pi}{4}\left(\frac{3}{12}\ \text{ft}\right)^2 (119\ \text{lb/ft}^2) = 5.84\ \text{lb} \qquad \textbf{(Ans)}$$

Similarly, the net viscous force, F_v, on that portion of the fluid is

$$F_v = 2\pi\left(\frac{D}{2}\right)\ell\tau_w$$

$$= 2\pi\left[\frac{3}{2(12)}\ \text{ft}\right](6\ \text{ft})(1.24\ \text{lb/ft}^2) = 5.84\ \text{lb} \qquad \textbf{(Ans)}$$

Note that the values of these two forces are the same. The net force is zero; there is no acceleration.

8.3 Fully Developed Turbulent Flow

Much remains to be learned about the nature of turbulent flow.

In the previous section various properties of fully developed laminar pipe flow were discussed. Since turbulent pipe flow is actually more likely to occur than laminar flow in practical situations, it is necessary to obtain similar information for turbulent pipe flow. However, turbulent flow is a very complex process. Numerous persons have devoted considerable effort in attempting to understand the variety of baffling aspects of turbulence. Although a considerable amount of knowledge about the topic has been developed, the field of turbulent flow still remains the least understood area of fluid mechanics. In this book we can provide only some of the very basic ideas concerning turbulence. The interested reader should consult some of the many books available for further reading (Refs. 1, 2, and 3).

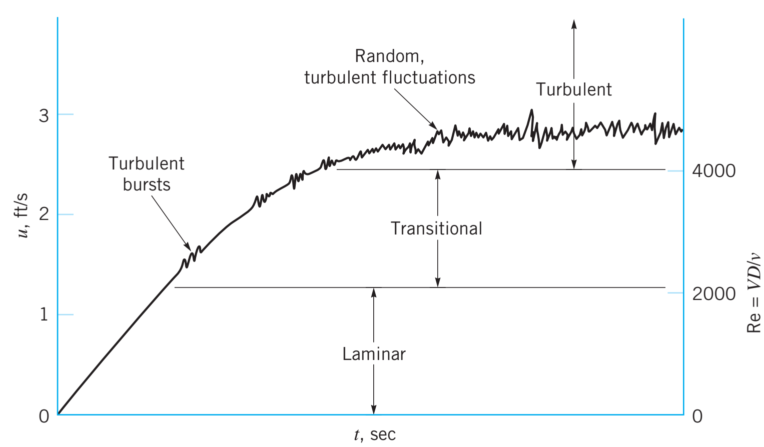
8.3.1 Transition from Laminar to Turbulent Flow

Flows are classified as laminar or turbulent. For any flow geometry, there is one (or more) dimensionless parameter such that with this parameter value below a particular value the flow is laminar, whereas with the parameter value larger than a certain value the flow is turbulent. The important parameters involved (i.e., Reynolds number, Mach number) and their critical values depend on the specific flow situation involved. For example, flow in a pipe and flow along a flat plate (boundary layer flow, as is discussed in Section 9.2.4) can be laminar or turbulent, depending on the value of the Reynolds number involved. For pipe flow the value of the Reynolds number must be less than approximately 2100 for laminar flow and greater than approximately 4000 for turbulent flow. For flow along a flat plate the transition between laminar and turbulent flow occurs at a Reynolds number of approximately 500,000 (see **Section 9.2.4**), where the length term in the Reynolds number is the distance measured from the leading edge of the plate.

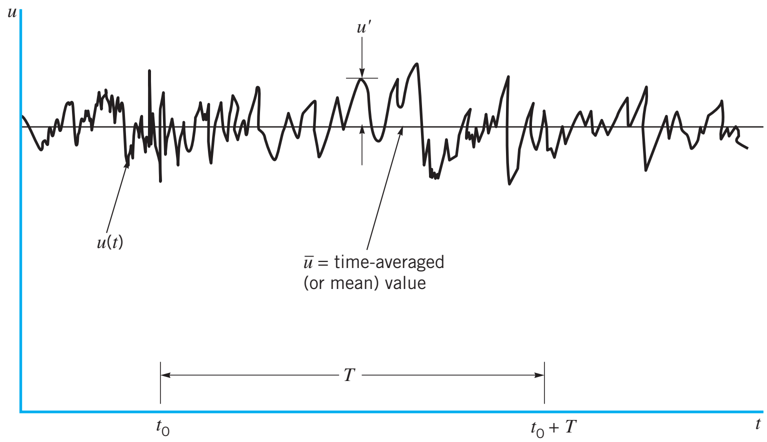
Consider a long section of pipe that is initially filled with a fluid at rest. As the valve is opened to start the flow, the flow velocity and, hence, the Reynolds number increase from zero (no flow) to their maximum steady-state flow values, as is shown in Fig. 8.11. Assume this transient process is slow enough so that unsteady effects are negligible (quasisteady flow). For an initial time period the Reynolds number is small enough for laminar flow to occur. At some time the Reynolds number reaches 2100, and the flow begins its transition to turbulent conditions. Intermittent spots or bursts of turbulence appear. As the Reynolds number is increased the entire flow field becomes turbulent. The flow remains turbulent as long as the Reynolds number exceeds approximately 4000.

Turbulent flows involve randomly fluctuating parameters.

A typical trace of the axial component of velocity measured at a given location in the flow, $u = u(t)$, is shown in Fig. 8.12. Its irregular, random nature is the distinguishing feature of turbulent flow. The character of many of the important properties of the flow (pressure drop, heat transfer, etc.) depends strongly on the existence and nature of the turbulent fluctuations or randomness indicated. In previous considerations involving inviscid flow, the Reynolds number is (strictly speaking) infinite (because the viscosity is zero), and the flow most surely would be turbulent. However, reasonable results were obtained by using the inviscid Bernoulli equation as the governing equation. The reason that such simplified inviscid analyses gave reasonable results is that viscous effects were not very important and the velocity used in the calculations was actually the time-averaged velocity, $\bar{u}$, indicated in Fig. 8.12. Calculation of the heat transfer, pressure drop, and many other parameters would



■ FIGURE 8.11 **Transition from laminar to turbulent flow in a pipe.**



■ **FIGURE 8.12** **The time-averaged, $\overline{u}$, and fluctuating, u', description of a parameter for turbulent flow.**

not be possible without inclusion of the seemingly small, but very important, effects associated with the randomness of the flow.

Consider flow in a pan of water placed on a stove. With the stove turned off, the fluid is stationary. The initial sloshing has died out because of viscous dissipation within the water. With the stove turned on, a temperature gradient in the vertical direction, $\partial T/\partial z$, is produced. The water temperature is greatest near the pan bottom and decreases toward the top of the fluid layer. If the temperature difference is very small, the water will remain stationary, even though the water density is smallest near the bottom of the pan because of the decrease in density with an increase in temperature. A further increase in the temperature gradient will cause a buoyancy-driven instability that results in fluid motion—the light, warm water rises to the top, and the heavy cold water sinks to the bottom. This slow, regular "turning over" increases the heat transfer from the pan to the water and promotes mixing within the pan. As the temperature gradient increases still further, the fluid motion becomes more vigorous and eventually turns into a chaotic, random, turbulent flow with considerable mixing and greatly increased heat transfer rate. The flow has progressed from a stationary fluid, to laminar flow, and finally to turbulent flow.

Mixing processes and heat and mass transfer processes are considerably enhanced in turbulent flow compared to laminar flow. This is due to the macroscopic scale of the randomness in turbulent flow. We are all familiar with the "rolling," vigorous eddy type motion of the water in a pan being heated on the stove (even if it is not heated to boiling). Such finitesized random mixing is very effective in transporting energy and mass throughout the flow field, thereby increasing the various rate processes involved. Laminar flow, on the other hand, can be thought of as very small but finite-sized fluid particles flowing smoothly in layers, one over another. The only randomness and mixing take place on the molecular scale and result in relatively small heat, mass, and momentum transfer rates.

Laminar (turbulent) flow involves randomness on the molecular (macroscopic) scale.

Without turbulence it would be virtually impossible to carry out life as we now know it. In some situations turbulent flow is desirable. To transfer the required heat between a solid and an adjacent fluid (such as in the cooling coils of an air conditioner or a boiler of a power plant) would require an enormously large heat exchanger if the flow were laminar. Similarly, the required mass transfer of a liquid state to a vapor state (such as is needed in the evaporated cooling system associated with sweating) would require very large surfaces if the fluid flowing past the surface were laminar rather than turbulent.

Turbulence is also of importance in the mixing of fluids. Smoke from a stack would continue for miles as a ribbon of pollutant without rapid dispersion within the surrounding air if the flow were laminar rather than turbulent. Under certain atmospheric conditions this is observed to occur. Although there is mixing on a molecular scale (laminar flow), it is several orders of magnitude slower and less effective than the mixing on a macroscopic scale (turbulent flow). It is considerably easier to mix cream into a cup of coffee (turbulent flow) than to thoroughly mix two colors of a viscous paint (laminar flow).

In other situations laminar (rather than turbulent) flow is desirable. The pressure drop in pipes (hence, the power requirements for pumping) can be considerably lower if the flow is laminar rather than turbulent. Fortunately, the blood flow through a person's arteries is normally laminar, except in the largest arteries with high blood flowrates. The aerodynamic drag on an airplane wing can be considerably smaller with laminar flow past it than with turbulent flow.

8.3.2 Turbulent Shear Stress

The fundamental difference between laminar and turbulent flow lies in the chaotic, random behavior of the various fluid parameters. Such variations occur in the three components of velocity, the pressure, the shear stress, the temperature, and any other variable that has a field description. Turbulent flow is characterized by random, three-dimensional vorticity (i.e., fluid particle rotation or spin; see **Section 6.1.3**). As is indicated in Fig. 8.12, such flows can be described in terms of their mean values (denoted with an overbar) on which are superimposed the fluctuations (denoted with a prime). Thus, if $u = u(x, y, z, t)$ is the x component of instantaneous velocity, then its time mean (or *time average*) value, $\overline{u}$, is

$$\overline{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(x, y, z, t)\, dt \qquad \textbf{(8.24)}$$

Turbulent flow parameters can be described in terms of mean and fluctuating portions.

where the time interval, T, is considerably longer than the period of the longest fluctuations, but considerably shorter than any unsteadiness of the average velocity. This is illustrated in Fig. 8.12.

The *fluctuating part* of the velocity, u' is that time-varying portion that differs from the average value

$$u = \overline{u} + u' \quad \text{or} \quad u' = u - \overline{u} \qquad \textbf{(8.25)}$$

Clearly, the time average of the fluctuations is zero, since

$$\overline{u'} = \frac{1}{T}\int_{t_0}^{t_0+T} (u - \overline{u})\, dt = \frac{1}{T}\left(\int_{t_0}^{t_0+T} u\, dt - \overline{u}\int_{t_0}^{t_0+T} dt\right)$$

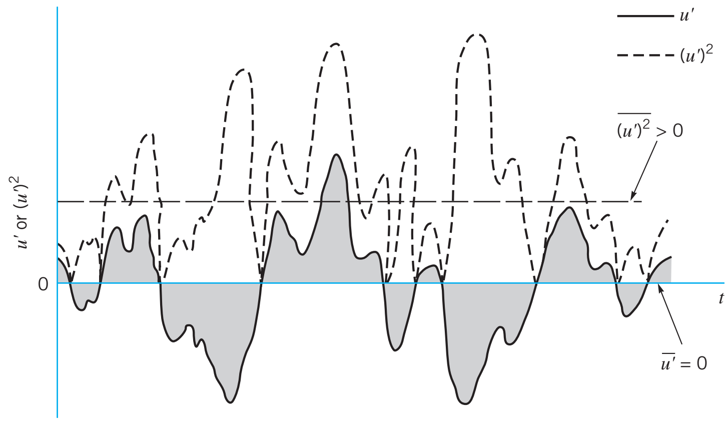
$$= \frac{1}{T}(T\overline{u} - T\overline{u}) = 0$$

The fluctuations are equally distributed on either side of the average. It is also clear, as is indicated in Fig. 8.13, that since the square of a fluctuation quantity cannot be negative $[(u')^2 \geq 0]$, its average value is positive. Thus,

$$\overline{(u')^2} = \frac{1}{T}\int_{t_0}^{t_0+T} (u')^2\, dt > 0$$

On the other hand, it may be that the average of products of the fluctuations, such as $\overline{u'v'}$, are zero or nonzero (either positive or negative).

The structure and characteristics of turbulence may vary from one flow situation to another. For example, the *turbulence intensity* (or the level of the turbulence) may be larger in a very gusty wind than it is in a relatively steady (although turbulent) wind. The turbulence



■ **FIGURE 8.13** **Average of the fluctuations and average of the square of the fluctuations.**

intensity, $\mathcal{I}$, is often defined as the square root of the mean square of the fluctuating velocity divided by the time-averaged velocity, or

$$\mathcal{I} = \frac{\sqrt{\overline{(u')^2}}}{\overline{u}} = \frac{\left[\frac{1}{T}\int_{t_0}^{t_0+T} (u')^2\, dt\right]^{1/2}}{\overline{u}}$$

The larger the turbulence intensity, the larger the fluctuations of the velocity (and other flow parameters). Well-designed wind tunnels have typical values of $\mathcal{I} \approx 0.01$, although with extreme care, values as low as $\mathcal{I} = 0.0002$ have been obtained. On the other hand, values of $\mathcal{I} \gtrsim 0.1$ are found for the flow in the atmosphere and rivers.

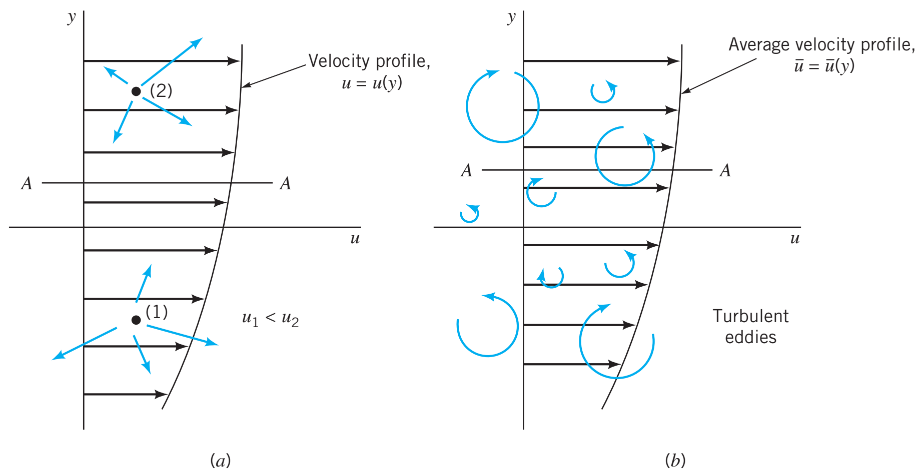
Another turbulence parameter that is different from one flow situation to another is the period of the fluctuations—the *time scale* of the fluctuations shown in Fig. 8.12. In many flows, such as the flow of water from a faucet, typical frequencies are on the order of 10, 100, or 1000 cycles per second (cps). For other flows, such as the Gulf Stream current in the Atlantic Ocean or flow of the atmosphere of Jupiter, characteristic random oscillations may have a period on the order of hours, days, or more.

The relationship between fluid motion and shear stress is very complex for turbulent flow.

It is tempting to extend the concept of viscous shear stress for laminar flow ($\tau = \mu\, du/dy$) to that of turbulent flow by replacing u, the instantaneous velocity, by $\overline{u}$, the time-averaged velocity. However, numerous experimental and theoretical studies have shown that such an approach leads to completely incorrect results. That is, $\tau \neq \mu\, d\overline{u}/dy$. A physical explanation for this behavior can be found in the concept of what produces a shear stress.

Laminar flow is modeled as fluid particles that flow smoothly along in layers, gliding past the slightly slower or faster ones on either side. As is discussed in Chapter 1, the fluid actually consists of numerous molecules darting about in an almost random fashion as is indicated in Fig. 8.14*a*. The motion is not entirely random—a slight bias in one direction produces the flowrate we associate with the motion of fluid particles, $\overline{u}$. As the molecules dart across a given plane (plane *A–A*, for example), the ones moving upward have come from an area of smaller average x component of velocity than the ones moving downward, which have come from an area of larger velocity.

The momentum flux in the x direction across plane *A–A* gives rise to a drag (to the left) of the lower fluid on the upper fluid and an equal but opposite effect of the upper fluid on the lower fluid. The sluggish molecules moving upward across plane *A–A* must be accelerated by the fluid above this plane. The rate of change of momentum in this process produces (on the macroscopic scale) a shear force. Similarly, the more energetic molecules moving down across plane *A–A* must be slowed down by the fluid below that plane. This



■ **FIGURE 8.14** **(*a*) Laminar flow shear stress caused by random motion of molecules. (*b*) Turbulent flow as a series of random, three-dimensional eddies.**

shear force is present only if there is a gradient in $u = u(y)$, otherwise the average x component of velocity (and momentum) of the upward and downward molecules is exactly the same. In addition, there are attractive forces between molecules. By combining these effects we obtain the well-known Newton viscosity law: $\tau = \mu \, du/dy$, where on a molecular basis μ is related to the mass and speed (temperature) of the random motion of the molecules.

Turbulent flows involve random motions of finite-sized fluid particles.

Although the above random motion of the molecules is also present in turbulent flow, there is another factor that is generally more important. A simplistic way of thinking about turbulent flow is to consider it as consisting of a series of random, three-dimensional eddy type motions as is depicted (in one dimension only) in Fig. 8.14*b*. (See the photograph at the beginning of **Chapter 8**.) These eddies range in size from very small diameter (on the order of the size of a fluid particle) to fairly large diameter (on the order of the size of the object or flow geometry considered). They move about randomly, conveying mass with an average velocity $\bar{u} = \bar{u}(y)$. This eddy structure greatly promotes mixing within the fluid. It also greatly increases the transport of x momentum across plane A–A. That is, finite parcels of fluid (not merely individual molecules as in laminar flow) are randomly transported across this plane, resulting in a relatively large (when compared with laminar flow) shear force.

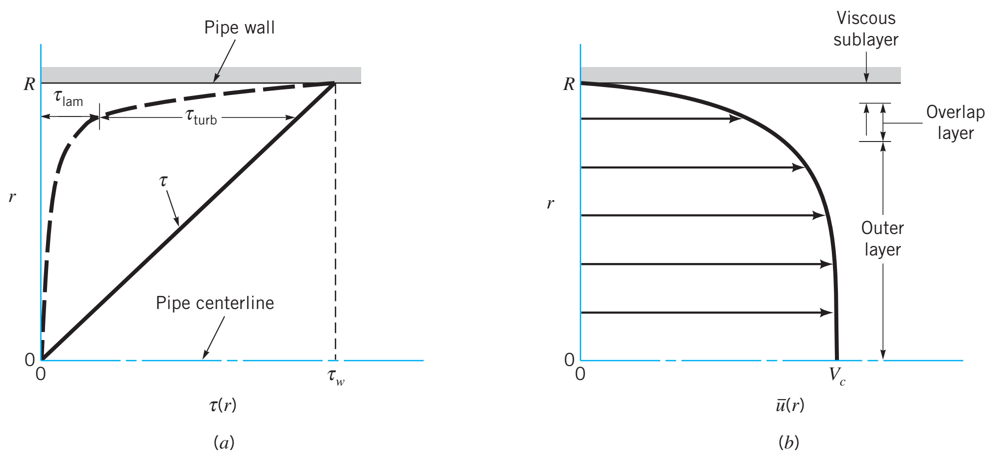
V8.2 Turbulence in a bowl

The random velocity components that account for this momentum transfer (hence, the shear force) are u' (for the x component of velocity) and v' (for the rate of mass transfer crossing the plane). A more detailed consideration of the processes involved will show that the apparent shear stress on plane A–A is given by the following (Ref. 2):

$$\tau = \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'} = \tau_{\text{lam}} + \tau_{\text{turb}} \qquad \textbf{(8.26)}$$

Note that if the flow is laminar, $u' = v' = 0$, so that $\overline{u'v'} = 0$ and Eq. 8.26 reduces to the customary random molecule-motion-induced *laminar shear stress*, $\tau_{\text{lam}} = \mu \, d\bar{u}/dy$. For turbulent flow it is found that the *turbulent shear stress*, $\tau_{\text{turb}} = -\rho\overline{u'v'}$, is positive. Hence, the shear stress is greater in turbulent flow than in laminar flow. Note the units on τ_{turb} are (density)(velocity)^2 = (slugs/ft^3)(ft/s)^2 = (slugs · ft/s^2)/ft^2 = lb/ft^2, or N/m^2, as expected. Terms of the form $-\rho\overline{u'v'}$ (or $-\rho\overline{v'w'}$, etc.) are called *Reynolds stresses* in honor of Osborne Reynolds who first discussed them in 1895.

It is seen from Eq. 8.26 that the shear stress in turbulent flow is not merely proportional to the gradient of the time-averaged velocity, $\bar{u}(y)$. It also contains a contribution due to the random fluctuations of the x and y components of velocity. The density is involved



■ **FIGURE 8.15** **Structure of turbulent flow in a pipe. (*a*) Shear stress. (*b*) Average velocity.**

The shear stress is the sum of a laminar portion and a turbulent portion.

because of the momentum transfer of the fluid within the random eddies. Although the relative magnitude of τ_{lam} compared to τ_{turb} is a complex function dependent on the specific flow involved, typical measurements indicate the structure shown in Fig. 8.15*a*. (Recall from Eq. 8.4 that the shear stress is proportional to the distance from the centerline of the pipe). In a very narrow region near the wall (the *viscous sublayer*), the laminar shear stress is dominant. Away from the wall (in the *outer layer*) the turbulent portion of the shear stress is dominant. The transition between these two regions occurs in the *overlap layer*. The corresponding typical velocity profile is shown in Fig. 8.15*b*.

The scale of the sketches shown in Fig. 8.15 is not necessarily correct. Typically the value of τ_{turb} is 100 to 1000 times greater than τ_{lam} in the outer region, while the converse is true in the viscous sublayer. A correct modeling of turbulent flow is strongly dependent on an accurate knowledge of τ_{turb}. This, in turn, requires an accurate knowledge of the fluctuations u' and v', or $\rho\overline{u'v'}$. As yet it is not possible to solve the governing equations (the Navier–Stokes equations) for these details of the flow, although numerical techniques using the largest and fastest computers available have produced important information about some of the characteristics of turbulence. Considerable effort has gone into the study of turbulence. Much remains to be learned. Perhaps studies in the new areas of chaos and fractal geometry will provide the tools for a better understanding of turbulence (see **Section 8.3.5**).

The vertical scale of Fig. 8.15 is also distorted. The viscous sublayer is usually a very thin layer adjacent to the wall. For example, for water flow in a 3-in.-diameter pipe with an average velocity of 10 ft/s, the viscous sublayer is approximately 0.002 in. thick. Since the fluid motion within this thin layer is critical in terms of the overall flow (the no-slip condition and the wall shear stress occur in this layer), it is not surprising to find that turbulent pipe flow properties can be quite dependent on the roughness of the pipe wall, unlike laminar pipe flow which is independent of roughness. Small roughness elements (scratches, rust, sand or dirt particles, etc.) can easily disturb this viscous sublayer (see Section 8.4), thereby affecting the entire flow.

An alternate form for the shear stress for turbulent flow is given in terms of the *eddy viscosity*, η, where

$$\tau = \eta \frac{d\bar{u}}{dy} \tag{8.27}$$

This extension of laminar flow terminology was introduced by J. Boussinesq, a French scientist, in 1877. Although the concept of an eddy viscosity is intriguing, in practice it is not an easy parameter to use. Unlike the absolute viscosity, μ, which is a known value for a given fluid, the eddy viscosity is a function of both the fluid and the flow conditions. That is, the eddy viscosity of water cannot be looked up in handbooks—its value changes from one turbulent flow condition to another and from one point in a turbulent flow to another.

The inability to accurately determine the Reynolds stress, $\rho\overline{u'v'}$, is equivalent to not knowing the eddy viscosity. Several semiempirical theories have been proposed (Ref. 3) to determine approximate values of η. **L. Prandtl** (1875–1953), a German physicist and aerodynamicist, proposed that the turbulent process could be viewed as the random transport of bundles of fluid particles over a certain distance, ℓ_m, the *mixing length*, from a region of one velocity to another region of a different velocity. By the use of some ad hoc assumptions and physical reasoning, it was concluded that the eddy viscosity was given by

Various ad hoc assumptions have been used to approximate turbulent shear stresses.

$$\eta = \rho\ell_m^2\left|\frac{d\bar{u}}{dy}\right|$$

Thus, the turbulent shear stress is

$$\tau_{\text{turb}} = \rho\ell_m^2\left(\frac{d\bar{u}}{dy}\right)^2 \tag{8.28}$$

The problem is thus shifted to that of determining the mixing length, ℓ_m. Further considerations indicate that ℓ_m is not a constant throughout the flow field. Near a solid surface the turbulence is dependent on the distance from the surface. Thus, additional assumptions are made regarding how the mixing length varies throughout the flow.

The net result is that as yet there is no general, all-encompassing, useful model that can accurately predict the shear stress throughout a general incompressible, viscous turbulent flow. Without such information it is impossible to integrate the force balance equation to obtain the turbulent velocity profile and other useful information, as was done for laminar flow.

8.3.3 Turbulent Velocity Profile

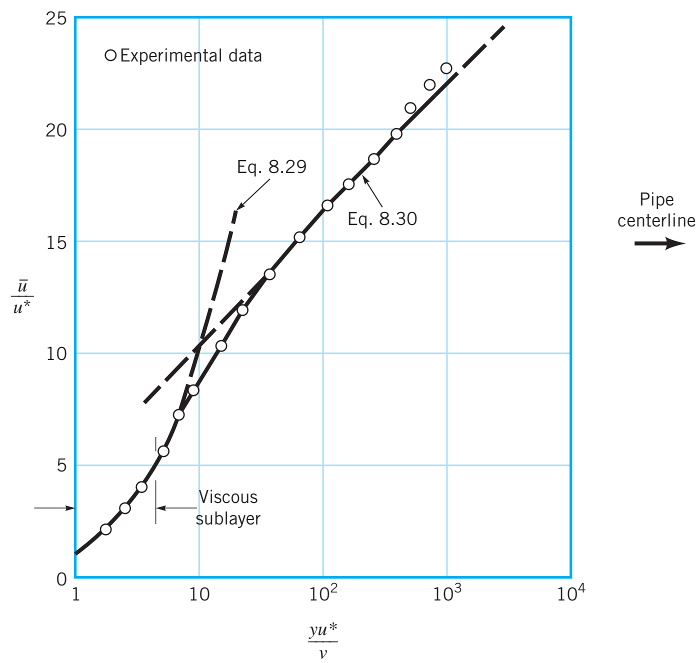
Considerable information concerning turbulent velocity profiles has been obtained through the use of dimensional analysis, experimentation, and semiempirical theoretical efforts. As is indicated in Fig. 8.15, fully developed turbulent flow in a pipe can be broken into three regions which are characterized by their distances from the wall: the viscous sublayer very near the pipe wall, the overlap region, and the outer turbulent layer throughout the center portion of the flow. Within the viscous sublayer the viscous shear stress is dominant compared with the turbulent (or Reynolds) stress, and the random, eddying nature of the flow is essentially absent. In the outer turbulent layer the Reynolds stress is dominant, and there is considerable mixing and randomness to the flow.

The character of the flow within these two regions is entirely different. For example, within the viscous sublayer the fluid viscosity is an important parameter; the density is unimportant. In the outer layer the opposite is true. By a careful use of dimensional analysis arguments for the flow in each layer and by a matching of the results in the common overlap layer, it has been possible to obtain the following conclusions about the turbulent velocity profile in a smooth pipe (Ref. 5).

In the viscous sublayer the velocity profile can be written in dimensionless form as

$$\frac{\bar{u}}{u^*} = \frac{yu^*}{\nu} \tag{8.29}$$

where $y = R - r$ is the distance measured from the wall, $\bar{u}$ is the time-averaged x component



■ **FIGURE 8.16** **Typical structure of the turbulent velocity profile in a pipe.**

of velocity, and $u^* = (\tau_w/\rho)^{1/2}$ is termed the *friction velocity*. Note that u^* is not an actual velocity of the fluid—it is merely a quantity that has dimensions of velocity. As is indicated in Fig. 8.16, Eq. 8.29 (commonly called the *law of the wall*) is valid very near the smooth wall, for $0 \le yu^*/\nu \lesssim 5$.

Dimensional analysis arguments indicate that in the overlap region the velocity should vary as the logarithm of y. Thus, the following expression has been proposed:

A turbulent flow velocity profile can be divided into various regions.

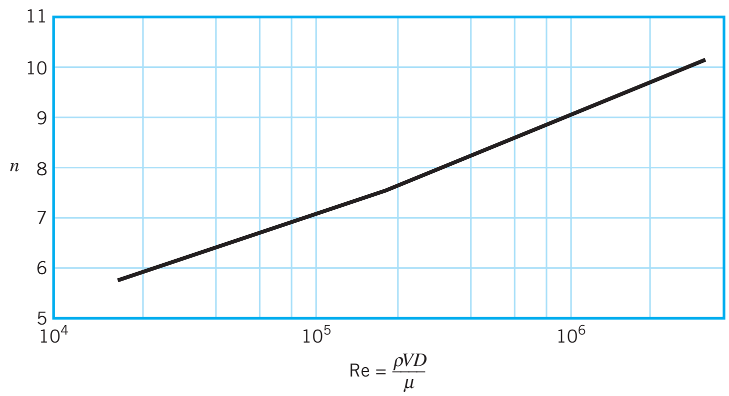
$$\frac{\overline{u}}{u^*} = 2.5 \ln\left(\frac{yu^*}{\nu}\right) + 5.0 \tag{8.30}$$

where the constants 2.5 and 5.0 have been determined experimentally. As is indicated in Fig. 8.16, for regions not too close to the smooth wall, but not all the way out to the pipe center, Eq. 8.30 gives a reasonable correlation with the experimental data. Note that the horizontal scale is a logarithmic scale. This tends to exaggerate the size of the viscous sublayer relative to the remainder of the flow. As is shown in **Example 8.4**, the viscous sublayer is usually quite thin. Similar results can be obtained for turbulent flow past rough walls (Ref. 17).

A number of other correlations exist for the velocity profile in turbulent pipe flow. In the central region (the outer turbulent layer) the expression $(V_c - \overline{u})/u^* = 2.5 \ln(R/y)$, where V_c is the centerline velocity, is often suggested as a good correlation with experimental data. Another often-used (and relatively easy to use) correlation is the empirical *power-law velocity profile*

$$\frac{\overline{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n} \tag{8.31}$$

In this representation, the value of n is a function of the Reynolds number, as is indicated in Fig. 8.17. The one-seventh power-law velocity profile ($n = 7$) is often used as a reasonable approximation for many practical flows. Typical turbulent velocity profiles based on this power-law representation are shown in Fig. 8.18.



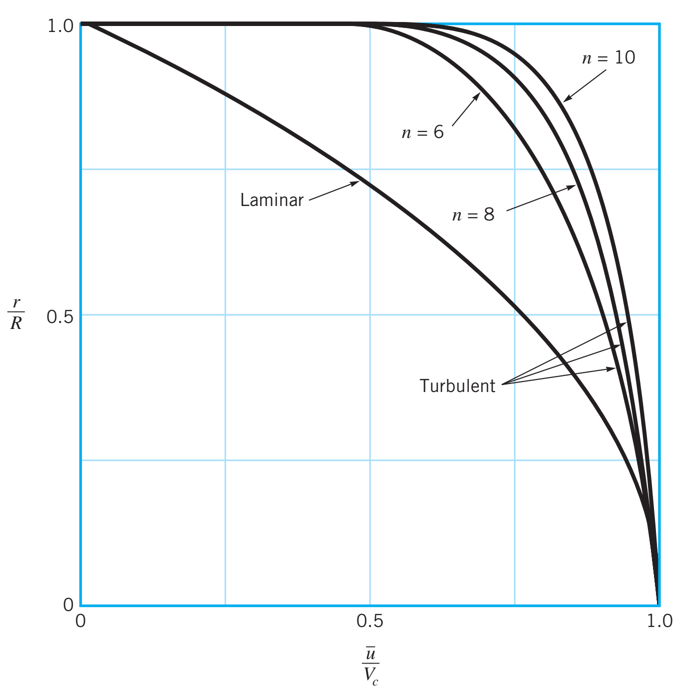
■ **FIGURE 8.17** **Exponent, *n*, for power-law velocity profiles. (Adapted from Ref. 1.)**

A power-law velocity profile approximates the actual turbulent velocity profile.

A closer examination of Eq. 8.31 shows that the power-law profile cannot be valid near the wall, since according to this equation the velocity gradient is infinite there. In addition, Eq. 8.31 cannot be precisely valid near the centerline because it does not give $d\bar{u}/dr = 0$ at $r = 0$. However, it does provide a reasonable approximation to the measured velocity profiles across most of the pipe.

Note from Fig. 8.18 that the turbulent profiles are much "flatter" than the laminar profile and that this flatness increases with Reynolds number (i.e., with n). Recall from **Chapter 3** that reasonable approximate results are often obtained by using the inviscid Bernoulli equation and by assuming a fictitious uniform velocity profile. Since most flows are turbulent and turbulent flows tend to have nearly uniform velocity profiles, the usefulness of the Bernoulli equation and the uniform profile assumption is not unexpected. Of course, many properties of the flow cannot be accounted for without including viscous effects.

V8.3 Laminar/turbulent velocity profiles



■ **FIGURE 8.18** **Typical laminar flow and turbulent flow velocity profiles.**

EXAMPLE 8.4

Water at 20 °C ($\rho = 998 \text{ kg/m}^3$ and $\nu = 1.004 \times 10^{-6} \text{ m}^2\text{/s}$) flows through a horizontal pipe of 0.1-m diameter with a flowrate of $Q = 4 \times 10^{-2} \text{ m}^3\text{/s}$ and a pressure gradient of 2.59 kPa/m. (a) Determine the approximate thickness of the viscous sublayer. (b) Determine the approximate centerline velocity, V_c. (c) Determine the ratio of the turbulent to laminar shear stress, $\tau_{\text{turb}}/\tau_{\text{lam}}$, at a point midway between the centerline and the pipe wall (i.e., at $r = 0.025$ m).

SOLUTION

(a) According to Fig. 8.16, the thickness of the viscous sublayer, δ_s, is approximately

$$\frac{\delta_s u^*}{\nu} = 5$$

or

$$\delta_s = 5\,\frac{\nu}{u^*}$$

where

$$u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2} \tag{1}$$

The wall shear stress can be obtained from the pressure drop data and Eq. 8.5, which is valid for either laminar or turbulent flow. Thus,

$$\tau_w = \frac{D\,\Delta p}{4\ell} = \frac{(0.1 \text{ m})(2.59 \times 10^3 \text{ N/m}^2)}{4(1 \text{ m})} = 64.8 \text{ N/m}^2$$

Hence, from Eq. 1 we obtain

$$u^* = \left(\frac{64.8 \text{ N/m}^2}{998 \text{ kg/m}^3}\right)^{1/2} = 0.255 \text{ m/s}$$

so that

$$\delta_s = \frac{5(1.004 \times 10^{-6} \text{ m}^2\text{/s})}{0.255 \text{ m/s}} = 1.97 \times 10^{-5} \text{ m} \simeq 0.02 \text{ mm} \qquad \textbf{(Ans)}$$

As stated previously, the viscous sublayer is very thin. Minute imperfections on the pipe wall will protrude into this sublayer and affect some of the characteristics of the flow (i.e., wall shear stress and pressure drop).

(b) The centerline velocity can be obtained from the average velocity and the assumption of a power-law velocity profile as follows. For this flow with

$$V = \frac{Q}{A} = \frac{0.04 \text{ m}^3\text{/s}}{\pi(0.1 \text{ m})^2/4} = 5.09 \text{ m/s}$$

the Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(5.09 \text{ m/s})(0.1 \text{ m})}{(1.004 \times 10^{-6} \text{ m}^2\text{/s})} = 5.07 \times 10^5$$

Thus, from Fig. 8.17, $n = 8.4$ so that

$$\frac{\bar{u}}{V_c} \approx \left(1 - \frac{r}{R}\right)^{1/8.4}$$

To determine the centerline velocity, V_c, we must know the relationship between V (the average velocity) and V_c. This can be obtained by integration of the power-law velocity profile as follows. Since the flow is axisymmetric,

$$Q = AV = \int \bar{u}\, dA = V_c \int_{r=0}^{r=R} \left(1 - \frac{r}{R}\right)^{1/n} (2\pi r)\, dr$$

which can be integrated to give

$$Q = 2\pi R^2 V_c \frac{n^2}{(n+1)(2n+1)}$$

Thus, since $Q = \pi R^2 V$, we obtain

$$\frac{V}{V_c} = \frac{2n^2}{(n+1)(2n+1)}$$

With $n = 8.4$ in the present case, this gives

$$V_c = \frac{(n+1)(2n+1)}{2n^2} V = 1.186V = 1.186\,(5.09 \text{ m/s})$$

$$= 6.04 \text{ m/s} \qquad \textbf{(Ans)}$$

Recall that $V_c = 2V$ for laminar pipe flow.

(c) From Eq. 8.4, which is valid for laminar or turbulent flow, the shear stress at $r = 0.025$ m is

$$\tau = \frac{2\tau_w r}{D} = \frac{2(64.8 \text{ N/m}^2)(0.025 \text{ m})}{(0.1 \text{ m})}$$

or

$$\tau = \tau_{\text{lam}} + \tau_{\text{turb}} = 32.4 \text{ N/m}^2$$

where $\tau_{\text{lam}} = -\mu\, d\bar{u}/dr$. From the power-law velocity profile (Eq. 8.31) we obtain the gradient of the average velocity as

$$\frac{d\bar{u}}{dr} = -\frac{V_c}{nR}\left(1 - \frac{r}{R}\right)^{(1-n)/n}$$

which gives

$$\frac{d\bar{u}}{dr} = -\frac{(6.04 \text{ m/s})}{8.4(0.05\text{m})}\left(1 - \frac{0.025 \text{ m}}{0.05 \text{ m}}\right)^{(1-8.4)/8.4} = -26.5/\text{s}$$

Thus,

$$\tau_{\text{lam}} = -\mu \frac{d\bar{u}}{dr} = -(\nu\rho) \frac{d\bar{u}}{dr}$$

$$= -(1.004 \times 10^{-6} \text{ m}^2/\text{s})(998 \text{ kg/m}^3)(-26.5/\text{s})$$

$$= 0.0266 \text{ N/m}^2$$

Thus, the ratio of turbulent to laminar shear stress is given by

$$\frac{\tau_{\text{turb}}}{\tau_{\text{lam}}} = \frac{\tau - \tau_{\text{lam}}}{\tau_{\text{lam}}} = \frac{32.4 - 0.0266}{0.0266} = 1220 \qquad \textbf{(Ans)}$$

As expected, most of the shear stress at this location in the turbulent flow is due to the turbulent shear stress.

The turbulent flow characteristics discussed in this section are not unique to turbulent flow in round pipes. Many of the characteristics introduced (i.e., the Reynolds stress, the viscous sublayer, the overlap layer, the outer layer, the general characteristics of the velocity profile, etc.) are found in other turbulent flows. In particular, turbulent pipe flow and turbulent flow past a solid wall (boundary layer flow) share many of these common traits. Such ideas are discussed more fully in **Chapter 9**.

8.3.4 Turbulence Modeling

Although it is not yet possible to theoretically predict the random, irregular details of turbulent flows, it would be useful to be able to predict the time-averaged flow fields (pressure, velocity, etc.) directly from the basic governing equations. To this end one can time average the governing Navier–Stokes equations (Eqs. 6.31 and **6.127**) to obtain equations for the average velocity and pressure. However, because the Navier–Stokes equations are nonlinear, the resulting time-averaged differential equations contain not only the desired average pressure and velocity as variables, but also averages of products of the fluctuations—terms of the type that one tried to eliminate by averaging the equations! For example, the Reynolds stress $-\rho\overline{u'v'}$ (see **Eq. 8.26**) occurs in the time-averaged momentum equation.

Thus, it is not possible to merely average the basic differential equations and obtain governing equations involving only the desired averaged quantities. This is the reason for the variety of ad hoc assumptions that have been proposed to provide "closure" to the equations governing the average flow. That is, the set of governing equations must be a complete or closed set of equations—the same number of equation as unknowns.

Various attempts have been made to solve this closure problem (Refs. 1, 32). Such schemes involving the introduction of an eddy viscosity or the mixing length (as introduced in **Section 8.3.2**) are termed algebraic or zero-equation models. Other methods, which are beyond the scope of this book, include the one-equation model and the two-equation model. These turbulence models are based on the equation for the turbulence kinetic energy and require significant computer usage.

Turbulence modeling is an important and extremely difficult topic. Although considerable progress has been made, much remains to be done in this area.

8.3.5 Chaos and Turbulence

Chaos theory may eventually provide a deeper understanding of turbulence.

Chaos theory is a relatively new branch of mathematical physics that may provide insight into the complex nature of turbulence. This method combines mathematics and numerical (computer) techniques to provide a new way to analyze certain problems. Chaos theory, which is quite complex and is only now being developed, involves the behavior of nonlinear dynamical systems and their response to initial and boundary conditions. The flow of a viscous fluid, which is governed by the nonlinear Navier–Stokes equations (Eq. 6.127), may be such a system.

To solve the Navier–Stokes equations for the velocity and pressure fields in a viscous flow, one must specify the particular flow geometry being considered (the boundary conditions) and the condition of the flow at some particular time (the initial conditions). If, as some researchers predict, the Navier–Stokes equations allow chaotic behavior, then the state of the flow at times after the initial time may be very, very sensitive to the initial conditions. A slight variation to the initial flow conditions may cause the flow at later times to be quite different than it would have been with the original, only slightly different initial conditions. When carried to the extreme, the flow may be "chaotic," "random," or perhaps (in current terminology), "turbulent."

The occurrence of such behavior would depend on the value of the Reynolds number. For example, it may be found that for sufficiently small Reynolds numbers the flow is not chaotic (i.e., it is laminar), while for large Reynolds numbers it is chaotic with turbulent characteristics.

Thus, with the advancement of chaos theory it may be found that the numerous ad hoc turbulence ideas mentioned in previous sections (i.e., eddy viscosity, mixing length, law of the wall, etc.) may not be needed. It may be that chaos theory can provide the turbulence properties and structure directly from the governing equations. As of now we must wait until this exciting topic is developed further. The interested reader is encouraged to consult Ref. 33 for a general introduction to chaos or Ref. 34 for additional material.

8.4 Dimensional Analysis of Pipe Flow

Most turbulent pipe flow information is based on experimental data.

As was discussed in previous sections, turbulent flow can be a very complex, difficult topic—one that as yet has defied a rigorous theoretical treatment. Thus, most turbulent pipe flow analyses are based on experimental data and semiempirical formulas, even if the flow is fully developed. These results are given in dimensionless form and cover a very wide range of flow parameters, including arbitrary fluids, pipes, and flowrates. In addition to these fully developed flow considerations, a variety of useful data are available regarding flow through pipe fittings, such as elbows, tees, valves, and the like. These data are conveniently expressed in dimensionless form.

8.4.1 The Moody Chart

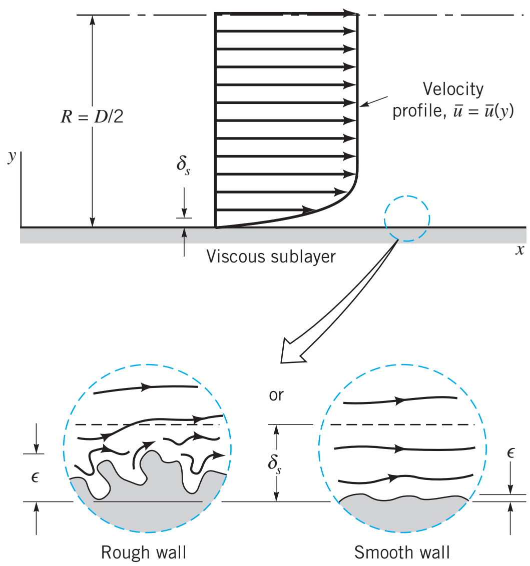
A dimensional analysis treatment of pipe flow provides the most convenient base from which to consider turbulent, fully developed pipe flow. An introduction to this topic was given in Section 8.3. As is discussed in **Sections 8.2.1** and 8.2.4, the pressure drop and head loss in a pipe are dependent on the wall shear stress, τ_w, between the fluid and pipe surface. A fundamental difference between laminar and turbulent flow is that the shear stress for turbulent flow is a function of the density of the fluid, ρ. For laminar flow, the shear stress is independent of the density, leaving the viscosity, μ, as the only important fluid property.

Thus, the pressure drop, Δp, for steady, incompressible turbulent flow in a horizontal round pipe of diameter D can be written in functional form as

$$\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho) \qquad \textbf{(8.32)}$$

where V is the average velocity, ℓ is the pipe length, and ε is a measure of the roughness of the pipe wall. It is clear that Δp should be a function of V, D, and ℓ. The dependence of Δp on the fluid properties μ and ρ is expected because of the dependence of τ on these parameters.

Although the pressure drop for laminar pipe flow is found to be independent of the roughness of the pipe, it is necessary to include this parameter when considering turbulent flow. As is discussed in **Section 8.3.3** and illustrated in Fig. 8.19, for turbulent flow there is a relatively thin viscous sublayer formed in the fluid near the pipe wall. In many instances this layer is very thin; $\delta_s/D \ll 1$, where δ_s is the sublayer thickness. If a typical wall roughness element protrudes sufficiently far into (or even through) this layer, the structure and properties of the viscous sublayer (along with Δp and τ_w) will be different than if the wall were smooth. Thus, for turbulent flow the pressure drop is expected to be a function of the wall roughness. For laminar flow there is no thin viscous layer–viscous effects are important across the entire pipe. Thus, relatively small roughness elements have completely negligible effects on laminar pipe flow. Of course, for pipes with very large wall "roughness," $(\varepsilon/D \gtrsim 0.1)$, such as that in corrugated pipes, the flowrate may be a function of the "roughness." We will consider only typical constant diameter pipes with relative roughnesses in the range



■ **FIGURE 8.19 Flow in the viscous sublayer near rough and smooth walls.**

$0 \leq \varepsilon/D \lesssim 0.05$. Analysis of flow in corrugated pipes does not fit into the standard constant diameter pipe category, although experimental results for such pipes are available (Ref. 30).

The list of parameters given in Eq. 8.32 is apparently a complete one. That is, experiments have shown that other parameters (such as surface tension, vapor pressure, etc.) do not affect the pressure drop for the conditions stated (steady, incompressible flow; round, horizontal pipe). Since there are seven variables ($k = 7$) which can be written in terms of the three reference dimensions MLT ($r = 3$), Eq. 8.32 can be written in dimensionless form in terms of $k - r = 4$ dimensionless groups. As was discussed in Section 7.9.1, one such representation is

Turbulent pipe flow properties depend on the fluid density and the pipe roughness.

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$$

This result differs from that used for laminar flow (see Eq. 8.17) in two ways. First, we have chosen to make the pressure dimensionless by dividing by the dynamic pressure, $\rho V^2/2$, rather than a characteristic viscous shear stress, $\mu V/D$. This convention was chosen in recognition of the fact that the shear stress for turbulent flow is normally dominated by τ_{turb}, which is a stronger function of the density than it is of viscosity. Second, we have introduced two additional dimensionless parameters, the Reynolds number, $\text{Re} = \rho V D/\mu$, and the *relative roughness*, ε/D, which are not present in the laminar formulation because the two parameters ρ and ε are not important in fully developed laminar pipe flow.

As was done for laminar flow, the functional representation can be simplified by imposing the reasonable assumption that the pressure drop should be proportional to the pipe length. (Such a step is not within the realm of dimensional analysis. It is merely a logical assumption supported by experiments.) The only way that this can be true is if the ℓ/D dependence is factored out as

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\ell}{D}\phi\left(\text{Re}, \frac{\varepsilon}{D}\right)$$

As was discussed in Section 8.2.3, the quantity $\Delta p D/(\ell \rho V^2/2)$ is termed the friction factor, f. Thus, for a horizontal pipe

$$\Delta p = f \frac{\ell}{D} \frac{\rho V^2}{2} \tag{8.33}$$

where

$$f = \phi\left(\text{Re}, \frac{\varepsilon}{D}\right)$$

For laminar fully developed flow, the value of f is simply $f = 64/\text{Re}$, independent of ε/D. For turbulent flow, the functional dependence of the friction factor on the Reynolds number and the relative roughness, $f = \phi(\text{Re}, \varepsilon/D)$, is a rather complex one that cannot, as yet, be obtained from a theoretical analysis. The results are obtained from an exhaustive set of experiments and usually presented in terms of a curve-fitting formula or the equivalent graphical form.

From **Eq. 5.89** the energy equation for steady incompressible flow is

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

where h_L is the head loss between sections (1) and (2). With the assumption of a constant diameter ($D_1 = D_2$ so that $V_1 = V_2$), horizontal ($z_1 = z_2$) pipe with fully developed flow ($\alpha_1 = \alpha_2$), this becomes $\Delta p = p_1 - p_2 = \gamma h_L$, which can be combined with Eq. 8.33 to give

The head loss in pipe flow is given in terms of the friction factor.

$$h_L = f \frac{\ell}{D} \frac{V^2}{2g} \tag{8.34}$$

Equation 8.34, called the *Darcy–Weisbach equation*, is valid for any fully developed, steady, incompressible pipe flow—whether the pipe is horizontal or on a hill. On the other hand, Eq. 8.33 is valid only for horizontal pipes. In general, with $V_1 = V_2$ the energy equation gives

$$p_1 - p_2 = \gamma(z_2 - z_1) + \gamma h_L = \gamma(z_2 - z_1) + f \frac{\ell}{D} \frac{\rho V^2}{2}$$

Part of the pressure change is due to the elevation change and part is due to the head loss associated with frictional effects, which are given in terms of the friction factor, f.

It is not easy to determine the functional dependence of the friction factor on the Reynolds number and relative roughness. Much of this information is a result of experiments conducted by J. Nikuradse in 1933 (Ref. 6) and amplified by many others since then. One difficulty lies in the determination of the roughness of the pipe. Nikuradse used artificially roughened pipes produced by gluing sand grains of known size onto pipe walls to produce pipes with sandpaper-type surfaces. The pressure drop needed to produce a desired flowrate was measured and the data were converted into the friction factor for the corresponding Reynolds number and relative roughness. The tests were repeated numerous times for a wide range of Re and ε/D to determine the $f = \phi(\text{Re}, \varepsilon/D)$ dependence.

In commercially available pipes the roughness is not as uniform and well defined as in the artificially roughened pipes used by Nikuradse. However, it is possible to obtain a measure of the effective relative roughness of typical pipes and thus to obtain the friction factor. Typical roughness values for various pipe surfaces are given in Table 8.1. Figure 8.20 shows the functional dependence of f on Re and ε/D and is called the *Moody chart* in honor of **L. F. Moody**, who, along with C. F. Colebrook, correlated the original data of Nikuradse in terms of the relative roughness of commercially available pipe materials. It should be noted that the values of ε/D do not necessarily correspond to the actual values obtained by a

■ TABLE 8.1
Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

microscopic determination of the average height of the roughness of the surface. They do, however, provide the correct correlation for $f = \phi(\text{Re}, \varepsilon/D)$.

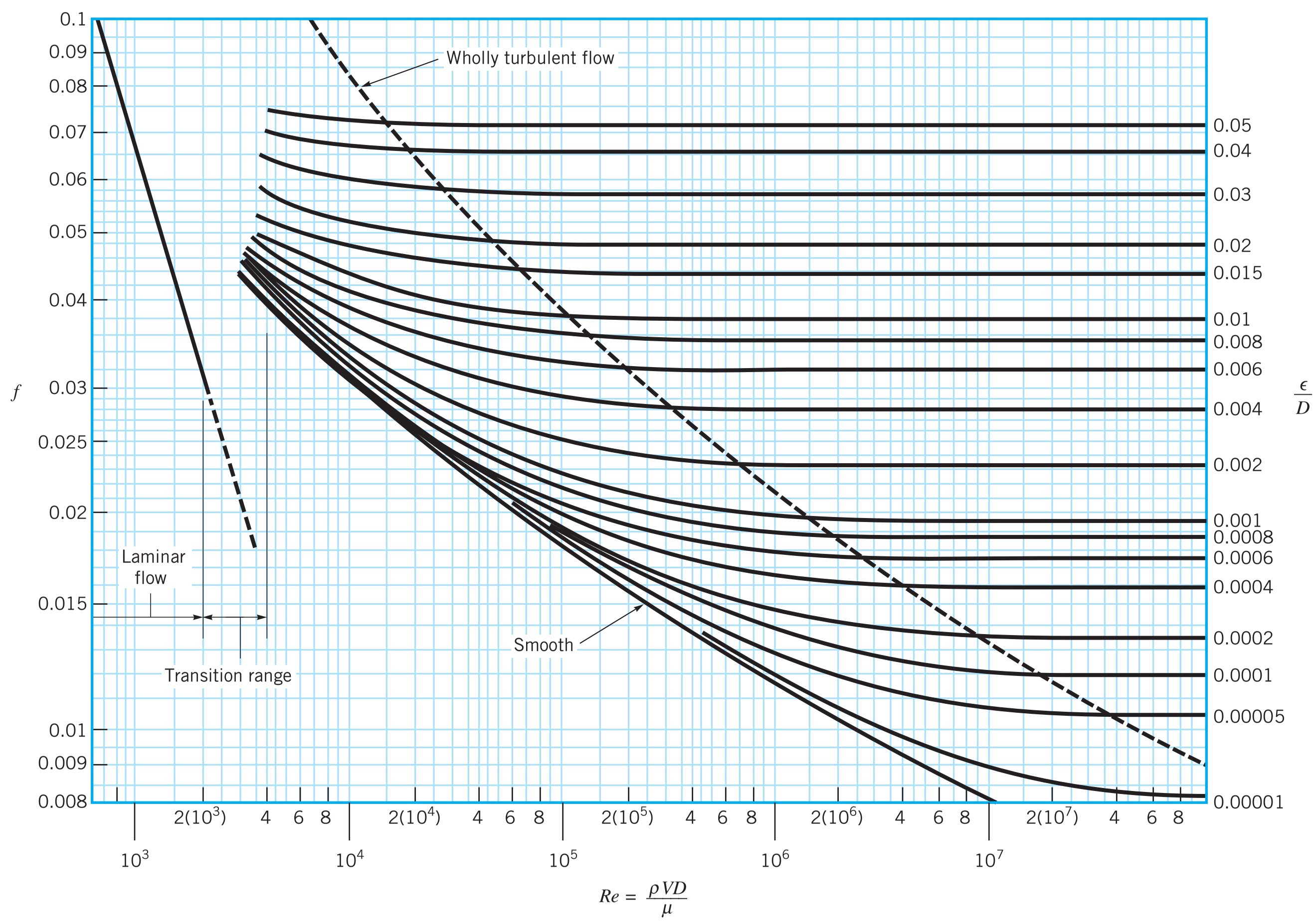
It is important to observe that the values of relative roughness given pertain to new, clean pipes. After considerable use, most pipes (because of a buildup of corrosion or scale) may have a relative roughness that is considerably larger (perhaps by an order of magnitude) than that given. Very old pipes may have enough scale buildup to not only alter the value of ε but also to change their effective diameter by a considerable amount.

The Moody chart gives the friction factor in terms of the Reynolds number and relative roughness.

The following characteristics are observed from the data of Fig. 8.20. For laminar flow, $f = 64/\text{Re}$, which is independent of relative roughness. For very large Reynolds numbers, $f = \phi(\varepsilon/D)$, which is independent of the Reynolds number. For such flows, commonly termed *completely turbulent flow* (or *wholly turbulent flow*), the laminar sublayer is so thin (its thickness decreases with increasing Re) that the surface roughness completely dominates the character of the flow near the wall. Hence, the pressure drop required is a result of an inertia-dominated turbulent shear stress rather than the viscosity-dominated laminar shear stress normally found in the viscous sublayer. For flows with moderate values of Re, the friction factor is indeed dependent on both the Reynolds number and relative roughness—$f = \phi(\text{Re}, \varepsilon/D)$. The gap in the figure for which no values of f are given (the $2100 < \text{Re} < 4000$ range) is a result of the fact that the flow in this transition range may be laminar or turbulent (or an unsteady mix of both) depending on the specific circumstances involved.

Note that even for smooth pipes ($\varepsilon = 0$) the friction factor is not zero. That is, there is a head loss in any pipe, no matter how smooth the surface is made. This is a result of the no-slip boundary condition that requires any fluid to stick to any solid surface it flows over. There is always some microscopic surface roughness that produces the no-slip behavior (and thus $f \neq 0$) on the molecular level, even when the roughness is considerably less than the viscous sublayer thickness. Such pipes are called *hydraulically smooth*.

Various investigators have attempted to obtain an analytical expression for $f = \phi(\text{Re}, \varepsilon/D)$. Note that the Moody chart covers an extremely wide range in flow parameters. The nonlaminar region covers more than four orders of magnitude in Reynolds number—from $\text{Re} = 4 \times 10^3$ to $\text{Re} = 10^8$. Obviously, for a given pipe and fluid, typical values of the average velocity do not cover this range. However, because of the large variety in pipes (D), fluids (ρ and μ), and velocities (V), such a wide range in Re is needed to accommodate nearly all applications of pipe flow. In many cases the particular pipe flow of interest is confined to a relatively small region of the Moody chart, and simple semiempirical expressions can



■ **FIGURE 8.20** **Friction factor as a function of Reynolds number and relative roughness for round pipes—the Moody chart. (Data from Ref. 7 with permission.)**

be developed for those conditions. For example, a company that manufactures cast iron water pipes with diameters between 2 and 12 in. may use a simple equation valid for their conditions only. The Moody chart, on the other hand, is universally valid for all steady, fully developed, incompressible pipe flows.

The turbulent portion of the Moody chart is represented by the Colebrook formula.

The following equation from Colebrook is valid for the entire nonlaminar range of the Moody chart

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \tag{8.35}$$

In fact, the Moody chart is a graphical representation of this equation, which is an empirical fit of the pipe flow pressure drop data. Equation 8.35 is called the *Colebrook formula*. A difficulty with its use is that it is implicit in the dependence of f. That is, for given conditions (Re and ε/D), it is not possible to solve for f without some sort of iterative scheme. With the use of modern computers and calculators, such calculations are not difficult. (As shown in **Problem 8.37** at the end of this chapter, it is possible to obtain an equation that adequately approximates the Colebrook/Moody chart relationship but does not require an iterative scheme.) A word of caution is in order concerning the use of the Moody chart or the equivalent Colebrook formula. Because of various inherent inaccuracies involved (uncertainty in the relative roughness, uncertainty in the experimental data used to produce the Moody chart, etc.), the use of several place accuracy in pipe flow problems is usually not justified. As a rule of thumb, a 10% accuracy is the best expected.

EXAMPLE 8.5

Air under standard conditions flows through a 4.0-mm-diameter drawn tubing with an average velocity of $V = 50$ m/s For such conditions the flow would normally be turbulent. However, if precautions are taken to eliminate disturbances to the flow (the entrance to the tube is very smooth, the air is dust free, the tube does not vibrate, etc.), it may be possible to maintain laminar flow. (a) Determine the pressure drop in a 0.1-m section of the tube if the flow is laminar. (b) Repeat the calculations if the flow is turbulent.

SOLUTION

Under standard temperature and pressure conditions the density and viscosity are $\rho = 1.23 \text{ kg/m}^3$ and $\mu = 1.79 \times 10^{-5} \text{ N} \cdot \text{s/m}^2$. Thus, the Reynolds number is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(1.23 \text{ kg/m}^3)(50 \text{ m/s})(0.004 \text{ m})}{1.79 \times 10^{-5} \text{ N} \cdot \text{s/m}^2} = 13{,}700$$

which would normally indicate turbulent flow.

(a) If the flow were laminar, then $f = 64/\text{Re} = 64/13{,}700 = 0.00467$ and the pressure drop in a 0.1-m-long horizontal section of the pipe would be

$$\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2 = (0.00467)\frac{(0.1 \text{ m})}{(0.004 \text{ m})}\frac{1}{2}(1.23 \text{ kg/m}^3)(50 \text{ m/s})^2$$

or

$$\Delta p = 0.179 \text{ kPa} \qquad \textbf{(Ans)}$$

Note that the same result is obtained from Eq. 8.8.

$$\Delta p = \frac{32\mu\ell}{D^2}V = \frac{32(1.79 \times 10^{-5} \text{ N} \cdot \text{s/m}^2)(0.1 \text{ m})(50 \text{ m/s})}{(0.004 \text{ m})^2} = 179 \text{ N/m}^2$$

(b) If the flow were turbulent, then $f = \phi(\text{Re}, \varepsilon/D)$, where from Table 8.1, $\varepsilon = 0.0015$ mm so that $\varepsilon/D = 0.0015 \text{ mm}/4.0 \text{ mm} = 0.000375$. From the Moody chart with $\text{Re} = 1.37 \times 10^4$ and $\varepsilon/D = 0.000375$ we obtain $f = 0.028$. Thus, the pressure drop in this case would be approximately

$$\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2 = (0.028)\frac{(0.1 \text{ m})}{(0.004 \text{ m})}\frac{1}{2}(1.23 \text{ kg/m}^3)(50 \text{ m/s})^2$$

or

$$\Delta p = 1.076 \text{ kPa} \qquad \textbf{(Ans)}$$

A considerable savings in effort to force the fluid through the pipe could be realized (0.179 kPa rather than 1.076 kPa) if the flow could be maintained as laminar flow at this Reynolds number. In general this is very difficult to do, although laminar flow in pipes has been maintained up to $\text{Re} \approx 100{,}000$ in rare instances.

An alternate method to determine the friction factor for the turbulent flow would be to use the Colebrook formula, Eq. 8.35. Thus,

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0 \log\left(\frac{0.000375}{3.7} + \frac{2.51}{1.37 \times 10^4\sqrt{f}}\right)$$

or

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(1.01 \times 10^{-4} + \frac{1.83 \times 10^{-4}}{\sqrt{f}}\right) \qquad \textbf{(1)}$$

An iterative procedure to obtain f can be done as follows. We assume a value of f ($f = 0.02$, for example), substitute it into the right-hand side of Eq. 1, and calculate a new f ($f = 0.0307$ in this case). Since the two values do not agree, the assumed value is not the solution. Hence, we try again. This time we assume $f = 0.0307$ (the last value calculated) and calculate the new value as $f = 0.0289$. Again this is still not the solution. Two more iterations show that the assumed and calculated values converge to the solution $f = 0.0291$, in agreement (within the accuracy of reading the graph) with the Moody chart method of $f = 0.028$.

Numerous other empirical formulas can be found in the literature (Ref. 5) for portions of the Moody chart. For example, an often-used equation, commonly referred to as the Blasius formula, for turbulent flow in smooth pipes ($\varepsilon/D = 0$) with $\text{Re} < 10^5$ is

$$f = \frac{0.316}{\text{Re}^{1/4}}$$

For our case this gives

$$f = 0.316(13{,}700)^{-0.25} = 0.0292$$

which is in agreement with the previous results. Note that the value of f is relatively insensitive to ε/D for this particular situation. Whether the tube was smooth glass ($\varepsilon/D = 0$) or the drawn tubing ($\varepsilon/D = 0.000375$) would not make much difference in the pressure drop. For this flow, an increase in relative roughness by a factor of 30 to $\varepsilon/D = 0.0113$ (equivalent to a commercial steel surface; see Table 8.1) would give $f = 0.043$. This would represent an increase in pressure drop and head loss by a factor of $0.043/0.0291 = 1.48$ compared with that for the original drawn tubing.

The pressure drop of 1.076 kPa in a length of 0.1 m of pipe corresponds to a change in absolute pressure [assuming $p = 101$ kPa (abs) at $x = 0$] of approximately

1.076/ 101 = 0.0107, or about 1%. Thus, the incompressible flow assumption on which the above calculations (and all of the formulas in this chapter) are based is reasonable. However, if the pipe were 2-m long the pressure drop would be 21.5 kPa, approximately 20% of the original pressure. In this case the density would not be approximately constant along the pipe, and a compressible flow analysis would be needed. Such considerations are discussed in **Chapter 11**.

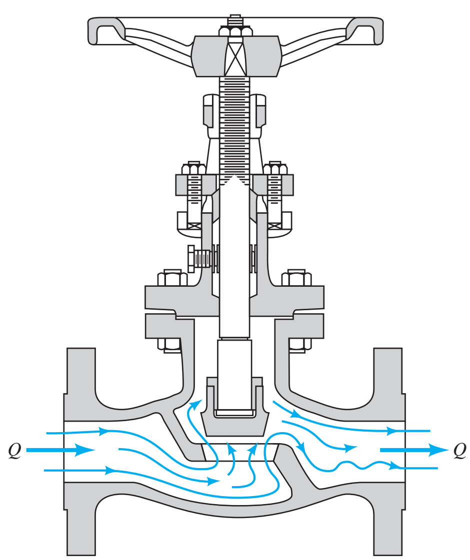
8.4.2 Minor Losses

Losses occur in straight pipes (major losses) and pipe system components (minor losses).

As discussed in the previous section, the head loss in long, straight sections of pipe can be calculated by use of the friction factor obtained from either the Moody chart or the Colebrook equation. Most pipe systems, however, consist of considerably more than straight pipes. These additional components (valves, bends, tees, and the like) add to the overall head loss of the system. Such losses are generally termed *minor losses*, with the apparent implication being that the majority of the system loss is associated with the friction in the straight portions of the pipes, the *major losses*. In many cases this is true. In other cases the minor losses are greater than the major losses. In this section we indicate how to determine the various minor losses that commonly occur in pipe systems.

The head loss associated with flow through a valve is a common minor loss. The purpose of a valve is to provide a means to regulate the flowrate. This is accomplished by changing the geometry of the system (i.e., closing or opening the valve alters the flow pattern through the valve), which in turn alters the losses associated with the flow through the valve. The flow resistance or head loss through the valve may be a significant portion of the resistance in the system. In fact, with the valve closed, the resistance to the flow is infinite—the fluid cannot flow. Such minor losses may be very important indeed. With the valve wide open the extra resistance due to the presence of the valve may or may not be negligible.

The flow pattern through a typical component such as a valve is shown in Fig. 8.21. It is not difficult to realize that a theoretical analysis to predict the details of such flows to obtain the head loss for these components is not, as yet, possible. Thus, the head loss information for essentially all components is given in dimensionless form and based on experi-



■ FIGURE 8.21 **Flow through a valve.**

Losses due to pipe system components are given in terms of loss coefficients.

mental data. The most common method used to determine these head losses or pressure drops is to specify the *loss coefficient*, K_L, which is defined as

$$K_L = \frac{h_L}{(V^2/2g)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

so that

$$\Delta p = K_L \tfrac{1}{2}\rho V^2$$

or

$$h_L = K_L \frac{V^2}{2g} \tag{8.36}$$

The pressure drop across a component that has a loss coefficient of $K_L = 1$ is equal to the dynamic pressure, $\rho V^2/2$.

The actual value of K_L is strongly dependent on the geometry of the component considered. It may also be dependent on the fluid properties. That is,

$$K_L = \phi(\text{geometry, Re})$$

where $\text{Re} = \rho VD/\mu$ is the pipe Reynolds number. For many practical applications the Reynolds number is large enough so that the flow through the component is dominated by inertia effects, with viscous effects being of secondary importance. This is true because of the relatively large accelerations and decelerations experienced by the fluid as it flows along a rather curved, variable-area (perhaps even torturous) path through the component (see Fig. 8.21). In a flow that is dominated by inertia effects rather than viscous effects, it is usually found that pressure drops and head losses correlate directly with the dynamic pressure. This is the reason why the friction factor for very large Reynolds number, fully developed pipe flow is independent of the Reynolds number. The same condition is found to be true for flow through pipe components. Thus, in most cases of practical interest the loss coefficients for components are a function of geometry only, $K_L = \phi(\text{geometry})$.

Minor losses are sometimes given in terms of an *equivalent length*, ℓ_{eq}. In this terminology, the head loss through a component is given in terms of the equivalent length of pipe that would produce the same head loss as the component. That is,

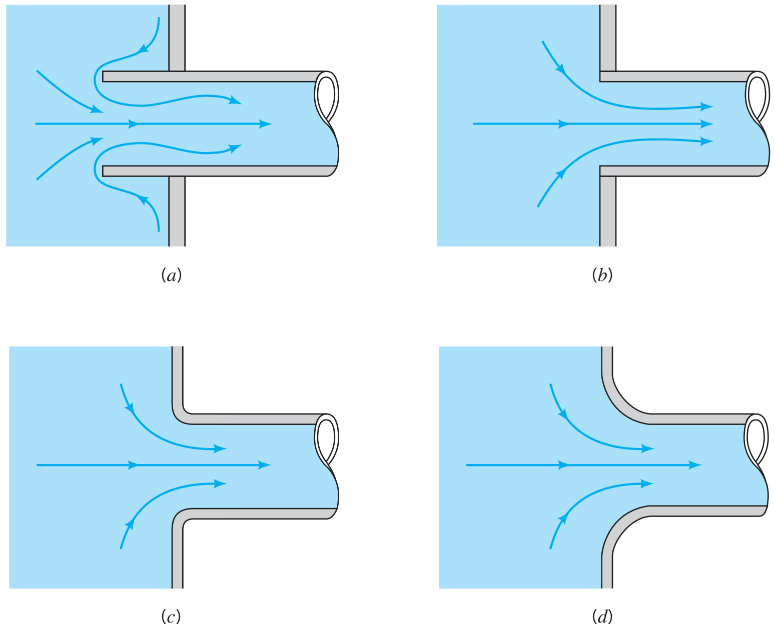
$$h_L = K_L \frac{V^2}{2g} = f \frac{\ell_{eq}}{D} \frac{V^2}{2g}$$

or

$$\ell_{eq} = \frac{K_L D}{f}$$

where D and f are based on the pipe containing the component. The head loss of the pipe system is the same as that produced in a straight pipe whose length is equal to the pipes of the original system plus the sum of the additional equivalent lengths of all of the components of the system. Most pipe flow analyses, including those in this book, use the loss coefficient method rather than the equivalent length method to determine the minor losses.

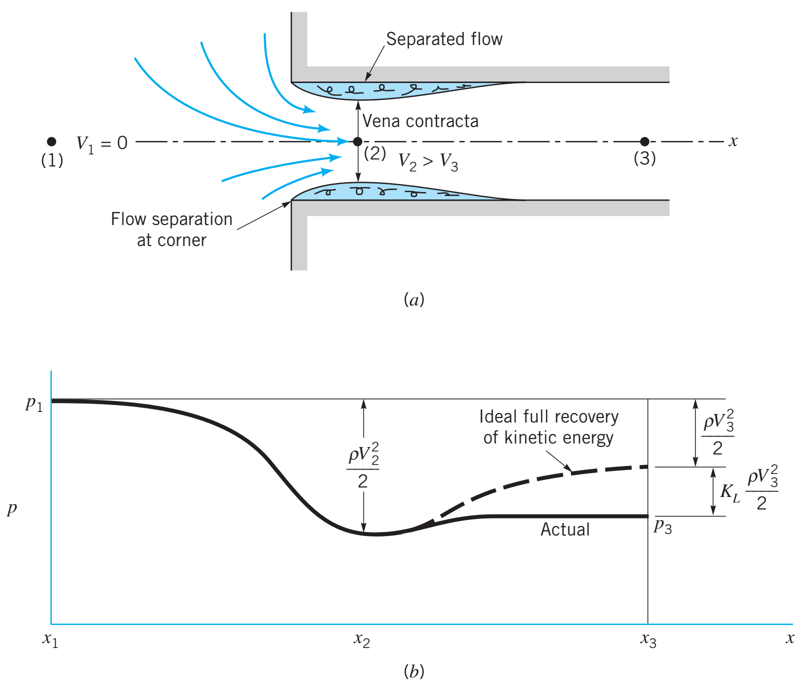
Many pipe systems contain various transition sections in which the pipe diameter changes from one size to another. Such changes may occur abruptly or rather smoothly through some type of area change section. Any change in flow area contributes losses that are not accounted for in the fully developed head loss calculation (the friction factor). The extreme cases involve flow into a pipe from a reservoir (an entrance) or out of a pipe into a reservoir (an exit).



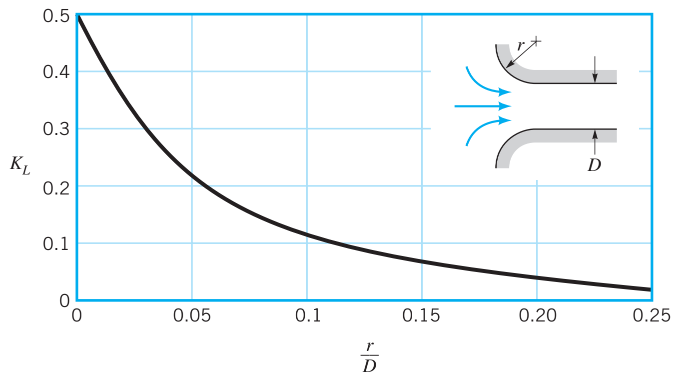
■ **FIGURE 8.22 Entrance flow conditions and loss coefficient (Refs. 28, 29). (*a*) Reentrant, $K_L = 0.8$, (*b*) sharp-edged, $K_L = 0.5$, (*c*) slightly rounded, $K_L = 0.2$ (see Fig. 8.24), (*d*) well-rounded, $K_L = 0.04$ (see Fig. 8.24).**

A vena contracta region is often developed at the entrance to a pipe.

A fluid may flow from a reservoir into a pipe through any number of different shaped entrance regions as are sketched in Fig. 8.22. Each geometry has an associated loss coefficient. A typical flow pattern for flow entering a pipe through a square-edged entrance is sketched in Fig. 8.23. As was discussed in **Chapter 3**, a vena contracta region may result



■ **FIGURE 8.23 Flow pattern and pressure distribution for a sharp-edged entrance.**



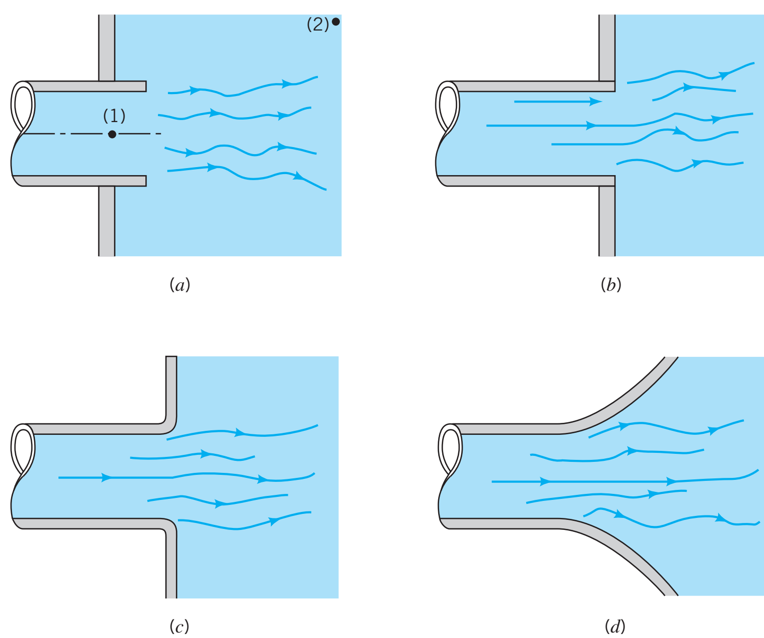
■ **FIGURE 8.24** **Entrance loss coefficient as a function of rounding of the inlet edge (Ref. 9).**

because the fluid cannot turn a sharp right-angle corner. The flow is said to separate from the sharp corner. The maximum velocity at section (2) is greater than that in the pipe at section (3), and the pressure there is lower. If this high-speed fluid could slow down efficiently, the kinetic energy could be converted into pressure (the Bernoulli effect), and the ideal pressure distribution indicated in Fig. 8.23 would result. The head loss for the entrance would be essentially zero.

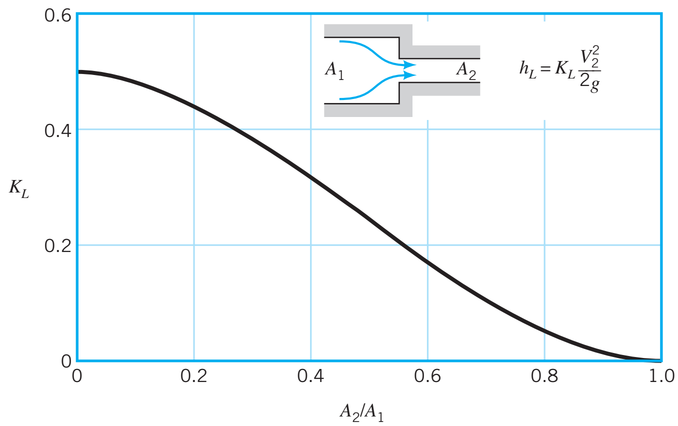
Minor head losses are often a result of the dissipation of kinetic energy.

Such is not the case. Although a fluid may be accelerated very efficiently, it is very difficult to slow down (decelerate) a fluid efficiently. Thus, the extra kinetic energy of the fluid at section (2) is partially lost because of viscous dissipation, so that the pressure does not return to the ideal value. An entrance head loss (pressure drop) is produced as is indicated in Fig. 8.23. The majority of this loss is due to inertia effects that are eventually dissipated by the shear stresses within the fluid. Only a small portion of the loss is due to the wall shear stress within the entrance region. The net effect is that the loss coefficient for a square-edged entrance is approximately $K_L = 0.50$. One-half of a velocity head is lost as the fluid enters the pipe. If the pipe protrudes into the tank (a reentrant entrance) as is shown in Fig. 8.22*a*, the losses are even greater.

V8.4 Entrance/exit flows



■ **FIGURE 8.25** **Exit flow conditions and loss coefficient. (*a*) Reentrant, $K_L = 1.0$, (*b*) sharp-edged, $K_L = 1.0$, (*c*) slightly rounded, $K_L = 1.0$, (*d*) well-rounded, $K_L = 1.0$.**



■ **FIGURE 8.26**
Loss coefficient for a sudden contraction (Ref. 10).

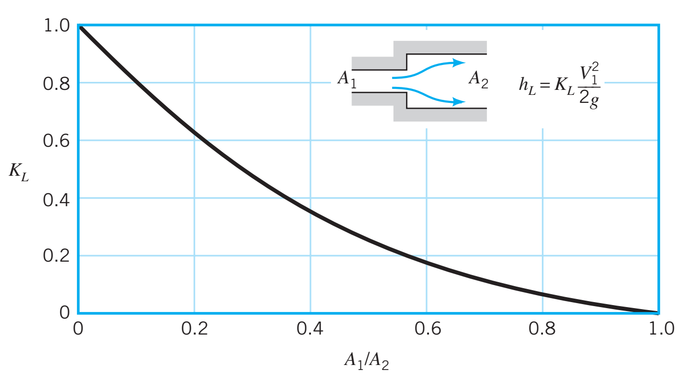
Pipe entrance losses can be relatively easily reduced by rounding the inlet.

An obvious way to reduce the entrance loss is to round the entrance region as is shown in Fig. 8.22*c*, thereby reducing or eliminating the vena contracta effect. Typical values for the loss coefficient for entrances with various amounts of rounding of the lip are shown in Fig. 8.24. A significant reduction in K_L can be obtained with only slight rounding.

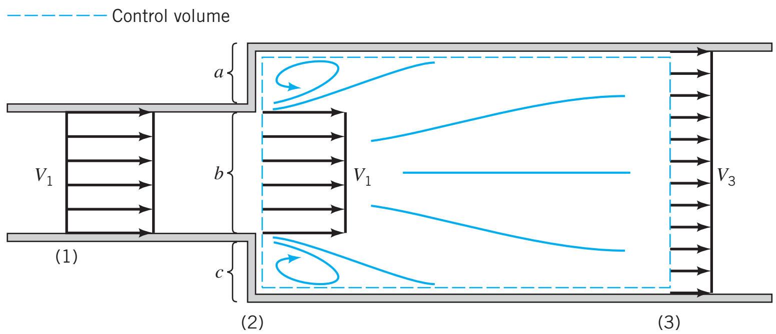
A head loss (the exit loss) is also produced when a fluid flows from a pipe into a tank as is shown in Fig. 8.25. In these cases the entire kinetic energy of the exiting fluid (velocity V_1) is dissipated through viscous effects as the stream of fluid mixes with the fluid in the tank and eventually comes to rest ($V_2 = 0$). The exit loss from points (1) and (2) is therefore equivalent to one velocity head, or $K_L = 1$.

Losses also occur because of a change in pipe diameter as is shown in Figs. 8.26 and 8.27. The sharp-edged entrance and exit flows discussed in the previous paragraphs are limiting cases of this type of flow with either $A_1/A_2 = \infty$, or $A_1/A_2 = 0$, respectively. The loss coefficient for a sudden contraction, $K_L = h_L/(V_2^2/2g)$, is a function of the area ratio, A_2/A_1, as is shown in Fig. 8.26. The value of K_L changes gradually from one extreme of a sharp-edged entrance ($A_2/A_1 = 0$ with $K_L = 0.50$) to the other extreme of no area change ($A_2/A_1 = 1$ with $K_L = 0$).

In many ways, the flow in a sudden expansion is similar to exit flow. As is indicated in Fig. 8.28, the fluid leaves the smaller pipe and initially forms a jet-type structure as it enters the larger pipe. Within a few diameters downstream of the expansion, the jet becomes dispersed across the pipe, and fully developed flow becomes established again. In this process [between sections (2) and (3)] a portion of the kinetic energy of the fluid is dissipated as a result of viscous effects. A square-edged exit is the limiting case with $A_1/A_2 = 0$.



■ **FIGURE 8.27**
Loss coefficient for a sudden expansion (Ref. 10).



■ **FIGURE 8.28 Control volume used to calculate the loss coefficient for a sudden expansion.**

A sudden expansion is one of the few components (perhaps the only one) for which the loss coefficient can be obtained by means of a simple analysis. To do this we consider the continuity and momentum equations for the control volume shown in Fig. 8.28 and the energy equation applied between (2) and (3). We assume that the flow is uniform at sections (1), (2), and (3) and the pressure is constant across the left-hand side of the control volume ($p_a = p_b = p_c = p_1$). The resulting three governing equations (mass, momentum, and energy) are

$$A_1V_1 = A_3V_3$$

$$p_1A_3 - p_3A_3 = \rho A_3V_3(V_3 - V_1)$$

The loss coefficient for a sudden expansion can be theoretically calculated.

and

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + h_L$$

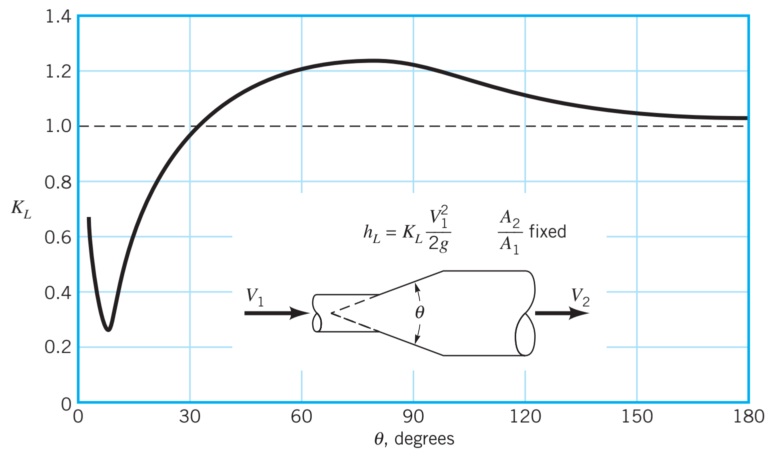
These can be rearranged to give the loss coefficient, $K_L = h_L/(V_1^2/2g)$, as

$$K_L = \left(1 - \frac{A_1}{A_2}\right)^2$$

where we have used the fact that $A_2 = A_3$. This result, plotted in Fig. 8.27, is in good agreement with experimental data. As with so many minor loss situations, it is not the viscous effects directly (i.e., the wall shear stress) that cause the loss. Rather, it is the dissipation of kinetic energy (another type of viscous effect) as the fluid decelerates inefficiently.

The losses may be quite different if the contraction or expansion is gradual. Typical results for a conical *diffuser* with a given area ratio, A_2/A_1, are shown in Fig. 8.29. (A diffuser is a device shaped to decelerate a fluid.) Clearly the included angle of the diffuser, θ, is a very important parameter. For very small angles, the diffuser is excessively long and most of the head loss is due to the wall shear stress as in fully developed flow. For moderate or large angles, the flow separates from the walls and the losses are due mainly to a dissipation of the kinetic energy of the jet leaving the smaller diameter pipe. In fact, for moderate or large values of θ (i.e., $\theta > 35°$ for the case shown in Fig. 8.29), the conical diffuser is, perhaps unexpectedly, less efficient than a sharp-edged expansion which has $K_L = 1$. There is an optimum angle ($\theta \approx 8°$ for the case illustrated) for which the loss coefficient is a minimum. The relatively small value of θ for the minimum K_L results in a long diffuser and is an indication of the fact that it is difficult to efficiently decelerate a fluid.

It must be noted that the conditions indicated in Fig. 8.29 represent typical results only. Flow through a diffuser is very complicated and may be strongly dependent on the area ratio



■ **FIGURE 8.29** **Loss coefficient for a typical conical diffuser (Ref. 5).**

A_2/A_1, specific details of the geometry, and the Reynolds number. The data are often presented in terms of a *pressure recovery coefficient*, $C_p = (p_2 - p_1)/(\rho V_1^2/2)$, which is the ratio of the static pressure rise across the diffuser to the inlet dynamic pressure. Considerable effort has gone into understanding this important topic (Refs. 11, 12).

Flow in a conical contraction (a nozzle; reverse the flow direction shown in Fig. 8.29) is less complex than that in a conical expansion. Typical loss coefficients based on the downstream (high-speed) velocity can be quite small, ranging from $K_L = 0.02$ for $\theta = 30°$, to $K_L = 0.07$ for $\theta = 60°$, for example. It is relatively easy to accelerate a fluid efficiently.

Bends in pipes produce a greater head loss than if the pipe were straight. The losses are due to the separated region of flow near the inside of the bend (especially if the bend is sharp) and the swirling secondary flow that occurs because of the imbalance of centripetal forces as a result of the curvature of the pipe centerline. These effects and the associated values of K_L for large Reynolds number flows through a 90° bend are shown in Fig. 8.30. The friction loss due to the axial length of the pipe bend must be calculated and added to that given by the loss coefficient of Fig. 8.30.

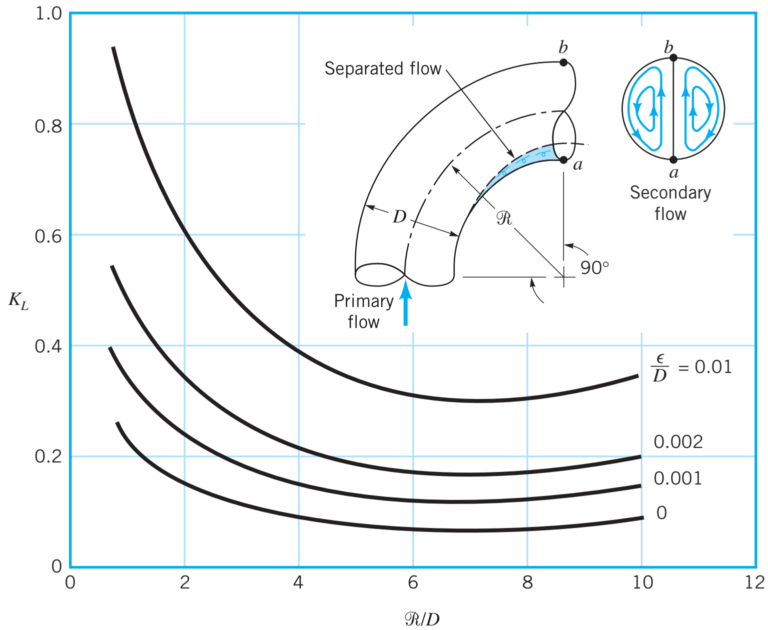
For situations in which space is limited, a flow direction change is often accomplished by use of miter bends, as is shown in Fig. 8.31, rather than smooth bends. The considerable losses in such bends can be reduced by the use of carefully designed guide vanes that help direct the flow with less unwanted swirl and disturbances.

V8.5 Car exhaust system

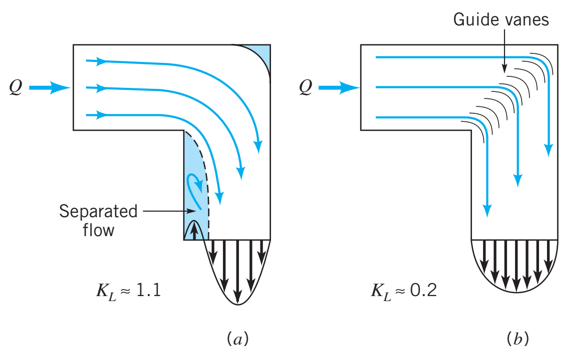
Another important category of pipe system components is that of commercially available pipe fittings such as elbows, tees, reducers, valves, and filters. The values of K_L for such components depend strongly on the shape of the component and only very weakly on the Reynolds number for typical large Re flows. Thus, the loss coefficient for a 90° elbow depends on whether the pipe joints are threaded or flanged but is, within the accuracy of the data, fairly independent of the pipe diameter, flow rate, or fluid properties (the Reynolds number effect). Typical values of K_L for such components are given in **Table 8.2**. These typical components are designed more for ease of manufacturing and costs than for reduction of the head losses that they produce. The flowrate from a faucet in a typical house is sufficient whether the value of K_L for an elbow is the typical $K_L = 1.5$, or it is reduced to $K_L = 0.2$ by use of a more expensive long-radius, gradual bend (Fig. 8.30).

Extensive tables are available for loss coefficients of standard pipe components.

Valves control the flowrate by providing a means to adjust the overall system loss coefficient to the desired value. When the valve is closed, the value of K_L is infinite and no



■ **FIGURE 8.30** **Character of the flow in a 90° bend and the associated loss coefficient (Ref. 5).**

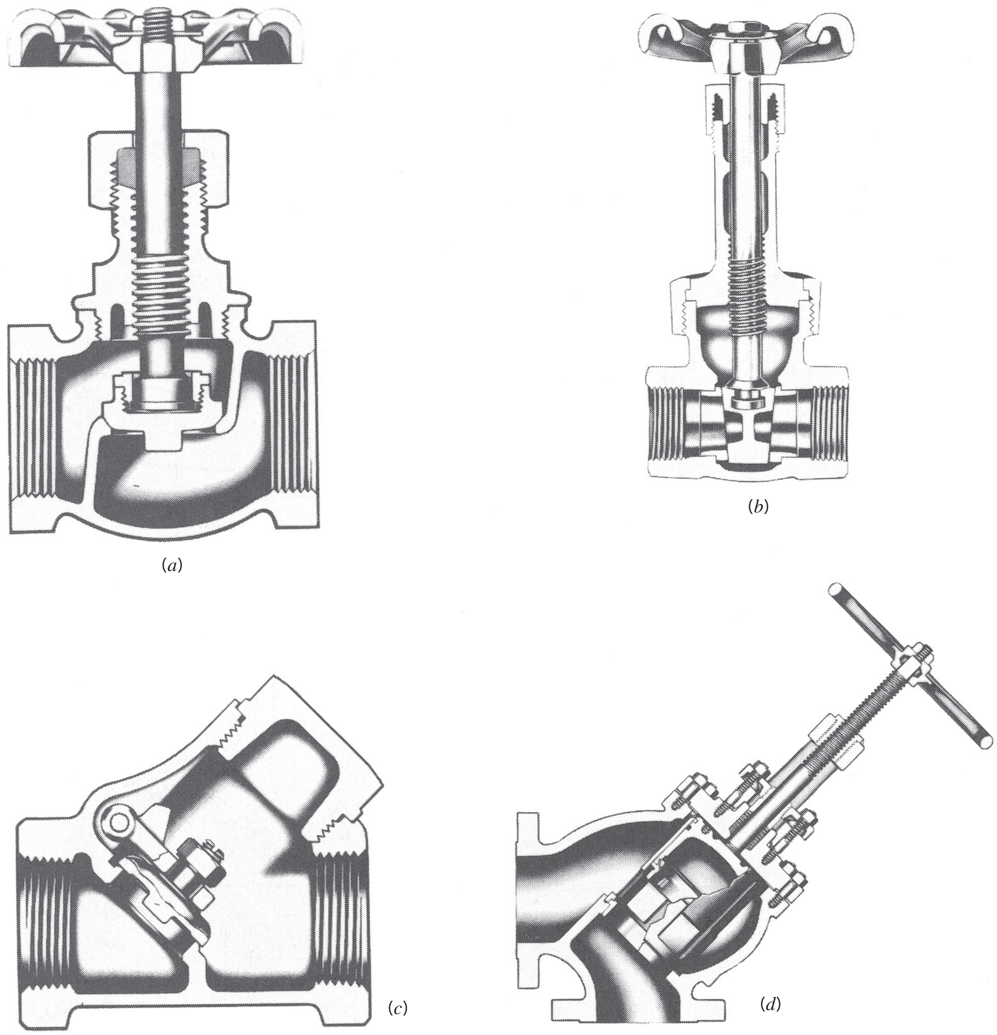


■ **FIGURE 8.31** **Character of the flow in a 90° mitered bend and the associated loss coefficient: (*a*) without guide vanes, (*b*) with guide vanes.**

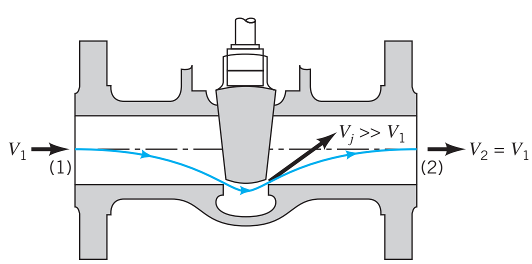
A valve is a variable resistance element in a pipe circuit.

fluid flows. Opening of the valve reduces K_L, producing the desired flowrate. Typical cross sections of various types of valves are shown in Fig. 8.32. Some valves (such as the conventional globe valve) are designed for general use, providing convenient control between the extremes of fully closed and fully open. Others (such as a needle valve) are designed to provide very fine control of the flowrate. The check valve provides a diode type operation that allows fluid to flow in one direction only.

Loss coefficients for typical valves are given in **Table 8.2**. As with many system components, the head loss in valves is mainly a result of the dissipation of kinetic energy of a high-speed portion of the flow. This is illustrated in Fig. 8.33.



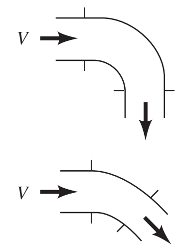
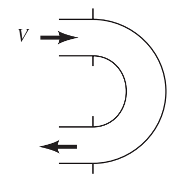
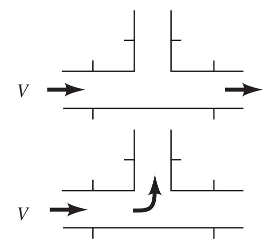
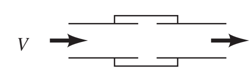
■ **FIGURE 8.32** **Internal structure of various valves: (*a*) globe valve, (*b*) gate valve, (*c*) swing check valve, (*d*) stop check valve. (Courtesy of Crane Co., Valve Division.)**



■ **FIGURE 8.33** **Head loss in a valve is due to dissipation of the kinetic energy of the large-velocity fluid near the valve seat.**

■ **TABLE 8.2**

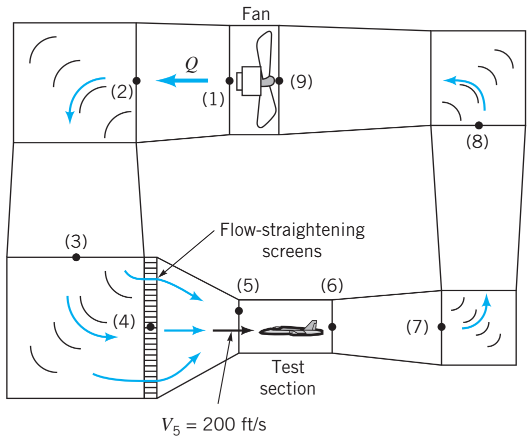
Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$ (Data from Refs. 5, 10, 27)

Component	K_L	
a. Elbows		
Regular 90°, flanged	0.3	
Regular 90°, threaded	1.5	
Long radius 90°, flanged	0.2	
Long radius 90°, threaded	0.7	
Long radius 45°, flanged	0.2	
Regular 45°, threaded	0.4	
b. 180° return bends		
180° return bend, flanged	0.2	
180° return bend, threaded	1.5	
c. Tees		
Line flow, flanged	0.2	
Line flow, threaded	0.9	
Branch flow, flanged	1.0	
Branch flow, threaded	2.0	
d. Union, threaded	0.08	
***e. Valves**		
Globe, fully open	10	
Angle, fully open	2	
Gate, fully open	0.15	
Gate, $\frac{1}{4}$ closed	0.26	
Gate, $\frac{1}{2}$ closed	2.1	
Gate, $\frac{3}{4}$ closed	17	
Swing check, forward flow	2	
Swing check, backward flow	∞	
Ball valve, fully open	0.05	
Ball valve, $\frac{1}{3}$ closed	5.5	
Ball valve, $\frac{2}{3}$ closed	210	

*See Fig. 8.36 for typical valve geometry

EXAMPLE 8.6

Air at standard conditions is to flow through the test section [between sections (5) and (6)] of the closed-circuit wind tunnel shown in Fig. E8.6 with a velocity of 200 ft/s. The flow is driven by a fan that essentially increases the static pressure by the amount $p_1 - p_9$ that is needed to overcome the head losses experienced by the fluid as it flows around the circuit. Estimate the value of $p_1 - p_9$ and the horsepower supplied to the fluid by the fan.



Location	Area (ft^2)	Velocity (ft/s)
1	22.0	36.4
2	28.0	28.6
3	35.0	22.9
4	35.0	22.9
5	4.0	200.0
6	4.0	200.0
7	10.0	80.0
8	18.0	44.4
9	22.0	36.4

■ FIGURE E8.6

SOLUTION

The maximum velocity within the wind tunnel occurs in the test section (smallest area). Thus, the maximum Mach number of the flow is $\text{Ma}_5 = V_5/c_5$, where $V_5 = 200$ ft/s and from Eq. 1.20 the speed of sound is $c_5 = (kRT_5)^{1/2} = \{1.4(1716 \text{ ft}\cdot\text{lb/slug}\cdot{}^\circ\text{R})[(460 + 59)^\circ\text{R}]\}^{1/2} = 1117$ ft/s. Thus, $\text{Ma}_5 = 200/1117 = 0.179$. As was indicated in **Chapter 3** and discussed fully in **Chapter 11**, most flows can be considered as incompressible if the Mach number is less than about 0.3. Hence, we can use the incompressible formulas for this problem.

The purpose of the fan in the wind tunnel is to provide the necessary energy to overcome the net head loss experienced by the air as it flows around the circuit. This can be found from the energy equation between points (1) and (9) as

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_9}{\gamma} + \frac{V_9^2}{2g} + z_9 + h_{L_{1-9}}$$

where $h_{L_{1-9}}$ is the total head loss from (1) to (9). With $z_1 = z_9$ and $V_1 = V_9$ this gives

$$\frac{p_1}{\gamma} - \frac{p_9}{\gamma} = h_{L_{1-9}} \qquad \textbf{(1)}$$

Similarly, by writing the energy equation (Eq. 5.84) across the fan, from (9) to (1), we obtain

$$\frac{p_9}{\gamma} + \frac{V_9^2}{2g} + z_9 + h_p = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_9$$

where h_p is the actual head rise supplied by the pump (fan) to the air. Again since $z_9 = z_1$ and $V_9 = V_1$ this, when combined with Eq. 1, becomes

$$h_p = \frac{(p_1 - p_9)}{\gamma} = h_{L_{1-9}}$$

The actual power supplied to the air (horsepower, $\mathcal{P}_a$) is obtained from the fan head by

$$\mathcal{P}_a = \gamma Q h_p = \gamma A_5 V_5 h_p = \gamma A_5 V_5 h_{L_{1-9}} \qquad \textbf{(2)}$$

Thus, the power that the fan must supply to the air depends on the head loss associated with the flow through the wind tunnel. To obtain a reasonable, approximate answer we make the following assumptions. We treat each of the four turning corners as a mitered bend with guide vanes so that from Fig. 8.31 $K_{L_{\text{corner}}} = 0.2$. Thus, for each corner

$$h_{L_{\text{corner}}} = K_L \frac{V^2}{2g} = 0.2 \frac{V^2}{2g}$$

where, because the flow is assumed incompressible, $V = V_5 A_5 / A$. The values of A and the corresponding velocities throughout the tunnel are given in Table E8.6.

We also treat the enlarging sections from the end of the test section (6) to the beginning of the nozzle (4) as a conical diffuser with a loss coefficient of $K_{L_{\text{dif}}} = 0.6$. This value is larger than that of a well-designed diffuser (see Fig. 8.29, for example). Since the wind tunnel diffuser is interrupted by the four turning corners and the fan, it may not be possible to obtain a smaller value of $K_{L_{\text{dif}}}$ for this situation. Thus,

$$h_{L_{\text{dif}}} = K_{L_{\text{dif}}} \frac{V_6^2}{2g} = 0.6 \frac{V_6^2}{2g}$$

The loss coefficients for the conical nozzle between section (4) and (5) and the flow-straightening screens are assumed to be $K_{L_{\text{noz}}} = 0.2$ and $K_{L_{\text{scr}}} = 4.0$ (Ref. 13), respectively. We neglect the head loss in the relatively short test section.

Thus, the total head loss is

$$h_{L_{1-9}} = h_{L_{\text{corner7}}} + h_{L_{\text{corner8}}} + h_{L_{\text{corner2}}} + h_{L_{\text{corner3}}} + h_{L_{\text{dif}}} + h_{L_{\text{noz}}} + h_{L_{\text{scr}}}$$

or

$$\begin{aligned} h_{L_{1-9}} &= [0.2(V_7^2 + V_8^2 + V_2^2 + V_3^2) + 0.6V_6^2 + 0.2V_5^2 + 4.0V_4^2]/2g \\ &= [0.2(80.0^2 + 44.4^2 + 28.6^2 + 22.9^2) + 0.6(200)^2 \\ &\quad + 0.2(200)^2 + 4.0(22.9)^2] \text{ ft}^2/\text{s}^2/[2(32.2 \text{ ft/s}^2)] \end{aligned}$$

or

$$h_{L_{1-9}} = 560 \text{ ft}$$

Hence, from Eq. 1 we obtain the pressure rise across the fan as

$$\begin{aligned} p_1 - p_9 = \gamma h_{L_{1-9}} &= (0.0765 \text{ lb/ft}^3)(560 \text{ ft}) \\ &= 42.8 \text{ lb/ft}^2 = 0.298 \text{ psi} \end{aligned}$$ **(Ans)**

From Eq. 2 we obtain the power added to the fluid as

$$\mathcal{P}_a = (0.0765 \text{ lb/ft}^3)(4.0 \text{ ft}^2)(200 \text{ ft/s})(560 \text{ ft}) = 34{,}300 \text{ ft} \cdot \text{lb/s}$$

or

$$\mathcal{P}_a = \frac{34{,}300 \text{ ft} \cdot \text{lb/s}}{550 \text{ (ft} \cdot \text{lb/s)/hp}} = 62.3 \text{ hp}$$ **(Ans)**

With a closed-return wind tunnel of this type, all of the power required to maintain the flow is dissipated through viscous effects, with the energy remaining within the closed tunnel. If heat transfer across the tunnel walls is negligible, the air temperature within the tunnel will increase in time. For steady state operations of such tunnels, it is often necessary to provide some means of cooling to maintain the temperature at acceptable levels.

It should be noted that the actual size of the motor that powers the fan must be greater than the calculated 62.3 hp because the fan is not 100% efficient. The power calculated above is that needed by the fluid to overcome losses in the tunnel, excluding those in the fan. If the fan were 60% efficient, it would require a shaft power of $\mathcal{P} = 62.3 \text{ hp}/(0.60) = 104 \text{ hp}$ to run the fan. Determination of fan (or pump) efficiencies can be a complex problem that depends on the specific geometry of the fan. Introductory material about fan performance is presented in **Chapter 12**; additional material can be found in various references (Refs. 14, 15, 16, for example).

It should also be noted that the above results are only approximate. Clever, careful design of the various components (corners, diffuser, etc.) may lead to improved (i.e., lower) values of the various loss coefficients, and hence lower power requirements. Since h_L is proportional to V^2, the components with the larger V tend to have the larger head loss. Thus, even though $K_L = 0.2$ for each of the four corners, the head loss for corner (7) is $(V_7/V_3)^2 = (80/22.9)^2 = 12.2$ times greater than it is for corner (3).

8.4.3 Noncircular Conduits

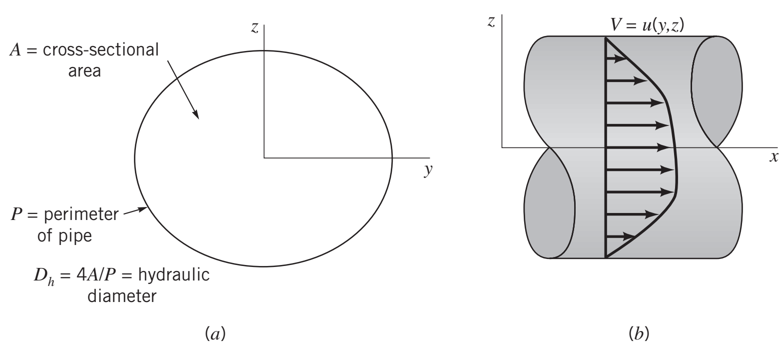
Many of the conduits that are used for conveying fluids are not circular in cross section. Although the details of the flows in such conduits depend on the exact cross-sectional shape, many round pipe results can be carried over, with slight modification, to flow in conduits of other shapes.

Theoretical results can be obtained for fully developed laminar flow in noncircular ducts, although the detailed mathematics often becomes rather cumbersome. For an arbitrary cross section, as is shown in Fig. 8.34, the velocity profile is a function of both y and z $[\mathbf{V} = u(y, z)\hat{\mathbf{i}}]$. This means that the governing equation from which the velocity profile is obtained (either the Navier–Stokes equations of motion or a force balance equation similar to that used for circular pipes, Eq. 8.6) is a partial differential equation rather than an ordinary differential equation. Although the equation is linear (for fully developed flow the convective acceleration is zero), its solution is not as straightforward as for round pipes. Typically the velocity profile is given in terms of an infinite series representation (Ref. 17).

The hydraulic diameter is used for noncircular duct calculations.

Practical, easy-to-use results can be obtained as follows. Regardless of the cross-sectional shape, there are no inertia effects in fully developed laminar pipe flow. Thus, the friction factor can be written as $f = C/\text{Re}_h$, where the constant C depends on the particular shape of the duct, and Re_h is the Reynolds number, $\text{Re}_h = \rho V D_h/\mu$, based on the hydraulic diameter. The *hydraulic diameter* defined as $D_h = 4A/P$ is four times the ratio of the cross-sectional flow area divided by the wetted perimeter, P, of the pipe as is illustrated in Fig. 8.34. It represents a characteristic length that defines the size of a cross section of a specified shape. The factor of 4 is included in the definition of D_h so that for round pipes the diameter and hydraulic diameter are equal $[D_h = 4A/P = 4(\pi D^2/4)/(\pi D) = D]$. The hydraulic diameter is also used in the definition of the friction factor, $h_L = f(\ell/D_h)V^2/2g$, and the relative roughness, ε/D_h.

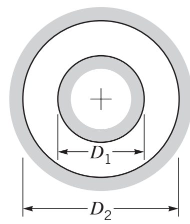
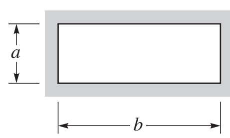
The values of $C = f\,\text{Re}_h$ for laminar flow have been obtained from theory and/or experiment for various shapes. Typical values are given in Table 8.3 along with the hydraulic diameter. Note that the value of C is relatively insensitive to the shape of the conduit. Unless the cross section is very "thin" in some sense, the value of C is not too different from its circular pipe value, $C = 64$. Once the friction factor is obtained, the calculations for noncircular conduits are identical to those for round pipes.



■ **FIGURE 8.34 Noncircular duct.**

■ **TABLE 8.3**

Friction Factors for Laminar Flow in Noncircular Ducts (Data from Ref. 18)

Shape	Parameter	$C = f\,\text{Re}_h$
I. Concentric Annulus	D_1/D_2	
$D_h = D_2 - D_1$	0.0001	71.8
	0.01	80.1
	0.1	89.4
	0.6	95.6
	1.00	96.0
II. Rectangle	a/b	
$D_h = \dfrac{2ab}{a+b}$	0	96.0
	0.05	89.9
	0.10	84.7
	0.25	72.9
	0.50	62.2
	0.75	57.9
	1.00	56.9

The Moody chart, developed for round pipes, can also be used for noncircular ducts.

Calculations for fully developed turbulent flow in ducts of noncircular cross section are usually carried out by using the Moody chart data for round pipes with the diameter replaced by the hydraulic diameter and the Reynolds number based on the hydraulic diameter. Such calculations are usually accurate to within about 15%. If greater accuracy is needed, a more detailed analysis based on the specific geometry of interest is needed.

EXAMPLE 8.7

Air at a temperature of 120 °F and standard pressure flows from a furnace through an 8-in.-diameter pipe with an average velocity of 10 ft/s. It then passes through a transition section and into a square duct whose side is of length a. The pipe and duct surfaces are smooth ($\varepsilon = 0$). Determine the duct size, a, if the head loss per foot is to be the same for the pipe and the duct.

SOLUTION

We first determine the head loss per foot for the pipe, $h_L/\ell = (f/D)\,V^2/2g$, and then size the square duct to give the same value. For the given pressure and temperature we obtain (from Table B.3) $\nu = 1.89 \times 10^{-4}\ \text{ft}^2/\text{s}$ so that

$$\text{Re} = \frac{VD}{\nu} = \frac{(10\ \text{ft/s})(\frac{8}{12}\ \text{ft})}{1.89 \times 10^{-4}\ \text{ft}^2/\text{s}} = 35{,}300$$

With this Reynolds number and with $\varepsilon/D = 0$ we obtain the friction factor from Fig. 8.20 as $f = 0.022$ so that

$$\frac{h_L}{\ell} = \frac{0.022}{(\frac{8}{12}\ \text{ft})}\,\frac{(10\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = 0.0512$$

Thus, for the square duct we must have

$$\frac{h_L}{\ell} = \frac{f}{D_h}\frac{V_s^2}{2g} = 0.0512 \tag{1}$$

where

$$D_h = 4A/P = 4a^2/4a = a \quad \text{and}$$

$$V_s = \frac{Q}{A} = \frac{\frac{\pi}{4}\left(\frac{8}{12}\text{ ft}\right)^2 (10\text{ ft/s})}{a^2} = \frac{3.49}{a^2} \tag{2}$$

is the velocity in the duct.

By combining Eqs. 1 and 2 we obtain

$$0.0512 = \frac{f}{a}\frac{(3.49/a^2)^2}{2(32.2)}$$

or

$$a = 1.30 f^{1/5} \tag{3}$$

where a is in feet. Similarly, the Reynolds number based on the hydraulic diameter is

$$\text{Re}_h = \frac{V_s D_h}{\nu} = \frac{(3.49/a^2)a}{1.89 \times 10^{-4}} = \frac{1.85 \times 10^4}{a} \tag{4}$$

We have three unknowns (a, f, and Re_h) and three equations (Eqs. 3, 4, and the third equation in graphical form, Fig. 8.20, the Moody chart). Thus, a trial and error solution is required.

As an initial attempt, assume the friction factor for the duct is the same as for the pipe. That is, assume $f = 0.022$. From Eq. 3 we obtain $a = 0.606$ ft, while from Eq. 4 we have $\text{Re}_h = 3.05 \times 10^4$. From **Fig. 8.20**, with this Reynolds number and the given smooth duct we obtain $f = 0.023$, which does not quite agree with the assumed value of f. Hence, we do not have the solution. We try again, using the latest calculated value of $f = 0.023$ as our guess. The calculations are repeated until the guessed value of f agrees with the value obtained from Fig. 8.20. The final result (after only two iterations) is $f = 0.023$, $\text{Re}_h = 3.03 \times 10^4$, and

$$a = 0.611 \text{ ft} = 7.34 \text{ in.} \qquad \textbf{(Ans)}$$

Note that the length of the side of the equivalent square duct is $a/D = 7.34/8 = 0.918$, or approximately 92% of the diameter of the equivalent duct. It can be shown that this value, 92%, is a very good approximation for any pipe flow—laminar or turbulent. The cross-sectional area of the duct ($A = a^2 = 53.9 \text{ in.}^2$) is greater than that of the round pipe ($A = \pi D^2/4 = 50.3 \text{ in.}^2$). Also, it takes less material to form the round pipe (perimeter $= \pi D =$ 25.1 in.) than the square duct (perimeter $= 4a = 29.4$ in.). Circles are very efficient shapes.

8.5 Pipe Flow Examples

Pipe systems may contain a single pipe with components or multiple interconnected pipes.

In the previous sections of this chapter, we discussed concepts concerning flow in pipes and ducts. The purpose of this section is to apply these ideas to the solutions of various practical problems. The application of the pertinent equations is straightforward, with rather simple calculations that give answers to problems of engineering importance. The main idea involved is to apply the energy equation between appropriate locations within the flow system, with the head loss written in terms of the friction factor and the minor loss coefficients. We will consider two classes of pipe systems: those containing a single pipe (whose length may be interrupted by various components), and those containing multiple pipes in parallel, series, or network configurations.

8.5.1 Single Pipes

The nature of the solution process for pipe flow problems can depend strongly on which of the various parameters are independent parameters (the "given") and which is the dependent parameter (the "determine"). The three most common types of problems are shown in Table 8.4 in terms of the parameters involved. We assume the pipe system is defined in terms of the length of pipe sections used and the number of elbows, bends, and valves needed to convey the fluid between the desired locations. In all instances we assume the fluid properties are given.

In a Type I problem we specify the desired flowrate or average velocity and determine the necessary pressure difference or head loss. For example, if a flowrate of 2.0 gal/min is required for a dishwasher that is connected to the water heater by a given pipe system, what pressure is needed in the water heater?

Pipe flow problems can be categorized by what parameters are given and what is to be calculated.

In a Type II problem we specify the applied driving pressure (or, alternatively, the head loss) and determine the flowrate. For example, how many gal/min of hot water are supplied to the dishwasher if the pressure within the water heater is 60 psi and the pipe system details (length, diameter, roughness of the pipe; number of elbows; etc.) are specified?

In a Type III problem we specify the pressure drop and the flowrate and determine the diameter of the pipe needed. For example, what diameter of pipe is needed between the water heater and dishwasher if the pressure in the water heater is 60 psi (determined by the city water system) and the flowrate is to be not less than 2.0 gal/min (determined by the manufacturer)?

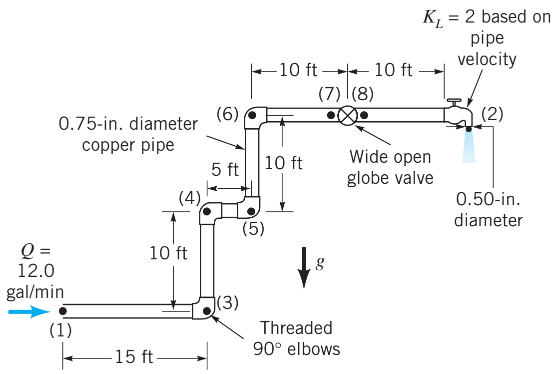
Several examples of these types of problems follow.

■ **TABLE 8.4**
Pipe Flow Types

Variable	Type I	Type II	Type III
a. Fluid			
Density	Given	Given	Given
Viscosity	Given	Given	Given
b. Pipe			
Diameter	Given	Given	Determine
Length	Given	Given	Given
Roughness	Given	Given	Given
c. Flow			
Flowrate or Average Velocity	Given	Determine	Given
d. Pressure			
Pressure Drop or Head Loss	Determine	Given	Given

EXAMPLE 8.8 (TYPE I, DETERMINE PRESSURE DROP)

Water at 60 °F flows from the basement to the second floor through the 0.75-in. (0.0625-ft)-diameter copper pipe (a drawn tubing) at a rate of $Q = 12.0$ gal/min $= 0.0267$ ft^3/s and exits through a faucet of diameter 0.50 in. as shown in Fig. E8.8*a*. Determine the pressure at point (1) if: (a) all losses are neglected, (b) the only losses included are major losses, or (c) all losses are included.



■ FIGURE E8.8a

SOLUTION

Since the fluid velocity in the pipe is given by $V_1 = Q/A_1 = Q/(\pi D^2/4) = (0.0267$ ft^3/s)/[$\pi(0.0625$ ft$)^2/4] = 8.70$ ft/s, and the fluid properties are $\rho = 1.94$ slugs/ft^3 and $\mu = 2.34 \times 10^{-5}$ lb · s/ft^2 (see Table B.1), it follows that $\text{Re} = \rho VD/\mu = (1.94$ slugs/ft^3)(8.70 ft/s)(0.0625 ft)/(2.34×10^{-5} lb · s/ft^2) = 45,000. Thus, the flow is turbulent. The governing equation for either case (a), (b), or (c) is Eq. 8.21,

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

where $z_1 = 0$, $z_2 = 20$ ft, $p_2 = 0$ (free jet), $\gamma = \rho g = 62.4$ lb/ft^3, and the outlet velocity is $V_2 = Q/A_2 = (0.0267 \text{ ft}^3/\text{s})/[\pi(0.50/12)^2\text{ft}^2/4] = 19.6$ ft/s. We assume that the kinetic energy coefficients α_1 and α_2 are unity. This is reasonable because turbulent velocity profiles are nearly uniform across the pipe. Thus,

$$p_1 = \gamma z_2 + \tfrac{1}{2}\rho(V_2^2 - V_1^2) + \gamma h_L \tag{1}$$

where the head loss is different for each of the three cases.

(a) If all losses are neglected ($h_L = 0$), Eq. 1 gives

$$p_1 = (62.4 \text{ lb/ft}^3)(20 \text{ ft}) + \frac{1.94 \text{ slugs/ft}^3}{2}\left[\left(19.6 \frac{\text{ft}}{\text{s}}\right)^2 - \left(8.70 \frac{\text{ft}}{\text{s}}\right)^2\right]$$
$$= (1248 + 299) \text{ lb/ft}^2 = 1547 \text{ lb/ft}^2$$

or

$$p_1 = 10.7 \text{ psi} \qquad \textbf{(Ans)}$$

Note that for this pressure drop, the amount due to elevation change (the hydrostatic effect) is $\gamma(z_2 - z_1) = 8.67$ psi and the amount due to the increase in kinetic energy is $\rho(V_2^2 - V_1^2)/2 = 2.07$ psi.

(b) If the only losses included are the major losses, the head loss is

$$h_L = f\frac{\ell}{D}\frac{V_1^2}{2g}$$

From **Table 8.1** the roughness for a 0.75-in.-diameter copper pipe (drawn tubing) is $\varepsilon = 0.00005$ ft so that $\varepsilon/D = 8 \times 10^{-5}$. With this ε/D and the calculated Reynolds number (Re = 45,000), the value of f is obtained from the Moody chart as $f = 0.0215$. Note that the Colebrook equation (**Eq. 8.35**) would give the same value of f. Hence, with the total length of the pipe as $\ell = (15 + 5 + 10 + 10 + 20)$ ft = 60 ft and the elevation and kinetic energy portions the same as for part (a), Eq. 1 gives

$$\begin{aligned} p_1 &= \gamma z_2 + \frac{1}{2}\rho(V_2^2 - V_1^2) + \rho f\frac{\ell}{D}\frac{V_1^2}{2} \\ &= (1248 + 299)\ \text{lb/ft}^2 \\ &\quad + (1.94\ \text{slugs/ft}^3)(0.0215)\left(\frac{60\ \text{ft}}{0.0625\ \text{ft}}\right)\frac{(8.70\ \text{ft/s})^2}{2} \\ &= (1248 + 299 + 1515)\ \text{lb/ft}^2 = 3062\ \text{lb/ft}^2 \end{aligned}$$

or

$$p_1 = 21.3\ \text{psi} \qquad \textbf{(Ans)}$$

Of this pressure drop, the amount due to pipe friction is approximately (21.3 − 10.7) psi = 10.6 psi.

(c) If major and minor losses are included, Eq. 1 becomes

$$p_1 = \gamma z_2 + \frac{1}{2}\rho(V_2^2 - V_1^2) + f\gamma\frac{\ell}{D}\frac{V_1^2}{2g} + \sum \rho K_L\frac{V^2}{2}$$

or

$$p_1 = 21.3\ \text{psi} + \sum \rho K_L\frac{V^2}{2} \qquad \textbf{(2)}$$

where the 21.3 psi contribution is due to elevation change, kinetic energy change, and major losses [part (b)], and the last term represents the sum of all of the minor losses. The loss coefficients of the components ($K_L = 1.5$ for each elbow and $K_L = 10$ for the wide-open globe valve) are given in **Table 8.2** (except for the loss coefficient of the faucet, which is given in Fig. E8.8*a* as $K_L = 2$). Thus,

$$\begin{aligned} \sum \rho K_L\frac{V^2}{2} &= (1.94\ \text{slugs/ft}^3)\frac{(8.70\ \text{ft})^2}{2}[10 + 4(1.5) + 2] \\ &= 1321\ \text{lb/ft}^2 \end{aligned}$$

or

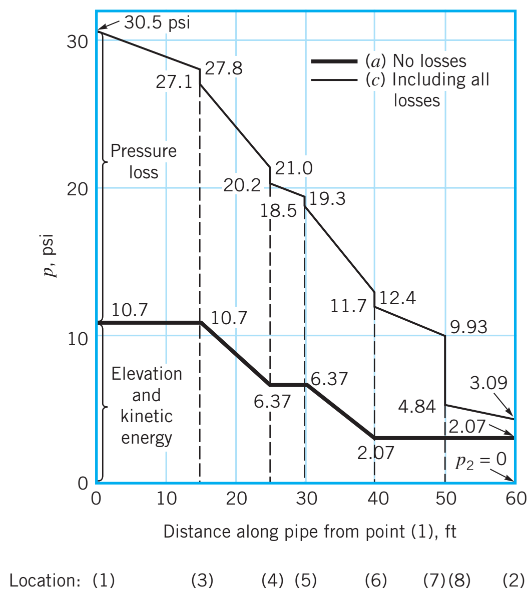
$$\sum \rho K_L\frac{V^2}{2} = 9.17\ \text{psi} \qquad \textbf{(3)}$$

Note that we did not include an entrance or exit loss because points (1) and (2) are located within the fluid streams, not within an attaching reservoir where the kinetic energy is zero. Thus, by combining Eqs. 2 and 3 we obtain the entire pressure drop as

$$p_1 = (21.3 + 9.17)\ \text{psi} = 30.5\ \text{psi} \qquad \textbf{(Ans)}$$

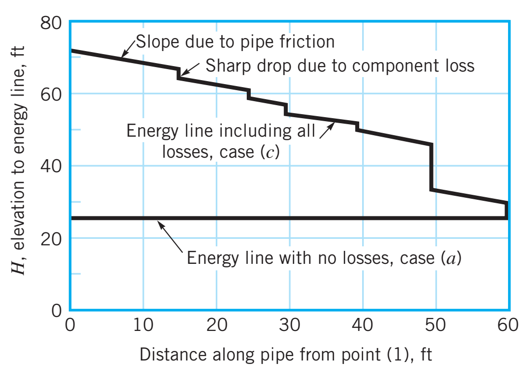
This pressure drop calculated by including all losses should be the most realistic answer of the three cases considered.

More detailed calculations will show that the pressure distribution along the pipe is as illustrated in Fig. E8.8*b* for cases (a) and (c)—neglecting all losses or including all losses. Note that not all of the pressure drop, $p_1 - p_2$, is a "pressure loss." The pressure change due to the elevation and velocity changes is completely reversible. The portion due to the major and minor losses is irreversible.



■ **FIGURE E8.8b**

This flow can be illustrated in terms of the energy line and hydraulic grade line concepts introduced in **Section 3.7**. As is shown in Fig. E8.8*c*, for case (a) there are no losses and the energy line (EL) is horizontal, one velocity head $(V^2/2g)$ above the hydraulic grade line (HGL), which is one pressure head (γz) above the pipe itself. For case (c) the energy line is not horizontal. Each bit of friction in the pipe or loss in a component reduces the available energy, thereby lowering the energy line. Thus, for case



■ **FIGURE E8.8c**

(a) the total head remains constant throughout the flow with a value of

$$H = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{(1547\ \text{lb/ft}^2)}{(62.4\ \text{lb/ft}^3)} + \frac{(8.70\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0$$

$$= 26.0\ \text{ft.}$$

$$= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{V_3^3}{2g} + z_3 = \ \ldots$$

For case (c) the energy line starts at

$$H_1 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{(30.5 \times 144)\text{lb/ft}^2}{(62.4\ \text{lb/ft}^3)} + \frac{(8.70\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 0 = 71.6\ \text{ft}$$

and falls to a final value of

$$H_2 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = 0 + \frac{(19.6\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 20\ \text{ft} = 26.0\ \text{ft}$$

The elevation of the energy line can be calculated at any point along the pipe. For example, at point (7), 50 ft from point (1),

$$H_7 = \frac{p_7}{\gamma} + \frac{V_7^2}{2g} + z_7 = \frac{(9.93 \times 144)\ \text{lb/ft}^2}{(62.4\ \text{lb/ft}^3)} + \frac{(8.70\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} + 20\ \text{ft} = 44.1\ \text{ft}$$

The head loss per foot of pipe is the same all along the pipe. That is,

$$\frac{h_L}{\ell} = f\frac{V^2}{2gD} = \frac{0.0215(8.70\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)(0.0625\ \text{ft})} = 0.404\ \text{ft/ft}$$

Thus, the energy line is a set of straight line segments of the same slope separated by steps whose height equals the head loss of the minor component at that location. As is seen from Fig. E8.8*c*, the globe valve produces the largest of all the minor losses.

Although the governing pipe flow equations are quite simple, they can provide very reasonable results for a variety of applications, as is shown in the next example.

EXAMPLE 8.9 (TYPE I, DETERMINE HEAD LOSS)

Crude oil at 140 °F with $\gamma = 53.7\ \text{lb/ft}^3$ and $\mu = 8 \times 10^{-5}\ \text{lb} \cdot \text{s/ft}^2$ (about four times the viscosity of water) is pumped across Alaska through the Alaskan pipeline, a 799-mile-long, 4-ft-diameter steel pipe, at a maximum rate of $Q = 2.4$ million barrels/day $= 117\ \text{ft}^3\text{/s}$, or $V = Q/A = 9.31$ ft/s. Determine the horsepower needed for the pumps that drive this large system.

SOLUTION

From the energy equation (Eq. 8.21) we obtain

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

where points (1) and (2) represent locations within the large holding tanks at either end of the line and h_p is the head provided to the oil by the pumps. We assume that $z_1 = z_2$ (pumped from sea level to sea level), $p_1 = p_2 = V_1 = V_2 = 0$ (large, open tanks) and $h_L = (f\ell/D)V^2/2g$). Minor losses are negligible because of the large length-to-diameter ratio of the relatively straight, uninterrupted pipe; $\ell/D = (799 \text{ mi})(5280 \text{ ft/mi})/(4 \text{ ft}) = 1.05 \times 10^6$. Thus,

$$h_p = h_L = f\frac{\ell}{D}\frac{V^2}{2g}$$

where from Fig. 8.20, $f = 0.0125$ since $\varepsilon/D = (0.00015 \text{ ft})/(4 \text{ ft}) = 0.0000375$ (see **Table 8.1**) and $\text{Re} = \rho VD/\mu = [(53.7/32.2) \text{ slugs/ft}^3](9.31 \text{ ft/s})(4.0 \text{ ft})/(8 \times 10^{-5} \text{ lb} \cdot \text{s/ft}^2) = 7.76 \times 10^5$. Thus,

$$h_p = 0.0125(1.05 \times 10^6)\frac{(9.31 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 17{,}700 \text{ ft}$$

and the actual power supplied to the fluid, $\mathcal{P}_a$, is

$$\begin{aligned}\mathcal{P}_a = \gamma Q h_p &= (53.7 \text{ lb/ft}^3)(117 \text{ ft}^3/\text{s})(17{,}700 \text{ ft}) \\ &= 1.11 \times 10^8 \text{ ft} \cdot \text{lb/s}\left(\frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb/s}}\right) \\ &= 202{,}000 \text{ hp}\end{aligned}$$

(Ans)

There are many reasons why it is not practical to drive this flow with a single pump of this size. First, there are no pumps this large! Second, the pressure at the pump outlet would need to be $p = \gamma h_L = (53.7 \text{ lb/ft}^3)(17{,}700 \text{ ft})(1 \text{ ft}^2/144 \text{ in.}^2) = 6600 \text{ psi}$. No practical 4-ft-diameter pipe would withstand this pressure. An equally unfeasible alternative would be to place the holding tank at the beginning of the pipe on top of a hill of height $h_L = 17{,}700$ ft and let gravity force the oil through the 799-mi pipe!

To produce the desired flow, the actual system contains 12 pumping stations positioned at strategic locations along the pipeline. Each station contains four pumps, three of which operate at any one time (the fourth is in reserve in case of emergency). Each pump is driven by a 13,500-hp motor, thereby producing a total horsepower of $\mathcal{P} = 12$ stations (3 pump/station) (13,500 hp/pump) = 486,000 hp. If we assume that the pump/motor combination is approximately 60% efficient, there is a total of 0.60 (486,000) hp = 292,000 hp available to drive the fluid. This number compares favorably with the 202,000-hp answer calculated above.

The assumption of a 140 °F oil temperature may not seem reasonable for flow across Alaska. Note, however, that the oil is warm when it is pumped from the ground and that the 202,000 hp needed to pump the oil is dissipated as a head loss (and therefore a temperature rise) along the pipe. However, if the oil temperature were 70 °F rather than 140 °F, the viscosity would be approximately $16 \times 10^{-5} \text{ lb} \cdot \text{s/ft}^2$ (twice as large), but the friction factor would only increase from $f = 0.0125$ at 140 °F ($\text{Re} = 7.76 \times 10^5$) to $f = 0.0140$ at 70 °F ($\text{Re} = 3.88 \times 10^5$). This doubling of viscosity would result in only an 11% increase in power (from 202,000 to 226,000 hp). Because of the large Reynolds numbers involved, the shear stress is due mostly to the turbulent nature of the flow. That is, the value of Re for this flow is large enough (on the relatively flat part of the Moody chart) so that f is nearly independent of Re (or viscosity).

Pipe flow problems in which it is desired to determine the flowrate for a given set of conditions (Type II problems) often require trial-and-error solution techniques. This is be-

Some pipe flow problems require a trial-and-error solution technique.

cause it is necessary to know the value of the friction factor to carry out the calculations, but the friction factor is a function of the unknown velocity (flowrate) in terms of the Reynolds number. The solution procedure is indicated in Example 8.10.

EXAMPLE 8.10 (TYPE II, DETERMINE FLOWRATE)

According to an appliance manufacturer, the 4-in.-diameter galvanized iron vent on a clothes dryer is not to contain more than 20 ft of pipe and four 90° elbows. Under these conditions determine the air flowrate if the pressure within the dryer is 0.20 inches of water. Assume a temperature of 100 °F and standard pressure.

SOLUTION

Application of the energy equation (Eq. 8.21) between the inside of the dryer, point (1), and the exit of the vent pipe, point (2), gives

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g} \quad \textbf{(1)}$$

where K_L for the entrance is assumed to be 0.5 and that for each elbow is assumed to be 1.5. In addition we assume that $V_1 = 0$ and $z_1 = z_2$. (The change in elevation is often negligible for gas flows.) Also, $p_2 = 0$, and $p_1/\gamma_{H_2O} = 0.2$ in., or

$$p_1 = (0.2 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)(62.4 \text{ lb/ft}^3) = 1.04 \text{ lb/ft}^2$$

Thus, with $\gamma = 0.0709$ lb/ft^3 (see **Table B.3**) and $V_2 = V$ (the air velocity in the pipe), Eq. 1 becomes

$$\frac{(1.04 \text{ lb/ft}^2)}{(0.0709 \text{ lb/ft}^3)} = \left[1 + f\frac{(20 \text{ ft})}{(\frac{4}{12} \text{ ft})} + 0.5 + 4(1.5)\right]\frac{V^2}{2(32.2 \text{ ft/s}^2)}$$

or

$$945 = (7.5 + 60f)V^2 \quad \textbf{(2)}$$

where V is in ft/s.

The value of f is dependent on Re, which is dependent on V, an unknown. However, from Table B.3, $\nu = 1.79 \times 10^{-4}$ ft^2/s and we obtain

$$\text{Re} = \frac{VD}{\nu} = \frac{(\frac{4}{12} \text{ ft})\, V}{1.79 \times 10^{-4} \text{ ft}^2/\text{s}}$$

or

$$\text{Re} = 1860\, V \quad \textbf{(3)}$$

where again V is in ft/s.

Also, since $\varepsilon/D = (0.0005 \text{ ft})/(4/12 \text{ ft}) = 0.0015$ (see Table 8.1 for the value of ε), we know which particular curve of the Moody chart is pertinent to this flow. Thus, we have three relationships (Eqs. 2, 3, and the $\varepsilon/D = 0.0015$ curve of Fig. 8.20) from which we can solve for the three unknowns f, Re, and V. This is done easily by an iterative scheme as follows.

It is usually simplest to assume a value of f, calculate V from Eq. 2, calculate Re from Eq. 3, and look up the appropriate value of f in the Moody chart for this value of Re. If the assumed f and the new f do not agree, the assumed answer is not correct—we do not have the solution to the three equations. Although values of either f, V, or Re could be assumed as starting values, it is usually simplest to assume a value of f because the correct value often lies on the relatively flat portion of the Moody chart for which f is quite insensitive to Re.

Thus, we assume $f = 0.022$, approximately the large Re limit for the given relative roughness. From Eq. 2 we obtain

$$V = \left[\frac{945}{7.5 + 60(0.022)}\right]^{1/2} = 10.4 \text{ ft/s}$$

and from Eq. 3

$$\text{Re} = 1860(10.4) = 19{,}300$$

With this Re and ε/D, Fig. 8.20 gives $f = 0.029$, which is not equal to the assumed solution $f = 0.022$ (although it is close!). We try again, this time with the newly obtained value of $f = 0.029$, which gives $V = 10.1$ ft/s and Re $= 18{,}800$. With these values, Fig. 8.20 gives $f = 0.029$, which agrees with the assumed value. Thus, the solution is $V = 10.1$ ft/s, or

$$Q = AV = \frac{\pi}{4}\left(\tfrac{4}{12}\text{ ft}\right)^2(10.1 \text{ ft/s}) = 0.881 \text{ ft}^3/\text{s} \qquad \textbf{(Ans)}$$

Note that the need for the iteration scheme is because one of the equations, $f = \phi(\text{Re}, \varepsilon/D)$, is in graphical form (the Moody chart). If the dependence of f on Re and ε/D is known in equation form, this graphical dependency is eliminated, and the solution technique may be easier. Such is the case if the flow is laminar so that the friction factor is simply $f = 64/\text{Re}$. For turbulent flow, we can use the Colebrook equation rather than the Moody chart, although this will normally require an iterative scheme also because of the complexity of the equation. As is shown below, such a formulation is ideally suited for an iterative computer solution.

We keep Eqs. 2 and 3 and use the Colebrook equation (Eq. 8.35, rather than the Moody chart) with $\varepsilon/D = 0.0015$ to give

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0 \log\left(4.05 \times 10^{-4} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad \textbf{(4)}$$

From Eq. 2 we have $V = [945/(7.5 + 60f)]^{1/2}$, which can be combined with Eq. 3 to give

$$\text{Re} = \frac{57{,}200}{\sqrt{7.5 + 60f}} \qquad \textbf{(5)}$$

The combination of Eqs. 4 and 5 provides a single equation for the determination of f

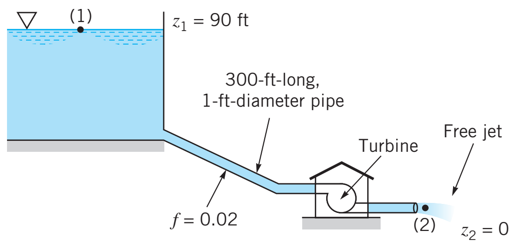
$$\frac{1}{\sqrt{f}} = -2.0 \log\left(4.05 \times 10^{-4} + 4.39 \times 10^{-5}\sqrt{60 + \frac{7.5}{f}}\right) \qquad \textbf{(6)}$$

A simple iterative solution of this equation gives $f = 0.029$, in agreement with the above solution which used the Moody chart. [This iterative solution using the Colebrook equation can be done as follows: (a) assume a value of f, (b) calculate a new value by using the assumed value in the right-hand side of Eq. 6, (c) use this new f to recalculate another value of f, and (d) repeat until the successive values agree.]

Note that unlike the Alaskan pipeline example (Example 8.9) in which we assumed minor losses are negligible, minor losses are of importance in this example because of the relatively small length-to-diameter ratio: $\ell/D = 20/(4/12) = 60$. The ratio of minor to major losses in this case is $K_L/(f\ell/D) = 6.5/[0.029\,(60)] = 3.74$. The elbows and entrance produce considerably more loss than the pipe itself.

EXAMPLE 8.11 (TYPE II, DETERMINE FLOWRATE)

The turbine shown in Fig. E8.11 extracts 50 hp from the water flowing through it. The 1-ft-diameter, 300-ft-long pipe is assumed to have a friction factor of 0.02. Minor losses are negligible. Determine the flowrate through the pipe and turbine.



■ FIGURE E8.11

SOLUTION

The energy equation (Eq. 8.21) can be applied between the surface of the lake (point (1)) and the outlet of the pipe as

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L + h_T \tag{1}$$

where $p_1 = V_1 = p_2 = z_2 = 0$, $z_1 = 90$ ft, and $V_2 = V$, the fluid velocity in the pipe. The head loss is given by

$$h_L = f\frac{\ell}{D}\frac{V^2}{2g} = 0.02\frac{(300\text{ ft})}{(1\text{ ft})}\frac{V^2}{2(32.2\text{ ft/s}^2)} = 0.0932V^2\text{ ft}$$

where V is in ft/s. Also, the turbine head is

$$h_T = \frac{\mathcal{P}_a}{\gamma Q} = \frac{\mathcal{P}_a}{\gamma(\pi/4)D^2V} = \frac{(50\text{ hp})[(550\text{ ft}\cdot\text{lb/s})/\text{hp}]}{(62.4\text{ lb/ft}^3)[(\pi/4)(1\text{ ft})^2V]} = \frac{561}{V}\text{ ft}$$

Thus, Eq. 1 can be written as

$$90 = \frac{V^2}{2(32.2)} + 0.0932V^2 + \frac{561}{V}$$

or

$$0.109V^3 - 90V + 561 = 0 \tag{2}$$

where V is in ft/s. The velocity of the water in the pipe is found as the solution of Eq. 2. Surprisingly, there are two real, positive roots: $V = 6.58$ ft/s or $V = 24.9$ ft/s. The third root is negative ($V = -31.4$ ft/s) and has no physical meaning for this flow. Thus, the two acceptable flowrates are

$$Q = \frac{\pi}{4}D^2V = \frac{\pi}{4}(1\text{ ft})^2(6.58\text{ ft/s}) = 5.17\text{ ft}^3\text{/s} \qquad \textbf{(Ans)}$$

or

$$Q = \frac{\pi}{4}(1\text{ ft})^2(24.9\text{ ft/s}) = 19.6\text{ ft}^3\text{/s} \qquad \textbf{(Ans)}$$

Either of these two flowrates gives the same power, $\mathcal{P}_a = \gamma Q h_T$. The reason for two possible solutions can be seen from the following. With the low flowrate ($Q = 5.17\ \text{ft}^3/\text{s}$), we obtain the head loss and turbine head as $h_L = 4.04$ ft and $h_T = 85.3$ ft. Because of the relatively low velocity there is a relatively small head loss and, therefore, a large head available for the turbine. With the large flowrate ($Q = 19.6\ \text{ft}^3/\text{s}$), we find $h_L = 57.8$ ft and $h_T = 22.5$ ft. The high-speed flow in the pipe produces a relatively large loss due to friction, leaving a relatively small head for the turbine. However, in either case the product of the turbine head times the flowrate is the same. That is, the power extracted ($\mathcal{P}_a = \gamma Q h_T$) is identical for each case. Although either flowrate will allow the extraction of 50 hp from the water, the details of the design of the turbine itself will depend strongly on which flowrate is to be used. Such information can be found in **Chapter 12** and various references about turbomachines (Refs. 14, 19, 20).

If the friction factor were not given, the solution to the problem would be much more lengthy. A trial-and-error solution similar to that in Example 8.10 would be required along with the solution of a cubic equation.

In pipe flow problems for which the diameter is the unknown (Type III), an iterative technique is required. This is, again, because the friction factor is a function of the diameter—through both the Reynolds number and the relative roughness. Thus, neither $\text{Re} = \rho V D/\mu = 4\rho Q/\pi \mu D$ nor ε/D are known unless D is known. Examples 8.12 and 8.13 illustrate this.

EXAMPLE 8.12 (TYPE III WITHOUT MINOR LOSSES, DETERMINE DIAMETER)

Air at standard temperature and pressure flows through a horizontal, galvanized iron pipe ($\varepsilon = 0.0005$ ft) at a rate of $2.0\ \text{ft}^3/\text{s}$. Determine the minimum pipe diameter if the pressure drop is to be no more than 0.50 psi per 100 ft of pipe.

SOLUTION

We assume the flow to be incompressible with $\rho = 0.00238\ \text{slugs/ft}^3$ and $\mu = 3.74 \times 10^{-7}\ \text{lb} \cdot \text{s/ft}^2$. Note that if the pipe were too long, the pressure drop from one end to the other, $p_1 - p_2$, would not be small relative to the pressure at the beginning, and compressible flow considerations would be required. For example, a pipe length of 200 ft gives $(p_1 - p_2)/p_1 = [(0.50\ \text{psi})/(100\ \text{ft})](200\ \text{ft})/14.7\ \text{psi} = 0.068 = 6.8\%$, which is probably small enough to justify the incompressible assumption.

With $z_1 = z_2$ and $V_1 = V_2$ the energy equation (Eq. 8.21) becomes

$$p_1 = p_2 + f\frac{\ell}{D}\frac{\rho V^2}{2} \tag{1}$$

where $V = Q/A = 4Q/(\pi D^2) = 4(2.0\ \text{ft}^3/\text{s})/\pi D^2$, or

$$V = \frac{2.55}{D^2}$$

where D is in feet. Thus, with $p_1 - p_2 = (0.5\ \text{lb/in.}^2)(144\ \text{in.}^2/\text{ft}^2)$ and $\ell = 100$ ft, Eq. 1

becomes

$$p_1 - p_2 = (0.5)(144)\ \text{lb/ft}^2$$
$$= f\frac{(100\ \text{ft})}{D}(0.00238\ \text{slugs/ft}^3)\frac{1}{2}\left(\frac{2.55}{D^2}\frac{\text{ft}}{\text{s}}\right)^2$$

or

$$D = 0.404 f^{1/5} \tag{2}$$

where D is in feet. Also $\text{Re} = \rho VD/\mu = (0.00238\ \text{slugs/ft}^3)\left[(2.55/D^2)\ \text{ft/s}\right]D/(3.74 \times 10^{-7}\ \text{lb}\cdot\text{s/ft}^2)$, or

$$\text{Re} = \frac{1.62 \times 10^4}{D} \tag{3}$$

and

$$\frac{\varepsilon}{D} = \frac{0.0005}{D} \tag{4}$$

Thus, we have four equations (Eqs. 2, 3, 4, and either the Moody chart or the Colebrook equation) and four unknowns (f, D, ε/D, and Re) from which the solution can be obtained by trial-and-error methods.

If we use the Moody chart, it is probably easiest to assume a value of f, use Eqs. 2, 3, and 4 to calculate D, Re, and ε/D, and then compare the assumed f with that from the Moody chart. If they do not agree, try again. Thus, we assume $f = 0.02$, a typical value, and obtain $D = 0.404(0.02)^{1/5} = 0.185$ ft, which gives $\varepsilon/D = 0.0005/0.185 = 0.0027$ and $\text{Re} = 1.62 \times 10^4/0.185 = 8.76 \times 10^4$. From the Moody chart we obtain $f = 0.027$ for these values of ε/D and Re. Since this is not the same as our assumed value of f, we try again. With $f = 0.027$, we obtain $D = 0.196$ ft, $\varepsilon/D = 0.0026$, and $\text{Re} = 8.27 \times 10^4$, which in turn give $f = 0.027$, in agreement with the assumed value. Thus, the diameter of the pipe should be

$$D = 0.196\ \text{ft} \qquad \textbf{(Ans)}$$

If we use the Colebrook equation (Eq. 8.35) with $\varepsilon/D = 0.0005/0.404 f^{1/5} = 0.00124/f^{1/5}$ and $\text{Re} = 1.62 \times 10^4/0.404 f^{1/5} = 4.01 \times 10^4/f^{1/5}$, we obtain

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

or

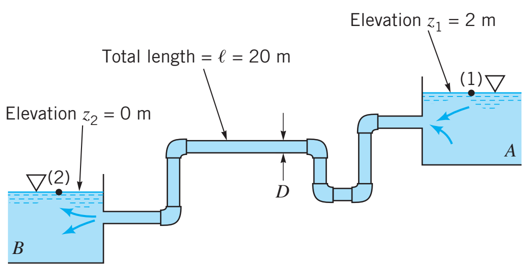
$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{3.35 \times 10^{-4}}{f^{1/5}} + \frac{6.26 \times 10^{-5}}{f^{3/10}}\right)$$

An iterative scheme (see solution of Eq. 6 in Example 8.10) to solve this equation for f gives $f = 0.027$, and hence $D = 0.196$ ft, in agreement with the Moody chart method.

In the previous example we only had to consider major losses. In some instances the inclusion of major and minor losses can cause a slightly more lengthy solution procedure, even though the governing equations are essentially the same. This is illustrated in Example 8.13.

EXAMPLE 8.13 (TYPE III WITH MINOR LOSSES, DETERMINE DIAMETER)

Water at 10 °C ($\nu = 1.307 \times 10^{-6}\ \text{m}^2/\text{s}$, see Table B.2) is to flow from reservoir A to reservoir B through a cast-iron pipe ($\varepsilon = 0.26$ mm) of length 20 m at a rate of $Q = 0.0020\ \text{m}^3/\text{s}$ as shown in Fig. E8.13. The system contains a sharp-edged entrance and six regular threaded 90° elbows. Determine the pipe diameter needed.



■ FIGURE E8.13

SOLUTION

The energy equation (Eq. 8.21) can be applied between two points on the surfaces of the reservoirs ($p_1 = p_2 = V_1 = V_2 = z_2 = 0$) as follows:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

or

$$z_1 = \frac{V^2}{2g}\left(f\frac{\ell}{D} + \sum K_L\right) \tag{1}$$

where $V = Q/A = 4Q/\pi D^2 = 4(2 \times 10^{-3}\ \text{m}^3/\text{s})/\pi D^2$, or

$$V = \frac{2.55 \times 10^{-3}}{D^2} \tag{2}$$

is the velocity within the pipe. (Note that the units on V and D are m/s and m, respectively.) The loss coefficients are obtained from Table 8.2 and Figs. 8.22 and 8.25 as $K_{L_{\text{ent}}} = 0.5$, $K_{L_{\text{elbow}}} = 1.5$, and $K_{L_{\text{exit}}} = 1$. Thus, Eq. 1 can be written as

$$2\ \text{m} = \frac{V^2}{2(9.81\ \text{m/s}^2)}\left\{\frac{20}{D}f + [6(1.5) + 0.5 + 1]\right\}$$

or, when combined with Eq. 2 to eliminate V,

$$6.03 \times 10^6 D^5 - 10.5D - 20f = 0 \tag{3}$$

To determine D we must know f, which is a function of Re and ε/D, where

$$\text{Re} = \frac{VD}{\nu} = \frac{[(2.55 \times 10^{-3})/D^2]D}{1.307 \times 10^{-6}} = \frac{1.95 \times 10^3}{D} \tag{4}$$

and

$$\frac{\varepsilon}{D} = \frac{2.6 \times 10^{-4}}{D} \tag{5}$$

where D is in meters. Again, we have four equations (Eqs. 3, 4, 5, and the Moody chart or the Colebrook equation) for the four unknowns D, f, Re, and ε/D.

Consider the solution by using the Moody chart. Although it is often easiest to assume a value of f and make calculations to determine if the assumed value is the correct one, with the inclusion of minor losses this may not be the simplest method. For example, if we assume $f = 0.02$ and calculate D from Eq. 3, we would have to solve a fifth-order equation. With only major losses (see **Example 8.12**), the term proportional to D in Eq. 3 is absent, and it is easy to solve for D if f is given. With both major and minor losses included (represented by the second and third terms in Eq. 3), this solution for D (given f) would require a trial-and-error or iterative technique.

Thus, for this type of problem it is perhaps easier to assume a value of D, calculate the corresponding f from Eq. 3, and with the values of Re and ε/D determined from Eqs. 4 and 5, look up the value of f in the Moody chart (or the Colebrook equation). The solution is obtained when the two values of f are in agreement. For example, assume $D = 0.05$ m, so that Eq. 3 gives $f = 0.0680$ and Eqs. 4 and 5 give $\text{Re} = 3.90 \times 10^4$ and $\varepsilon/D = 5.2 \times 10^{-3}$. With these values of Reynolds number and relative roughness, the Moody chart gives $f = 0.033$, which does not coincide with that obtained from Eq. 3 ($f = 0.0680$). Thus, $D \neq 0.05$ m.

A few more rounds of calculation will reveal that the solution is given by $D \approx 0.045$ m with $f = 0.032$.

$$D \approx 45 \text{ mm} \qquad \textbf{(Ans)}$$

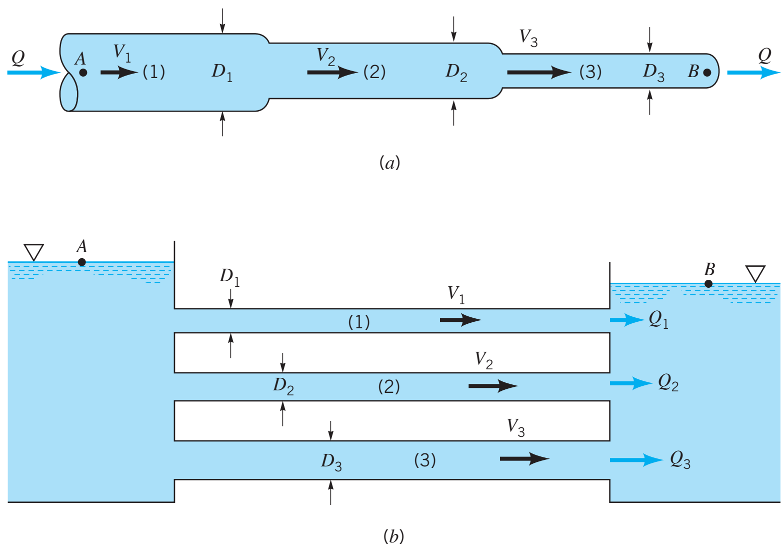
It is interesting to attempt to solve this example if all losses are neglected so that Eq. 1 becomes $z_1 = 0$. Clearly from Fig. E8.13, $z_1 = 2$ m. Obviously something is wrong. A fluid cannot flow from one elevation, beginning with zero pressure and velocity, and end up at a lower elevation with zero pressure and velocity unless energy is removed (i.e., a head loss or a turbine) somewhere between the two locations. If the pipe is short (negligible friction) and the minor losses are negligible, there is still the kinetic energy of the fluid as it leaves the pipe and enters the reservoir. After the fluid meanders around in the reservoir for some time, this kinetic energy is lost and the fluid is stationary. No matter how small the viscosity is, the exit loss cannot be neglected. The same result can be seen if the energy equation is written from the free surface of the upstream tank to the exit plane of the pipe, at which point the kinetic energy is still available to the fluid. In either case the energy equation becomes $z_1 = V^2/2g$ in agreement with the inviscid results of **Chapter 3** (the Bernoulli equation).

8.5.2 Multiple Pipe Systems

In many pipe systems there is more than one pipe involved. The complex system of tubes in our lungs (beginning with the relatively large-diameter trachea and ending in minute bronchi after numerous branchings) and the maze of pipes in a city's water distribution system are typical of such systems. The governing mechanisms for the flow in multiple pipe systems are the same as for the single pipe systems discussed in this chapter. However, because of the numerous unknowns involved, additional complexities may arise in solving for the flow in multiple pipe systems. Some of these complexities are discussed in this section.

An analogy between pipe systems and electrical circuits can be made.

The simplest multiple pipe systems can be classified into series or parallel flows, as are shown in Fig. 8.35. The nomenclature is similar to that used in electrical circuits. Indeed, an analogy between fluid and electrical circuits is often made as follows. In a simple electrical circuit, there is a balance between the voltage (e), current (i), and resistance (R) as given by Ohm's law: $e = iR$. In a fluid circuit there is a balance between the pressure drop (Δp), the flowrate or velocity (Q or V), and the flow resistance as given in terms of the friction factor and minor loss coefficients (f and K_L). For a simple flow $[\Delta p = f(\ell/D)(\rho V^2/2)]$, it follows that $\Delta p = Q^2\widetilde{R}$, where $\widetilde{R}$, a measure of the resistance to the flow, is proportional to f.



■ **FIGURE 8.35** **Series (*a*) and parallel (*b*) pipe systems.**

The main differences between the solution methods used to solve electrical circuit problems and those for fluid circuit problems lie in the fact that Ohm's law is a linear equation (doubling the voltage doubles the current), while the fluid equations are generally nonlinear (doubling the pressure drop does not double the flowrate unless the flow is laminar). Thus, although some of the standard electrical engineering methods can be carried over to help solve fluid mechanics problems, others cannot.

One of the simplest multiple pipe systems is that containing pipes in *series*, as is shown in Fig. 8.35*a*. Every fluid particle that passes through the system passes through each of the pipes. Thus, the flowrate (but not the velocity) is the same in each pipe, and the head loss from point A to point B is the sum of the head losses in each of the pipes. The governing equations can be written as follows:

$$Q_1 = Q_2 = Q_3$$

and

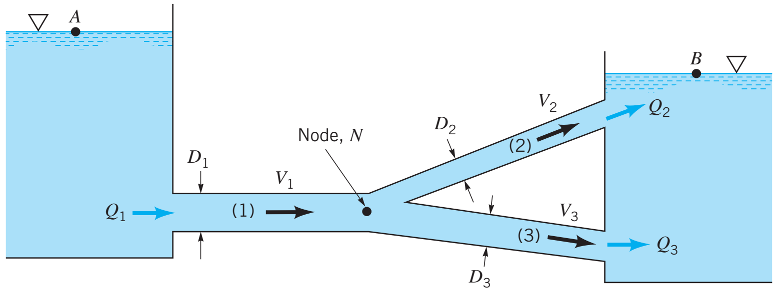
$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

Series and parallel pipe systems are often encountered.

where the subscripts refer to each of the pipes. In general, the friction factors will be different for each pipe because the Reynolds numbers ($\text{Re}_i = \rho V_i D_i / \mu$) and the relative roughnesses (ε_i / D_i) will be different. If the flowrate is given, it is a straightforward calculation to determine the head loss or pressure drop (Type I problem). If the pressure drop is given ad the flowerate is to be calculated (Type II problem), an iteration scheme is needed. In this situation none of the friction factors, f_i, are known, so the calculations may involve more trial-and-error attempts than for corresponding single pipe systems. The same is true for problems in which the pipe diameter (or diameters) is to be determined (Type III problems).

Another common multiple pipe system contains pipes in *parallel*, as is shown in Fig. 8.35*b*. In this system a fluid particle traveling from A to B may take any of the paths available, with the total flowrate equal to the sum of the flowrates in each pipe. However, by writing the energy equation between points A and B it is found that the head loss experienced by any fluid particle traveling between these locations is the same, independent of the path taken. Thus, the governing equations for parallel pipes are

$$Q = Q_1 + Q_2 + Q_3$$



■ FIGURE 8.36 **Multiple pipe loop system.**

and

$$h_{L_1} = h_{L_2} = h_{L_3}$$

Again, the method of solution of these equations depends on what information is given and what is to be calculated.

Another type of multiple pipe system called a *loop* is shown in Fig. 8.36. In this case the flowrate through pipe (1) equals the sum of the flowrates through pipes (2) and (3), or $Q_1 = Q_2 + Q_3$. As can be seen by writing the energy equation between the surfaces of each reservoir, the head loss for pipe (2) must equal that for pipe (3), even though the pipe sizes and flowrates may be different for each. That is,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_2}$$

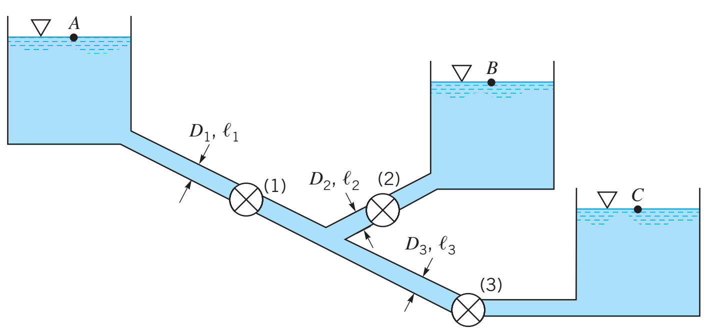
for a fluid particle traveling through pipes (1) and (2), while

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_3}$$

for fluid that travels through pipes (1) and (3). These can be combined to give $h_{L_2} = h_{L_3}$. This is a statement of the fact that fluid particles that travel through pipe (2) and particles that travel through pipe (3) all originate from common conditions at the junction (or node, N) of the pipes and all end up at the same final conditions.

The three-reservoir problem can be quite complex.

The flow in a relatively simple looking multiple pipe system may be more complex than it appears initially. The branching system termed the *three-reservoir problem* shown in Fig. 8.37 is such a system. Three reservoirs at known elevations are connected together with three pipes of known properties (lengths, diameters, and roughnesses). The problem is to determine the flowrates into or out of the reservoirs. If valve (1) were closed, the fluid would flow from reservoir B to C, and the flowrate could be easily calculated. Similar calculations could be carried out if valves (2) or (3) were closed with the others open.



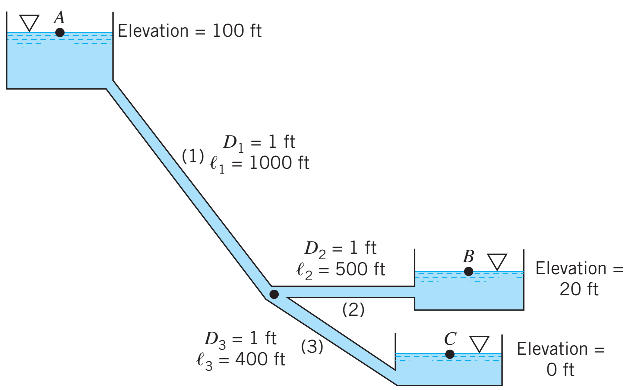
■ FIGURE 8.37 **A three-reservoir system.**

For some pipe systems, the direction of flow is not known a priori.

With all valves open, however, it is not necessarily obvious which direction the fluid flows. For the conditions indicated in Fig. 8.37, it is clear that fluid flows from reservoir *A* because the other two reservoir levels are lower. Whether the fluid flows into or out of reservoir *B* depends on the elevation of reservoirs *B* and *C* and the properties (length, diameter, roughness) of the three pipes. In general, the flow direction is not obvious, and the solution process must include the determination of this direction. This is illustrated in Example 8.14.

EXAMPLE 8.14

Three reservoirs are connected by three pipes as are shown in Fig. E8.14. For simplicity we assume that the diameter of each pipe is 1 ft, the friction factor for each is 0.02, and because of the large length-to-diameter ratio, minor losses are negligible. Determine the flowrate into or out of each reservoir.



■ FIGURE E8.14

SOLUTION

It is not obvious which direction the fluid flows in pipe (2). However, we assume that it flows out of reservoir *B*, write the governing equations for this case, and check our assumption. The continuity equation requires that $Q_1 + Q_2 = Q_3$, which, since the diameters are the same for each pipe, becomes simply

$$V_1 + V_2 = V_3 \tag{1}$$

The energy equation for the fluid that flows from *A* to *C* in pipes (1) and (3) can be written as

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

By using the fact that $p_A = p_C = V_A = V_C = z_C = 0$, this becomes

$$z_A = f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

For the given conditions of this problem we obtain

$$100 \text{ ft} = \frac{0.02}{2(32.2 \text{ ft/s}^2)} \frac{1}{(1 \text{ ft})} [(1000 \text{ ft})V_1^2 + (400 \text{ ft})V_3^2]$$

or

$$322 = V_1^2 + 0.4V_3^2 \tag{2}$$

where V_1 and V_3 are in ft/s. Similarly the energy equation for fluid flowing from B and C is

$$\frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_2 \frac{\ell_2}{D_2} \frac{V_2^2}{2g} + f_3 \frac{\ell_3}{D_3} \frac{V_3^2}{2g}$$

or

$$z_B = f_2 \frac{\ell_2}{D_2} \frac{V_2^2}{2g} + f_3 \frac{\ell_3}{D_3} \frac{V_3^2}{2g}$$

For the given conditions this can be written as

$$64.4 = 0.5V_2^2 + 0.4V_3^2 \quad \textbf{(3)}$$

Equations 1, 2, and 3 (in terms of the three unknowns V_1, V_2, and V_3) are the governing equations for this flow, provided the fluid flows from reservoir B. It turns out, however, that there is no solution for these equations with positive, real values of the velocities. Although these equations do not appear to be complicated, there is no simple way to solve them directly. Thus, a trial-and-error solution is suggested. This can be accomplished as follows. Assume a value of $V_1 > 0$, calculate V_3 from Eq. 2, and then V_2 from Eq. 3. It is found that the resulting V_1, V_2, V_3 trio does not satisfy Eq. 1 for any value of V_1 assumed. There is no solution to Eqs. 1, 2, and 3 with real, positive values of V_1, V_2, and V_3. Thus, our original assumption of flow out of reservoir B must be incorrect.

To obtain the solution, assume the fluid flows into reservoirs B and C and out of A. For this case the continuity equation becomes

$$Q_1 = Q_2 + Q_3$$

or

$$V_1 = V_2 + V_3 \quad \textbf{(4)}$$

Application of the energy equation between points A and B and A and C gives

$$z_A = z_B + f_1 \frac{\ell_1}{D_1} \frac{V_1^2}{2g} + f_2 \frac{\ell_2}{D_2} \frac{V_2^2}{2g}$$

and

$$z_A = z_C + f_1 \frac{\ell_1}{D_1} \frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3} \frac{V_3^2}{2g}$$

which, with the given data, become

$$258 = V_1^2 + 0.5\,V_2^2 \quad \textbf{(5)}$$

and

$$322 = V_1^2 + 0.4\,V_3^2 \quad \textbf{(6)}$$

Equations 4, 5, and 6 can be solved as follows. By substracting Eq. 5 from 6 we obtain

$$V_3 = \sqrt{160 + 1.25V_2^2}$$

Thus, Eq. 5 can be written as

$$258 = (V_2 + V_3)^2 + 0.5V_2^2 = (V_2 + \sqrt{160 + 1.25V_2^2})^2 + 0.5V_2^2$$

or

$$2V_2\sqrt{160 + 1.25V_2^2} = 98 - 2.75V_2^2 \quad \textbf{(7)}$$

which, upon squaring both sides, can be written as

$$V_2^4 - 460\,V_2^2 + 3748 = 0$$

By using the quadratic formula we can solve for V_2^2 to obtain either $V_2^2 = 452$ or $V_2^2 = 8.30$. Thus, either $V_2 = 21.3$ ft/s or $V_2 = 2.88$ ft/s. The value $V_2 = 21.3$ ft/s is not a root of the original equations. It is an extra root introduced by squaring Eq. 7, which with $V_2 = 21.3$ becomes "1140 = −1140." Thus, $V_2 = 2.88$ ft/s and from Eq. 5, $V_1 = 15.9$ ft/s. The corresponding flowrates are

$$Q_1 = A_1V_1 = \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} (1 \text{ ft})^2 (15.9 \text{ ft/s})$$

$$= 12.5 \text{ ft}^3\text{/s from } A \qquad \textbf{(Ans)}$$

$$Q_2 = A_2V_2 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4} (1 \text{ ft})^2 (2.88 \text{ ft/s}) \qquad \textbf{(Ans)}$$

$$= 2.26 \text{ ft}^3\text{/s into } B$$

and

$$Q_3 = Q_1 - Q_2 = (12.5 - 2.26) \text{ ft}^3\text{/s} = 10.2 \text{ ft}^3\text{/s into } C \qquad \textbf{(Ans)}$$

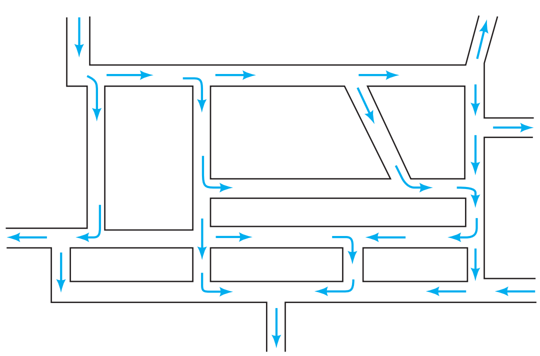
Note the slight differences in the governing equations depending on the direction of the flow in pipe (2)—compare Eqs. 1, 2, and 3 with Eqs. 4, 5, and 6.

If the friction factors were not given, a trial-and-error procedure similar to that needed for Type II problems (see **Section 8.5.1**) would be required.

The ultimate in multiple pipe systems is a *network* of pipes such as that shown in Fig. 8.38. Networks like these often occur in city water distribution systems and other systems that may have multiple "inlets" and "outlets." The direction of flow in the various pipes is by no means obvious—in fact, it may vary in time, depending on how the system is used from time to time.

Pipe network problems can be solved using node and loop concepts.

The solution for pipe network problems is often carried out by use of node and loop equations similar in many ways to that done in electrical circuits. For example, the continuity equation requires that for each *node* (the junction of two or more pipes) the net flowrate is zero. What flows into a node must flow out at the same rate. In addition, the net pressure difference completely around a *loop* (starting at one location in a pipe and returning to that location) must be zero. By combining these ideas with the usual head loss and pipe flow equations, the flow throughout the entire network can be obtained. Of course, trial-and-error solutions are usually required because the direction of flow and the friction factors may not be known. Such a solution procedure using matrix techniques is ideally suited for computer use (Refs. 21, 22).



■ **FIGURE 8.38 A general pipe network.**

8.6 Pipe Flowrate Measurement

It is often necessary to determine experimentally the flowrate in a pipe. In **Chapter 3** we introduced various types of flow-measuring devices (Venturi meter, nozzle meter, orifice meter, etc.) and discussed their operation under the assumption that viscous effects were not important. In this section we will indicate how to account for the ever-present viscous effects in these flow meters. We will also indicate other types of commonly used flow meters.

8.6.1 Pipe Flowrate Meters

Three of the most common devices used to measure the instantaneous flowrate in pipes are the orifice meter, the nozzle meter, and the Venturi meter. As was discussed in **Section 3.6.3**, each of these meters operates on the principle that a decrease in flow area in a pipe causes an increase in velocity that is accompanied by a decrease in pressure. Correlation of the pressure difference with the velocity provides a means of measuring the flowrate. In the absence of viscous effects and under the assumption of a horizontal pipe, application of the Bernoulli equation (Eq. 3.7) between points (1) and (2) shown in Fig. 8.39 gave

$$Q_{\text{ideal}} = A_2 V_2 = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}} \tag{8.37}$$

where $\beta = D_2/D_1$. Based on the results of the previous sections of this chapter, we anticipate that there is a head loss between (1) and (2) so that the governing equations become

$$Q = A_1 V_1 = A_2 V_2$$

and

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

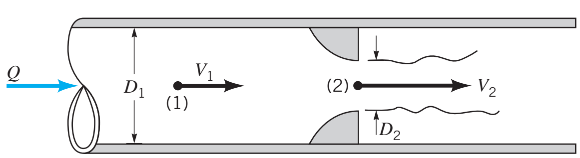
The ideal situation has $h_L = 0$ and results in Eq. 8.37. The difficulty in including the head loss is that there is no accurate expression for it. The net result is that empirical coefficients are used in the flowrate equations to account for the complex real world effects brought on by the nonzero viscosity. The coefficients are discussed below.

An orifice discharge coefficient is used to account for nonideal effects.

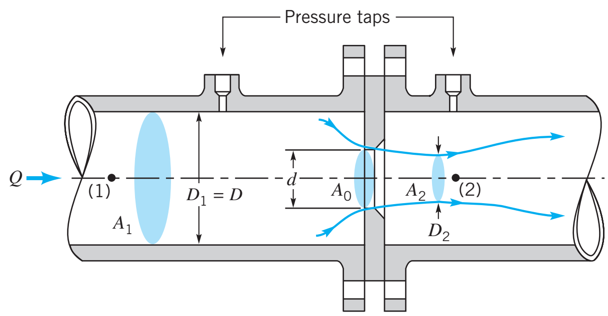
A typical *orifice meter* is constructed by inserting between two flanges of a pipe a flat plate with a hole, as shown in Fig. 8.40. The pressure at point (2) within the vena contracta is less than that at point (1). Nonideal effects occur for two reasons. First, the vena contracta area, A_2, is less than the area of the hole, A_o, by an unknown amount. Thus, $A_2 = C_c A_o$, where C_c is the contraction coefficient ($C_c < 1$). Second, the swirling flow and turbulent motion near the orifice plate introduce a head loss that cannot be calculated theoretically. Thus, an *orifice discharge coefficient*, C_o, is used to take these effects into account. That is,

$$Q = C_o Q_{\text{ideal}} = C_o A_o \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}} \tag{8.38}$$

where $A_o = \pi d^2/4$ is the area of the hole in the orifice plate. The value of C_o is a function of $\beta = d/D$ and the Reynolds number $\text{Re} = \rho VD/\mu$, where $V = Q/A_1$. Typical values of C_o



■ **FIGURE 8.39 Typical pipe flow meter geometry.**



■ **FIGURE 8.40 Typical orifice meter construction.**

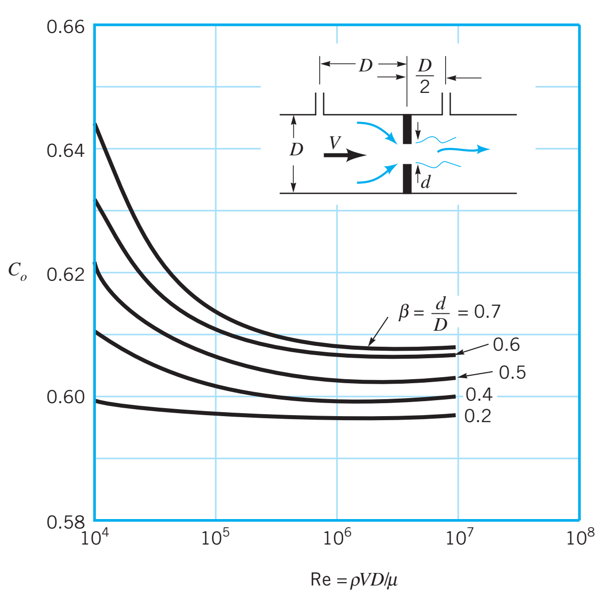
are given in Fig. 8.41. Note that the value of C_o depends on the specific construction of the orifice meter (i.e., the placement of the pressure taps, whether the orifice plate edge is square or beveled, etc.). Very precise conditions governing the construction of standard orifice meters have been established to provide the greatest accuracy possible (Refs. 23, 24).

The nozzle meter is more efficient than the orifice meter.

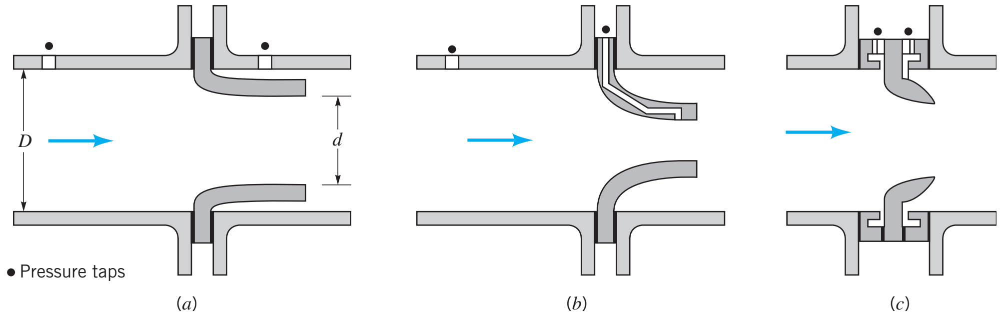
Another type of pipe flow meter that is based on the same principles used in the orifice meter is the *nozzle meter*, three variations of which are shown in Fig. 8.42. This device uses a contoured nozzle (typically placed between flanges of pipe sections) rather than a simple (and less expensive) plate with a hole as in an orifice meter. The resulting flow pattern for the nozzle meter is closer to ideal than the orifice meter flow. There is only a slight vena contracta and the secondary flow separation is less severe, but there still are viscous effects. These are accounted for by use of the *nozzle discharge coefficient*, C_n, where

$$Q = C_n Q_{\text{ideal}} = C_n A_n \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}} \qquad \textbf{(8.39)}$$

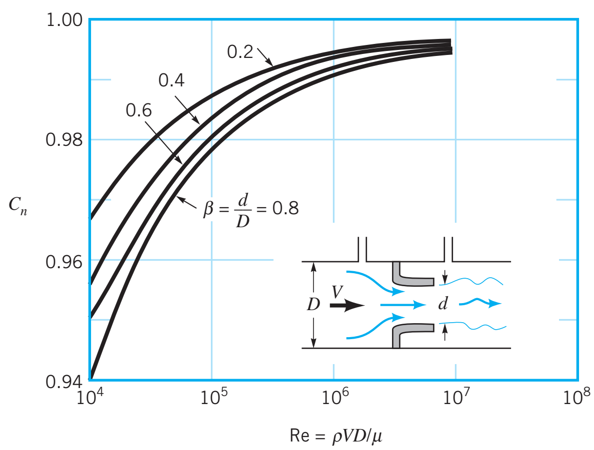
with $A_n = \pi d^2/4$. As with the orifice meter, the value of C_n is a function of the diameter ratio, $\beta = d/D$, and the Reynolds number, $\text{Re} = \rho VD/\mu$. Typical values obtained from experiments are shown in Fig. 8.43. Again, precise values of C_n depend on the specific details of the nozzle design. Accepted standards have been adopted (Ref. 24). Note that $C_n > C_o$; the nozzle meter is more efficient (less energy dissipated) than the orifice meter.



■ **FIGURE 8.41 Orifice meter discharge coefficient (Ref. 24).**



■ **FIGURE 8.42** **Typical nozzle meter construction.**



■ **FIGURE 8.43** **Nozzle meter discharge coefficient (Ref. 24).**

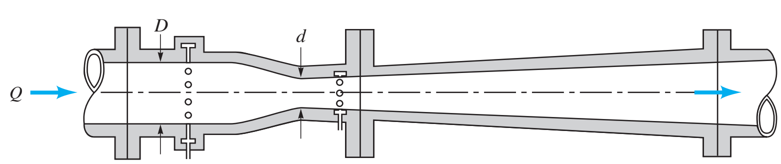
The most precise and most expensive of the three obstruction-type flow meters is the *Venturi meter* shown in Fig. 8.44 (**G. B. Venturi** (1746–1822)). Although the operating principle for this device is the same as for the orifice or nozzle meters, the geometry of the Venturi meter is designed to reduce head losses to a minimum. This is accomplished by providing a relatively streamlined contraction (which eliminates separation ahead of the throat) and a very gradual expansion downstream of the throat (which eliminates separation in this decelerating portion of the device). Most of the head loss that occurs in a well-designed Venturi meter is due to friction losses along the walls rather than losses associated with separated flows and the inefficient mixing motion that accompanies such flow.

Thus, the flowrate through a Venturi meter is given by

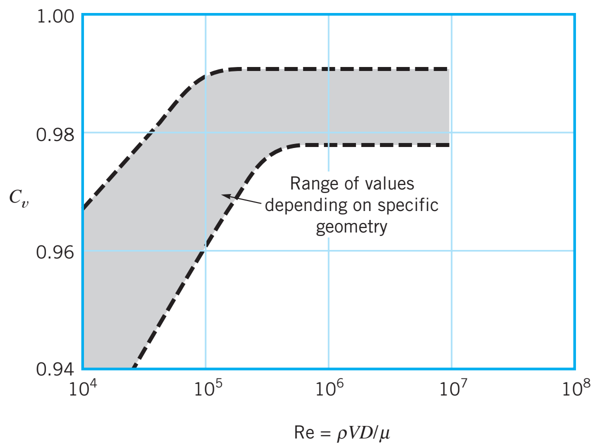
$$Q = C_v Q_{\text{ideal}} = C_v A_T \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}}$$

The Venturi discharge coefficient is a function of the specific geometry of the meter.

where $A_T = \pi d^2/4$ is the throat area. The range of values of C_v, the *Venturi discharge coefficient*, is given in Fig. 8.45. The throat-to-pipe diameter ratio ($\beta = d/D$), the Reynolds



■ **FIGURE 8.44** **Typical Venturi meter construction.**



■ **FIGURE 8.45 Venturi meter discharge coefficient (Ref. 23).**

Precise standards exist for the design of accurate flow meters.

number, and the shape of the converging and diverging sections of the meter are among the parameters that affect the value of C_v.

Again, the precise values of C_n, C_o, and C_v depend on the specific geometry of the devices used. Considerable information concerning the design, use, and installation of standard flow meters can be found in various books (Refs. 23, 24, 25, 26, 31).

EXAMPLE 8.15

Ethyl alcohol flows through a pipe of diameter $D = 60$ mm in a refinery. The pressure drop across the nozzle meter used to measure the flowrate is to be $\Delta p = 4.0$ kPa when the flowrate is $Q = 0.003 \text{ m}^3/\text{s}$. Determine the diameter, d, of the nozzle.

SOLUTION

From Table 1.6 the properties of ethyl alcohol are $\rho = 789 \text{ kg/m}^3$ and $\mu = 1.19 \times 10^{-3} \text{ N} \cdot \text{s/m}^2$. Thus,

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi D\mu} = \frac{4(789 \text{ kg/m}^3)(0.003 \text{ m}^3/\text{s})}{\pi(0.06 \text{ m})(1.19 \times 10^{-3} \text{ N} \cdot \text{s/m}^2)} = 42{,}200$$

From Eq. 8.39 the flowrate through the nozzle is

$$Q = 0.003 \text{ m}^3/\text{s} = C_n \frac{\pi}{4} d^2 \sqrt{\frac{2(4 \times 10^3 \text{ N/m}^2)}{789 \text{ kg/m}^3(1 - \beta^4)}}$$

or

$$1.20 \times 10^{-3} = \frac{C_n d^2}{\sqrt{1 - \beta^4}} \tag{1}$$

where d is in meters. Note that $\beta = d/D = d/0.06$. Equation 1 and Fig. 8.43 represent two equations for the two unknowns d and C_n that must be solved by trial and error.

As a first approximation we assume that the flow is ideal, or $C_n = 1.0$, so that Eq. 1 becomes

$$d = (1.20 \times 10^{-3} \sqrt{1 - \beta^4})^{1/2} \tag{2}$$

In addition, for many cases $1 - \beta^4 \approx 1$, so that an approximate value of d can be obtained from Eq. 2 as

$$d = (1.20 \times 10^{-3})^{1/2} = 0.0346 \text{ m}$$

Hence, with an initial guess of $d = 0.0346$ m or $\beta = d/D = 0.0346/0.06 = 0.577$, we obtain from Fig. 8.43 (using Re = 42,200) a value of $C_n = 0.972$. Clearly this does not agree with our initial assumption of $C_n = 1.0$. Thus, we do not have the solution to Eq. 1 and Fig. 8.43. Next we assume $\beta = 0.577$ and $C_n = 0.972$ and solve for d from Eq. 1 to obtain

$$d = \left(\frac{1.20 \times 10^{-3}}{0.972} \sqrt{1 - 0.577^4}\right)^{1/2}$$

or $d = 0.0341$ m. With the new value of $\beta = 0.0341/0.060 = 0.568$ and Re = 42,200, we obtain (from Fig. 8.43) $C_n \approx 0.972$ in agreement with the assumed value. Thus,

$$d = 34.1 \text{ mm} \qquad \textbf{(Ans)}$$

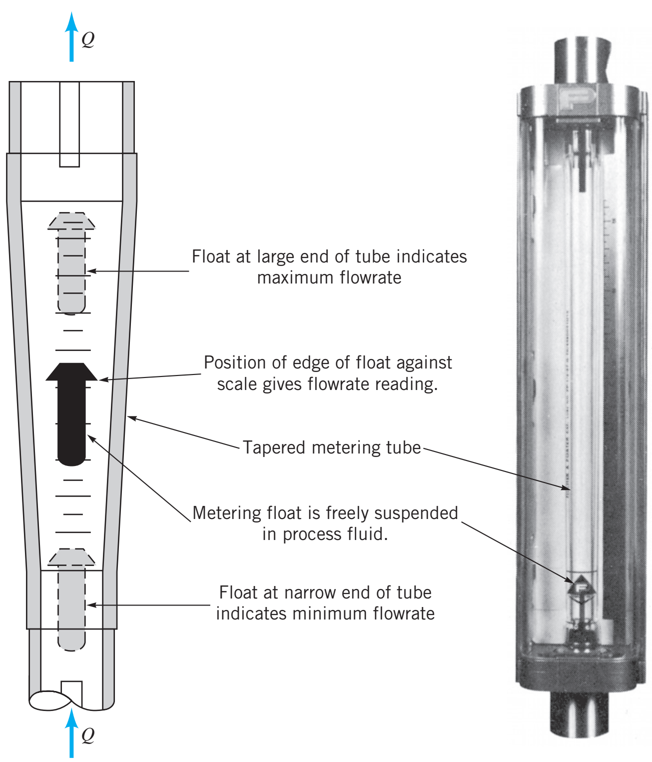
If numerous cases are to be investigated, it may be much easier to replace the discharge coefficient data of Fig. 8.43 by the equivalent equation, $C_n = \phi(\beta, \text{Re})$, and use a computer to iterate for the answer. Such equations are available in the literature (Ref. 24). This would be similar to using the Colebrook equation rather than the Moody chart for pipe friction problems.

There are many types of flow meters.

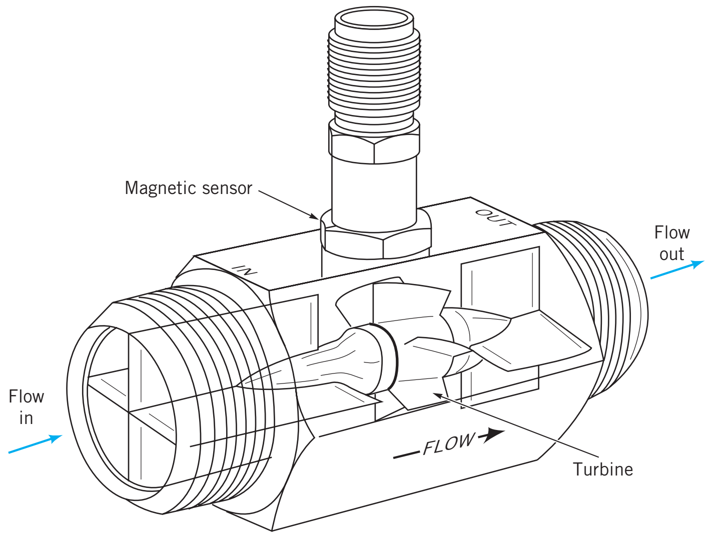
Numerous other devices are used to measure the flowrate in pipes. Many of these devices use principles other than the high-speed/low-pressure concept of the orifice, nozzle, and Venturi meters.

A quite common, accurate, and relatively inexpensive flow meter is the *rotameter*, or variable area meter as is shown in Fig. 8.46. In this device a float is contained within a tapered, transparent metering tube that is attached vertically to the pipeline. As fluid flows

V8.6 Rotameter



■ **FIGURE 8.46**
Rotameter-type flow meter. (Courtesy of Fischer & Porter Co.).



■ **FIGURE 8.47**
Turbine-type flow meter. (Courtesy of E G & G Flow Technology, Inc.)

through the meter (entering at the bottom), the float will rise within the tapered tube and reach an equilibrium height that is a function of the flowrate. This height corresponds to an equilibrium condition for which the net force on the float (buoyancy, float weight, fluid drag) is zero. A calibration scale in the tube provides the relationship between the float position and the flowrate.

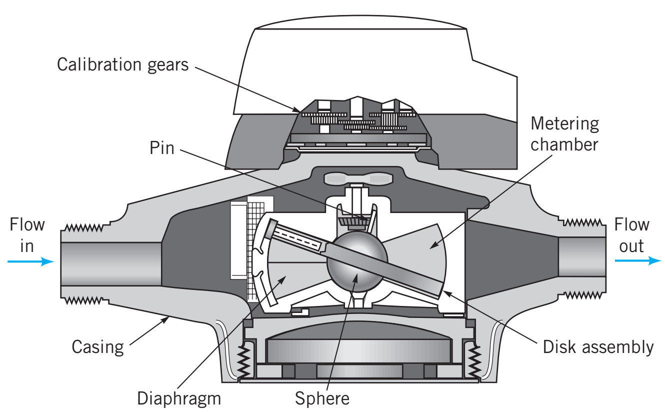
Another useful pipe flowrate meter is a *turbine meter* as is shown in Fig. 8.47. A small, freely rotating propeller or turbine within the turbine meter rotates with an angular velocity that is a function of (nearly proportional to) the average fluid velocity in the pipe. This angular velocity is picked up magnetically and calibrated to provide a very accurate measure of the flowrate through the meter.

8.6.2 Volume Flow Meters

Volume flow meters measure volume rather than volume flowrate.

In many instances it is necessary to know the amount (volume or mass) of fluid that has passed through a pipe during a given time period, rather than the instantaneous flowrate. For example, we are interested in how many gallons of gasoline are pumped into the tank in our car rather than the rate at which it flows into the tank. There are numerous quantity-measuring devices that provide such information.

V8.7 Water meter

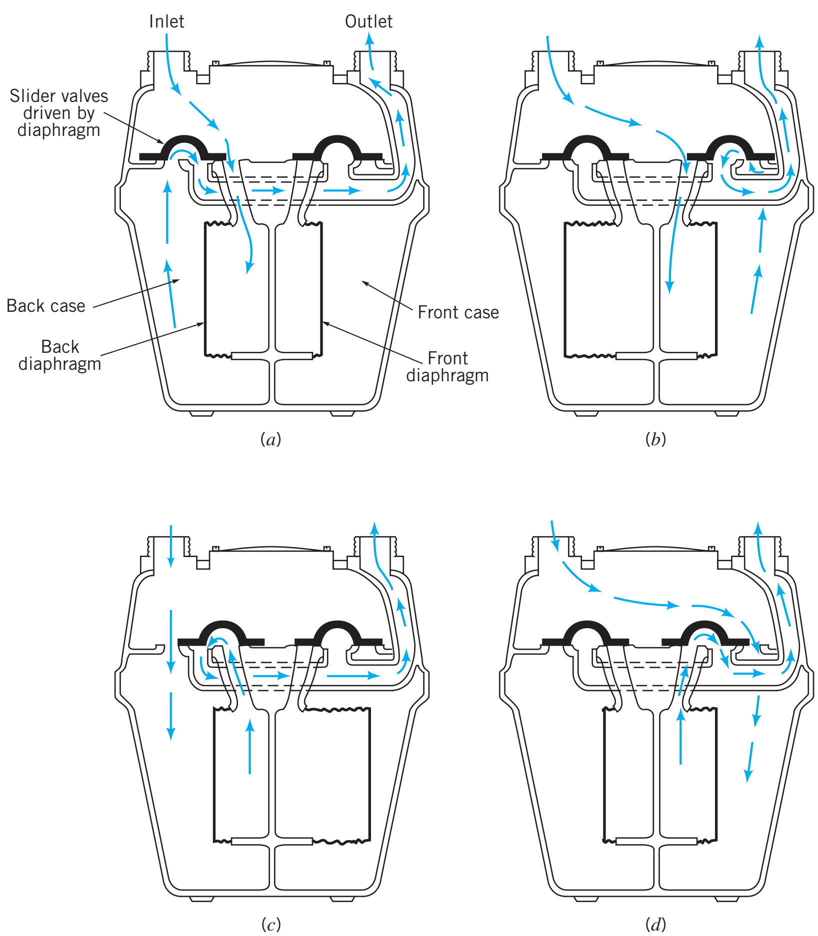


■ **FIGURE 8.48**
Nutating disk flow meter. (Courtesy of Badger Meter, Inc.)

The *nutating disk meter* shown in Fig. 8.48 is widely used to measure the net amount of water used in domestic and commercial water systems as well as the amount of gasoline delivered to your gas tank. This meter contains only one essential moving part and is relatively inexpensive and accurate. Its operating principle is very simple, but it may be difficult to understand its operation without actually inspecting the device firsthand. The device consists of a metering chamber with spherical sides and conical top and bottom. A disk passes through a central sphere and divides the chamber into two portions. The disk is constrained to be at an angle not normal to the axis of symmetry of the chamber. A radial plate (diaphragm) divides the chamber so that the entering fluid causes the disk to wobble (nutate), with fluid flowing alternately above or below the disk. The fluid exits the chamber after the disk has completed one wobble, which corresponds to a specific volume of fluid passing through the chamber. During each wobble of the disk, the pin attached to the tip of the center sphere, normal to the disk, completes one circle. The volume of fluid that has passed through the meter can be obtained by counting the number of revolutions completed.

The nutating disk meter is very simple in design, using one moving part.

Another quantity-measuring device that is used for gas flow measurements is the *bellows meter* as shown in Fig. 8.49. It contains a set of bellows that alternately fill and empty as a result of the pressure of the gas and the motion of a set of inlet and outlet valves. The



■ **FIGURE 8.49** **Bellows-type flow meter. (Courtesy of BTR—Rockwell Gas Products). (*a*) Back case emptying, back diaphragm filling. (*b*) Front diaphragm filling, front case emptying. (*c*) Back case filling, back diaphragm emptying. (*d*) Front diaphragm emptying, front case filling.**

The bellows meter contains a complex set of moving parts.

common household natural gas meter is of this type. For each cycle [(*a*) through (*d*)] a known volume of gas passes through the meter.

The nutating disk meter (water meter) is an example of extreme simplicity—one cleverly designed moving part. The bellows meter (gas meter), on the other hand, is relatively complex—it contains many moving, interconnected parts. This difference is dictated by the application involved. One measures a common, safe-to-handle, relatively high-pressure liquid, whereas the other measures a relatively dangerous, low-pressure gas. Each device does its intended job very well.

There are numerous devices used to measure fluid flow, only a few of which have been discussed here. The reader is encouraged to review the literature to gain familiarity with other useful, clever devices (Refs. 25, 26).

Key Words and Topics

In the E-book, click on any key word or topic to go to that subject.

Colebrook formula
Entrance region
Equivalent length
Friction factor
Fully developed flow
Hagen–Poiseuille flow
Laminar flow
Loss coefficient
Major losses
Minor losses
Moody chart
Multiple pipe systems
Noncircular conduits
Nozzle meter
Orifice meter
Poiseuille's law
Relative roughness
Reynolds number
Transitional flow
Turbulent flow
Turbulent shear stress
Venturi meter
Volume flow meters

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Review Problems

In the E-book, ***click here*** to go to a set of review problems complete with answers and detailed solutions.

Problems

Note: Unless otherwise indicated use the values of fluid properties found in the tables on the inside of the front cover. Problems designated with an (*) are intended to be solved with the aid of a programmable calculator or a computer. Problems designated with a (†) are "open-ended" problems and require critical thinking in that to work them one must make various assumptions and provide the necessary data. There is not a unique answer to these problems.

In the E-book, answers to the even-numbered problems can be obtained by clicking on the problem number. In the E-book, access to the videos that accompany problems can be obtained by clicking on the "video" segment (i.e., **Video 8.3**) of the problem statement. The lab-type problems can be accessed by clicking on the "*click here*" segment of the problem statement.

8.1 Rainwater runoff from a parking lot flows through a 3-ft-diameter pipe, completely filling it. Whether flow in a pipe is laminar or turbulent depends on the value of the Reynolds number. (See **Video V8.1**.) Would you expect the flow to be laminar or turbulent? Support your answer with appropriate calculations.

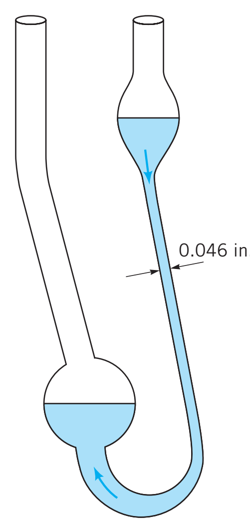
† **8.2** Under normal circumstances is the air flow through your trachea (your windpipe) laminar or turbulent? List all assumptions and show all calculations.

8.3 The flow of water in a 3-mm-diameter pipe is to remain laminar. Plot a graph of the maximum flowrate allowed as a function of temperature for $0 < T < 100$ °C.

8.4 Air at 100 °F flows at standard atmospheric pressure in a pipe at a rate of 0.08 lb/s. Determine the maximum diameter allowed if the flow is to be turbulent.

8.5 Carbon dioxide at 20 °C and a pressure of 550 kPa (abs) flows in a pipe at a rate of 0.04 N/s. Determine the maximum diameter allowed if the flow is to be turbulent.

8.6 It takes 20 seconds for a 0.5 cubic inch of water to flow through the 0.046-in. diameter tube of the capillary tube viscometer shown in **Video V1.3** and Fig. P8.6. Is the flow in the tube laminar or turbulent? Explain.



■ **FIGURE P8.6**

8.7 To cool a given room it is necessary to supply 5 ft^3/s of air through an 8-in.-diameter pipe. Approximately how long is the entrance length in this pipe?

8.8 The wall shear stress in a fully developed flow portion of a 1.2-in.-diameter pipe carrying water is 1.85 lb/ft^2. Determine the pressure gradient, $\partial p/\partial x$, where x is in the flow direction, if the pipe is **(a)** horizontal, **(b)** vertical with flow up, or **(c)** vertical with flow down.

8.9 The pressure drop needed to force water through a horizontal 1-in.-diameter pipe is 0.60 psi for every 12-ft length of pipe. Determine the shear stress on the pipe wall. Determine the shear stress at distances 0.3 and 0.5 in. away from the pipe wall.

8.10 Repeat Problem 8.9 if the pipe is on a 20° hill. Is the flow up or down the hill? Explain.

8.11 Water flows in a constant diameter pipe with the following conditions measured: At section (a) $p_a = 32.4$ psi and $z_a = 56.8$ ft; at section (b) $p_b = 29.7$ psi and $z_b = 68.2$ ft. Is the flow from (a) to (b) or from (b) to (a)? Explain.

8.12 Water flows downhill through a 3-in.-diameter steel pipe. The slope of the hill is such that for each mile (5280 ft) of horizontal distance, the change in elevation is Δz ft. Determine the maximum value of Δz if the flow is to remain laminar and the pressure all along the pipe is constant.

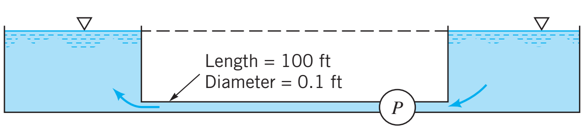
8.13 Some fluids behave as a non-Newtonian power-law fluid characterized by $\tau = -C(du/dr)^n$, where $n = 1, 3, 5$, and so on, and C is a constant. (If $n = 1$, the fluid is the customary Newtonian fluid.) For flow in a round pipe of a diameter D, integrate the force balance equation (Eq. 8.3) to obtain the velocity profile

$$u(r) = \frac{-n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{1/n}\left[r^{(n+1)/n} - \left(\frac{D}{2}\right)^{(n+1)/n}\right]$$

***8.14** For the flow discussed in Problem 8.13, plot the dimensionless velocity profile u/V_c, where V_c is the centerline velocity (at $r = 0$), as a function of the dimensionless radial coordinate $r/(D/2)$, where D is the pipe diameter. Consider values of $n = 1, 3, 5$, and 7.

8.15 A fluid of density $\rho = 1000 \text{ kg/m}^3$ and viscosity $\mu = 0.30 \text{ N} \cdot \text{s/m}^2$ flows steadily down a vertical 0.10-m-diameter pipe and exits as a free jet from the lower end. Determine the maximum pressure allowed in the pipe at a location 10 m above the pipe exit if the flow is to be laminar.

8.16 Water is pumped steadily from one large, open tank to another at the same elevation as shown in Fig. P8.16. Determine the maximum power the pump can add to the water if the flow is to remain laminar.



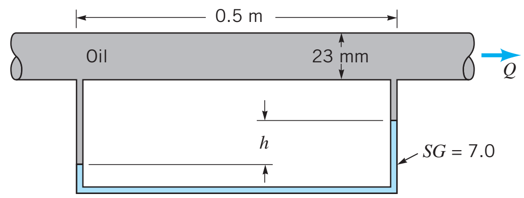
■ **FIGURE P8.16**

8.17 Glycerin at 20 °C flows upward in a vertical 75-mm-diameter pipe with a centerline velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.

8.18 A fluid flows through a horizontal 0.1-in.-diameter pipe. When the Reynolds number is 1500, the head loss over a 20-ft length of the pipe is 6.4 ft. Determine the fluid velocity.

8.19 A viscous fluid flows in a 0.10-m-diameter pipe such that its velocity measured 0.012 m away from the pipe wall is 0.8 m/s. If the flow is laminar, determine the centerline velocity and the flowrate.

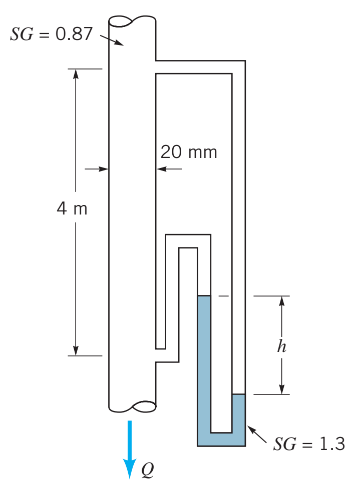
8.20 Oil (specific weight $= 8900 \text{ N/m}^3$, viscosity $= 0.10 \text{ N} \cdot \text{s/m}^2$) flows through a horizontal 23-mm-diameter tube as shown in Fig. P8.20. A differential U-tube manometer is used to measure the pressure drop along the tube. Determine the range of values for h for laminar flow.



■ **FIGURE P8.20**

8.21 A fluid flows in a smooth pipe with a Reynolds number of 6000. By what percent would the head loss be reduced if the flow could be maintained as laminar flow rather than the expected turbulent flow?

8.22 Oil of $SG = 0.87$ and a kinematic viscosity $\nu = 2.2 \times 10^{-4} \text{ m}^2/\text{s}$ flows through the vertical pipe shown in Fig. P8.22 at a rate of $4 \times 10^{-4} \text{ m}^3/\text{s}$. Determine the manometer reading, h.



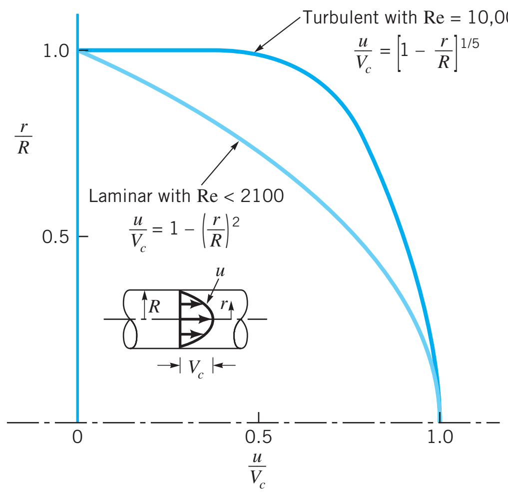
■ **FIGURE P8.22**

8.23 Determine the manometer reading, h, for Problem 8.22 if the flow is up rather than down the pipe. Note: The manometer reading will be reversed.

8.24 For Problem 8.22, what flowrate (magnitude and direction) will cause $h = 0$?

8.25 The kinetic energy coefficient, α, is defined in Eq. 5.86. Show that its value for a power-law turbulent velocity profile (Eq. 8.31) is given by $\alpha = (n + 1)^3(2n + 1)^3/[4n^4(n + 3)(2n + 3)]$.

8.26 As shown in **Video V8.3** and Fig. P8.26, the velocity profile for laminar flow in a pipe is quite different from that for turbulent flow. With laminar flow the velocity profile is parabolic; with turbulent flow at Re = 10,000 the velocity profile can be approximated by the power-law profile shown in the figure. **(a)** For laminar flow, determine at what radial location you would place a Pitot tube if it is to measure the average velocity in the pipe. **(b)** Repeat part **(a)** for turbulent flow with Re = 10,000.



■ **FIGURE P8.26**

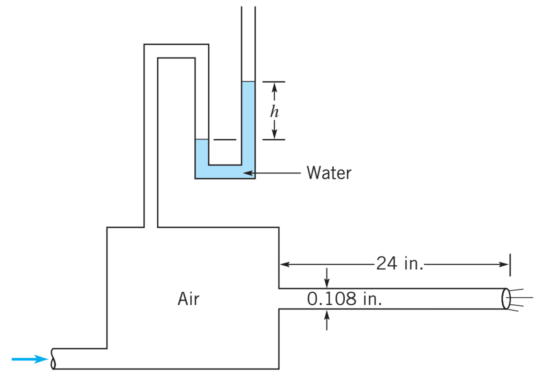
8.27 Water at 80 °F flows in a 6-in.-diameter pipe with a flowrate of 2.0 cfs. What is the approximate velocity at a distance 2.0 in. away from the wall? Determine the centerline velocity.

8.28 During a heavy rainstorm, water from a parking lot completely fills an 18-in.-diameter, smooth, concrete storm sewer. If the flowrate is 10 ft^3/s, determine the pressure drop in a 100-ft horizontal section of the pipe. Repeat the problem if there is a 2-ft change in elevation of the pipe per 100 ft of its length.

8.29 Carbon dioxide at a temperature of 0 °C and a pressure of 600 kPa (abs) flows through a horizontal 40-mm-diameter pipe with an average velocity of 2 m/s. Determine the friction factor if the pressure drop is 235 N/m^2 per 10-m length of pipe.

8.30 Water flows through a 6-in.-diameter horizontal pipe at a rate of 2.0 cfs and a pressure drop of 4.2 psi per 100 ft of pipe. Determine the friction factor.

8.31 Air flows through the 0.108-in.-diameter, 24-in.-long tube shown in Fig. P8.31. Determine the friction factor if the flowrate is $Q = 0.00191$ cfs when $h = 1.70$ in. Compare your results with the expression $f = 64/\text{Re}$. Is the flow laminar or turbulent?



■ **FIGURE P8.31**

8.32 When soup is stirred in a bowl, there is considerable turbulence in the resulting motion (see **Video V8.2**). From a very simplistic standpoint, this turbulence consists of numerous intertwined swirls, each involving a characteristic diameter and velocity. As time goes by, the smaller swirls (the fine scale structure) die out relatively quickly, leaving the large swirls that continue for quite some time. Explain why this is to be expected.

8.33 Determine the thickness of the viscous sublayer in a smooth 8-in.-diameter pipe if the Reynolds number is 25,000.

8.34 Water at 60 °F flows through a 6-in.-diameter pipe with an average velocity of 15 ft/s. Approximately what is the height of the largest roughness element allowed if this pipe is to be classified as smooth?

8.35 A 70-ft-long, 0.5-in.-diameter hose with a roughness of $\varepsilon = 0.0009$ ft is fastened to a water faucet where the pressure is p_1. Determine p_1 if there is no nozzle attached and the average velocity in the hose is 6 ft/s. Neglect minor losses and elevation changes.

8.36 Repeat Problem 8.35 if there is a nozzle of diameter 0.25 in. attached to the end of the hose.

***8.37** The following equation is sometimes used in place of the Colebrook equation (Eq. 8.35):

$$f = \frac{1.325}{\{\ln[(\varepsilon/3.7D) + (5.74/\text{Re}^{0.9})]\}^2}$$

for $10^{-6} < \varepsilon/D < 10^{-2}$ and $5000 < \text{Re} < 10^{+8}$ (Ref. 22, pg. 220). An advantage of this equation is that given Re and ε/D, it does not require an iteration procedure to obtain f. Plot a graph of the percent difference in f as given by this equation and the original Colebrook equation for Reynolds numbers in the range of validity of the above equation, with $\varepsilon/D = 10^{-4}$.

8.38 Water flows at a rate of 10 gallons per minute in a new horizontal 0.75-in.-diameter galvanized iron pipe. Determine the pressure gradient, $\Delta p/\ell$, along the pipe.

8.39 As shown in Fig. 8.3 and **Video V8.1** (and quantified by Reynolds), the character of the flow within a pipe depends strongly on whether the flow is laminar or turbulent. One advantage of laminar flow is that the friction factor and head loss are less than that for turbulent flow at the same Reynolds number. Under what circumstances might it be advantageous to have turbulent rather than laminar pipe flow?

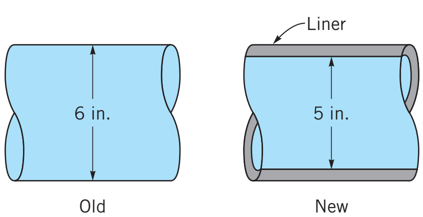
8.40 A garden hose is attached to a faucet that is fully opened. Without a nozzle on the end of the hose, the water does not shoot very far. However, if you place your thumb over a portion of the end of the hose, it is possible to shoot the water a considerable distance. Explain this phenomenon. (*Note*: The flowrate decreases as the area covered by your thumb increases.)

8.41 Air at standard temperature and pressure flows through a 1-in.-diameter galvanized iron pipe with an average velocity of 8 ft/s. What length of pipe produces a head loss equivalent to **(a)** a flanged 90° elbow, **(b)** a wide-open angle valve, or **(c)** a sharp-edged entrance?

***8.42** Water at 40 °C flows through drawn tubings with diameters of 0.025, 0.050, or 0.075 m. Plot the head loss in each meter length of pipe for flowrates between 5×10^{-4} m^3/s and 50×10^{-4} m^3/s. In your solution obtain the friction factor from the Colebrook formula.

8.43 Air at standard temperature and pressure flows at a rate of 7.0 cfs through a horizontal, galvanized iron duct that has a rectangular cross-sectional shape of 12 in. by 6 in. Estimate the pressure drop per 200 ft of duct.

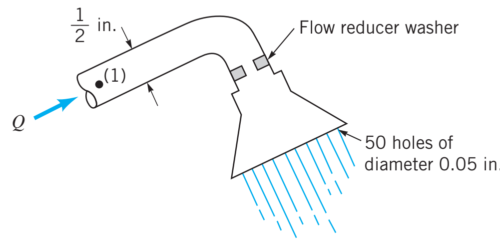
8.44 Water flows at a rate of 2.0 ft^3/s in an old, rusty 6-in.-diameter pipe that has a relative roughness of 0.010. It is proposed that by inserting a smooth plastic liner with an inside diameter of 5 in. into the old pipe as shown in Fig. P8.44, the pressure drop per mile can be reduced. Is it true that the lined pipe can carry the required 2.0 ft^3/s at a lower pressure drop than in the old pipe? Support your answer with appropriate calculations.



■ **FIGURE P8.44**

† **8.45** Consider the process of donating blood. Blood flows from a vein in which the pressure is greater than atmospheric, through a long small-diameter tube, and into a plastic bag that is essentially at atmospheric pressure. Based on fluid mechanics principles, estimate the amount of time it takes to donate a pint of blood. List all assumptions and show calculations.

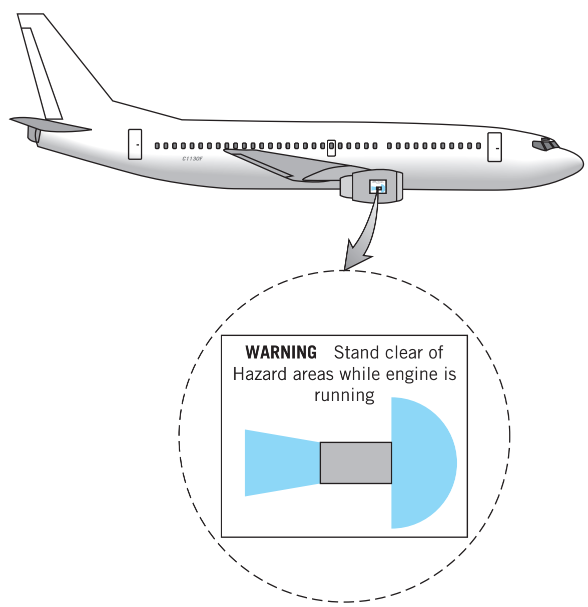
8.46 To conserve water and energy, a "flow reducer" is installed in the shower head as shown in Fig. P8.46. If the pressure at point (1) remains constant and all losses except for that in the "flow reducer" are neglected, determine the value of the loss coefficient (based on the velocity in the pipe) of the "flow reducer" if its presence is to reduce the flowrate by a factor of 2. Neglect gravity.



■ **FIGURE P8.46**

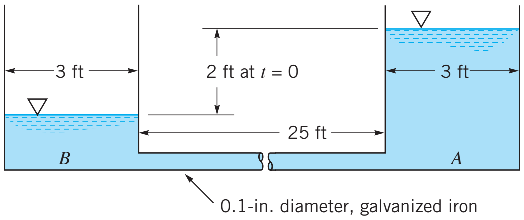
8.47 Water flows at a rate of 0.040 m^3/s in a 0.12-m-diameter pipe that contains a sudden contraction to a 0.06-m-diameter pipe. Determine the pressure drop across the contraction section. How much of this pressure difference is due to losses and how much is due to kinetic energy changes?

8.48 A sign like the one shown in Fig. P8.48 is often attached to the side of a jet engine as a warning to airport workers. Based on **Video V8.4** or Figs. 8.22 and 8.25, explain why the danger areas (indicated in color) are the shape they are.



■ **FIGURE P8.48**

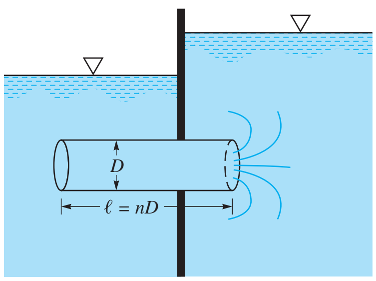
8.49 At time $t = 0$ the level of water in tank A shown in Fig. P8.49 is 2 ft above that in tank B. Plot the elevation of the water in tank A as a function of time until the free surfaces in both tanks are at the same elevation. Assume quasisteady conditions—that is, the steady pipe flow equations are assumed valid at any time, even though the flowrate does change (slowly) in time. Neglect minor losses. *Note*: Verify and use the fact that the flow is laminar.



■ FIGURE P8.49

***8.50** Repeat Problem 8.49 if the pipe diameter is changed to 0.1 ft rather than 0.1 in. *Note*: The flow may not be laminar for this case.

8.51 As shown in Fig. P8.51, water flows from one tank to another through a short pipe whose length is n times the pipe diameter. Head losses occur in the pipe and at the entrance and exit. (See **Video V8.4**.) Determine the maximum value of n if the major loss is to be no more than 10% of the minor loss and the friction factor is 0.02.



■ FIGURE P8.51

8.52 Gasoline flows in a smooth pipe of 40-mm diameter at a rate of 0.001 m^3/s. If it were possible to prevent turbulence from occurring, what would be the ratio of the head loss for the actual turbulent flow compared to that if it were laminar flow?

8.53 A 3-ft-diameter duct is used to carry ventilating air into a vehicular tunnel at a rate of 9000 ft^3/min. Tests show that the pressure drop is 1.5 in. of water per 1500 ft of duct. What is the value of the friction factor for this duct and the approximate size of the equivalent roughness of the surface of the duct?

8.54 Natural gas ($\rho = 0.0044$ slugs/ft^3 and $\nu = 5.2 \times 10^{-5}$ ft^2/s) is pumped through a horizontal 6-in.-diameter cast-iron pipe at a rate of 800 lb/hr. If the pressure at section (1) is 50 psi (abs), determine the pressure at section (2) 8 mi downstream if the flow is assumed incompressible. Is the incompressible assumption reasonable? Explain.

***8.55** Water flows in a 20-mm-diameter galvanized iron pipe with average velocities between 0.01 and 10.0 m/s. Plot the head loss per meter of pipe length over this velocity range. Discuss.

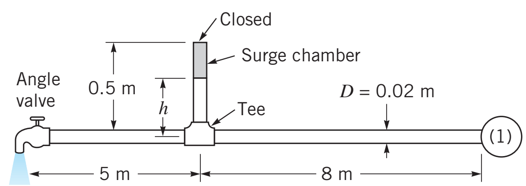
8.56 A fluid flows through a smooth horizontal 2-m-long tube of diameter 2 mm with an average velocity of 2.1 m/s. Determine the head loss and the pressure drop if the fluid is **(a)** air, **(b)** water, or **(c)** mercury.

8.57 Air at standard temperature and pressure flows through a horizontal 2 ft by 1.3 ft rectangular galvanized iron duct with a flowrate of 8.2 cfs. Determine the pressure drop in inches of water per 200-ft length of duct.

8.58 Air flows through a rectangular galvanized iron duct of size 0.30 m by 0.15 m at a rate of 0.068 m^3/s. Determine the head loss in 12 m of this duct.

8.59 Air at standard conditions flows through a horizontal 1 ft by 1.5 ft rectangular wooden duct at a rate of 5000 ft^3/min. Determine the head loss, pressure drop, and power supplied by the fan to overcome the flow resistance in 500 ft of the duct.

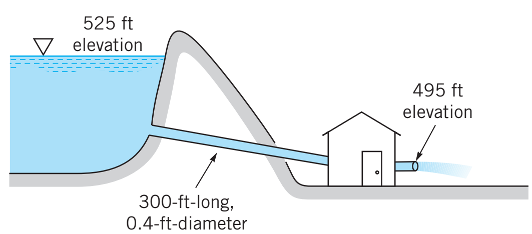
8.60 When the valve is closed, the pressure throughout the horizontal pipe shown in Fig. P8.60 is 400 kPa, and the water level in the closed, air-filled surge chamber is $h = 0.4$ m. If the valve is fully opened and the pressure at point (1) remains 400 kPa, determine the new level of the water in the surge chamber. Assume the friction factor is $f = 0.02$ and the fittings are threaded fittings.



■ FIGURE P8.60

8.61 What horsepower is added to water to pump it vertically through a 200-ft-long, 1.0-in.-diameter drawn tubing at a rate of 0.060 ft^3/s if the pressures at the inlet and outlet are the same?

8.62 Water flows from a lake as is shown in Fig. P8.62 at a rate of 4.0 cfs. Is the device inside the building a pump or a turbine? Explain and determine the horsepower of the device. Neglect all minor losses and assume the friction factor is 0.025.

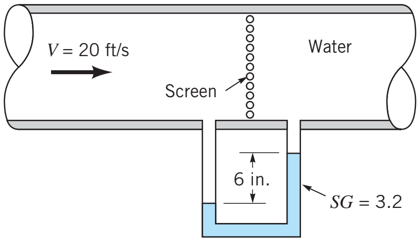


■ FIGURE P8.62

8.63 Repeat Problem 8.62 if the flowrate is 1.0 cfs.

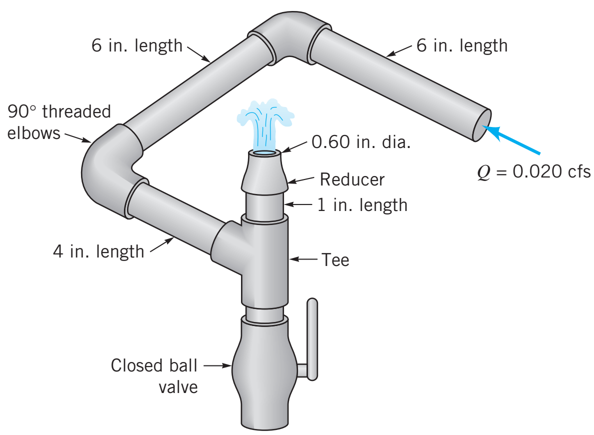
8.64 At a ski resort, water at 40 °F is pumped through a 3-in.-diameter, 2000-ft-long steel pipe from a pond at an elevation of 4286 ft to a snow-making machine at an elevation of 4623 ft at a rate of 0.26 ft^3/s. If it is necessary to maintain a pressure of 180 psi at the snow-making machine, determine the horsepower added to the water by the pump. Neglect minor losses.

8.65 Water flows through the screen in the pipe shown in Fig. P8.65 as indicated. Determine the loss coefficient for the screen.



■ **FIGURE P8.65**

8.66 Water flows steadily through the 0.75-in-diameter galvanized iron pipe system shown in **Video V8.6** and Fig. P8.66 at a rate of 0.020 cfs. Your boss suggests that friction losses in the straight pipe sections are negligible compared to losses in the threaded elbows and fittings of the system. Do you agree or disagree with your boss? Support your answer with appropriate calculations.



■ **FIGURE P8.66**

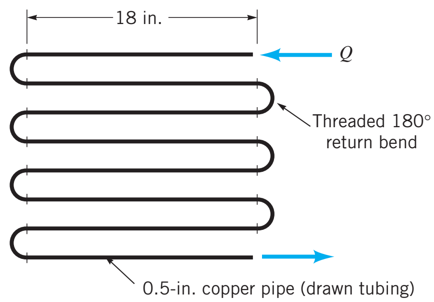
8.67 Because of a worn-out washer in a kitchen sink faucet, water drips at a steady rate even though the faucet is "turned off." Readings from a water meter of the type shown in **Video V8.7** indicate that during a one-week time period when the home owners were away, 200 gallons of water dripped from the faucet. **(a)** If the pressure within the 0.50-in-diameter pipe is 50 psi, determine the loss coefficient for the leaky faucet. **(b)** What length of the pipe would be needed to produce a head loss equivalent to the leaky faucet?

8.68 Assume a car's exhaust system can be approximated as 14 ft of 0.125-ft-diameter cast-iron pipe with the equivalent of six 90° flanged elbows and a muffler. (See **Video V8.5**.) The muffler acts as a resistor with a loss coefficient of $K_L = 8.5$. Determine the pressure at the beginning of the exhaust system if the flowrate is 0.10 cfs, the temperature is 250 °F, and the exhaust has the same properties as air.

8.69 Air is to flow through a smooth, horizontal, rectangular duct at a rate of 100 m^3/s with a pressure drop of not more than 40 mm of water per 50 m of duct. If the aspect ratio (width to height) is 3 to 1, determine the size of the duct.

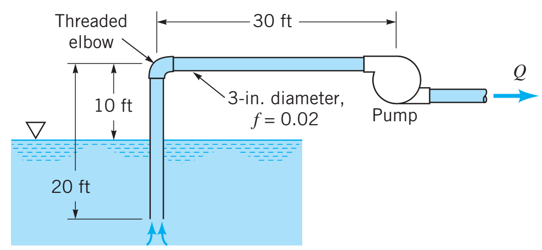
8.70 Repeat Problem 3.14 if all head losses are included. The pipes are 1-in. copper pipes with regular flanged fittings. The faucets are globe valves.

8.71 Water at 40 °F flows through the coils of the heat exchanger as shown in Fig. P8.71 at a rate of 0.9 gal/min. Determine the pressure drop between the inlet and outlet of the horizontal device.



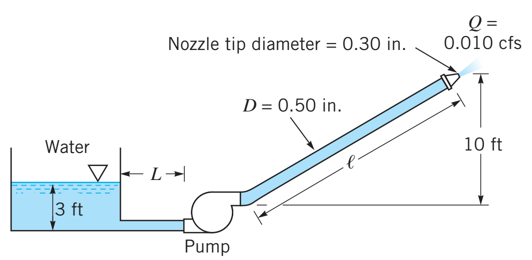
■ **FIGURE P8.71**

8.72 Water at 40 °F is pumped from a lake as shown in Fig. P8.72. What is the maximum flowrate possible without cavitation occurring?



■ **FIGURE P8.72**

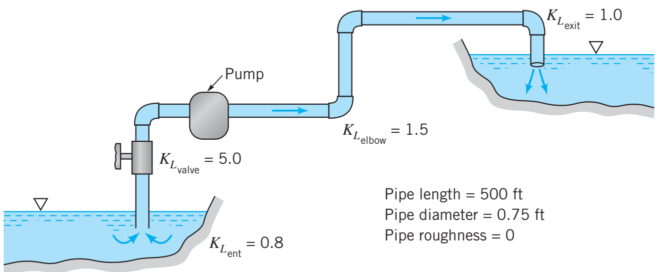
8.73 The $\frac{1}{2}$-in.-diameter hose shown in Fig. P8.73 can withstand a maximum pressure of 200 psi without rupturing. Determine the maximum length, ℓ, allowed if the friction factor is 0.022 and the flowrate is 0.010 cfs. Neglect minor losses.



■ **FIGURE P8.73**

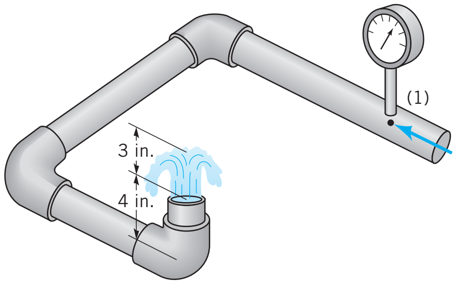
8.74 The hose shown in Fig. P8.73 will collapse if the pressure within it is lower than 10 psi below atmospheric pressure. Determine the maximum length, L, allowed if the friction factor is 0.015 and the flowrate is 0.010 cfs. Neglect minor losses.

8.75 The pump shown in Fig. P8.75 delivers a head of 250 ft to the water. Determine the power that the pump adds to the water. The difference in elevation of the two ponds is 200 ft.



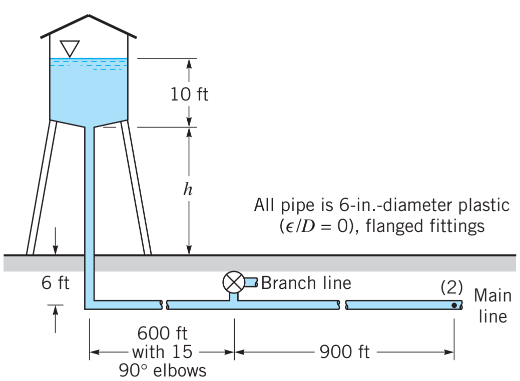
■ **FIGURE P8.75**

8.76 As shown in **Video V8.6** and Fig. P8.76, water "bubbles up" 3 in. above the exit of the vertical pipe attached to three horizontal pipe segments. The total length of the 0.75-in.-diameter galvanized iron pipe between point (1) and the exit is 21 in. Determine the pressure needed at point (1) to produce this flow.



■ **FIGURE P8.76**

8.77 The pressure at section (2) shown in Fig. P8.77 is not to fall below 60 psi when the flowrate from the tank varies from 0 to 1.0 cfs and the branch line is shut off. Determine the minimum height, h, of the water tank under the assumption that **(a)** minor losses are negligible, **(b)** minor losses are not negligible.

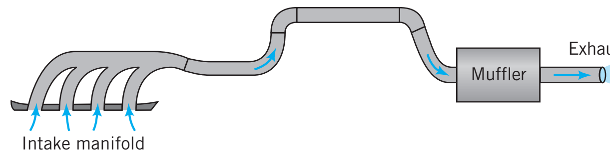


■ **FIGURE P8.77**

8.78 Repeat Problem 8.77 with the assumption that the branch line is open so that half of the flow from the tank goes into the branch, and half continues in the main line.

8.79 Repeat Problem 3.43 if head losses are included.

8.80 The exhaust from your car's engine flows through a complex pipe system as shown in Fig. P8.80 and **Video V8.5**. Assume that the pressure drop through this system is Δp_1 when the engine is idling at 1000 rpm at a stop sign. Estimate the pressure drop (in terms of Δp_1) with the engine at 3000 rpm when you are driving on the highway. List all the assumptions that you made to arrive at your answer.



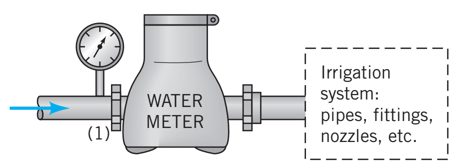
■ **FIGURE P8.80**

8.81 Water flows from a large open tank, through a 50-ft-long, 0.10-ft-diameter pipe and exits with a velocity of 5 ft/s when the water level in the tank is 10 ft above the pipe exit. The sum of the minor loss coefficients for the pipe system is 12. Determine the new water level needed in the tank if the velocity is to remain 5 ft/s when 20 ft of the pipe is removed (i.e., when the length is reduced to 30 ft). The minor loss coefficients remain the same.

8.82 Water is to flow at a rate of 3.5 ft^3/s in a horizontal aluminum pipe ($\varepsilon = 5 \times 10^{-6}$ ft). The inlet and outlet pressures are 65 psi and 30 psi, respectively, and the pipe length is 500 ft. Determine the diameter of this water pipe.

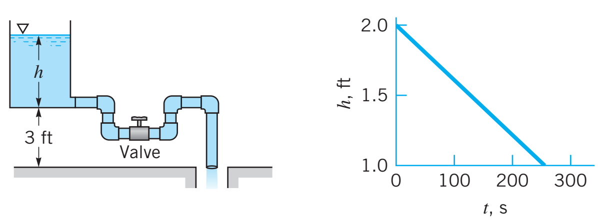
8.83 Water flows downward through a vertical smooth pipe. When the flowrate is 0.5 ft^3/s there is no change in pressure along the pipe. Determine the diameter of the pipe.

8.84 As shown in Fig. P8.84, a standard household water meter is incorporated into a lawn irrigation system to measure the volume of water applied to the lawn. Note that these meters measure volume, not volume flowrate. (See **Video V8.7**.) With an upstream pressure of p_1 = 50 psi the meter registered that 120 ft^3 of water was delivered to the lawn during an "on" cycle. Estimate the upstream pressure, p_1, needed if it is desired to have 150 ft^3 delivered during an "on" cycle. List any assumptions needed to arrive at your answer.



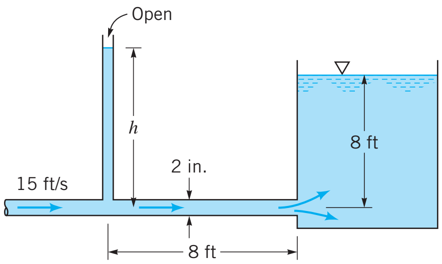
■ **FIGURE P8.84**

8.85 When water flows from the tank shown in Fig. P8.85, the water depth in the tank as a function of time is as indicated. Determine the cross-sectional area of the tank. The total length of the 0.60-in.-diameter pipe is 20 ft, and the friction factor is 0.03. The loss coefficients are: 0.50 for the entrance, 1.5 for each elbow, and 10 for the valve.



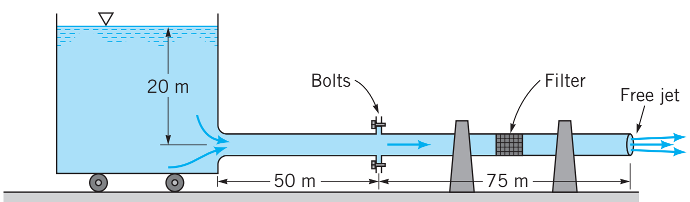
■ **FIGURE P8.85**

8.86 Water flows through a 2-in.-diameter pipe with a velocity of 15 ft/s as shown in Fig. P8.86. The relative roughness of the pipe is 0.004, and the loss coefficient for the exit is 1.0. Determine the height, h, to which the water rises in the piezometer tube.



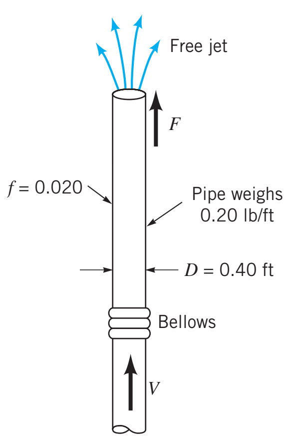
■ **FIGURE P8.86**

8.87 Water flows from a large tank that sits on frictionless wheels as shown in Fig. P8.87. The pipe has a diameter of 0.50 m and a roughness of 9.2×10^{-5} m. The loss coefficient for the filter is 8; other minor losses are negligible. The tank and the first 50-m section of the pipe are bolted to the last 75-m section of the pipe which is clamped firmly to the floor. Determine the tension in the bolts.



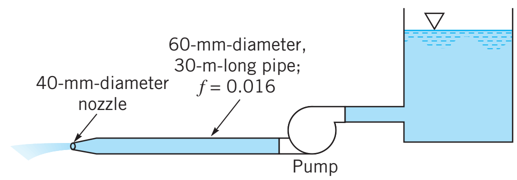
■ **FIGURE P8.87**

8.88 Water flows through two sections of the vertical pipe shown in Fig. P8.88. The bellows connection cannot support any force in the vertical direction. The 0.4-ft-diameter pipe weighs 0.2 lb/ft, and the friction factor is assumed to be 0.02. At what velocity will the force, F, required to hold the pipe be zero?



■ **FIGURE P8.88**

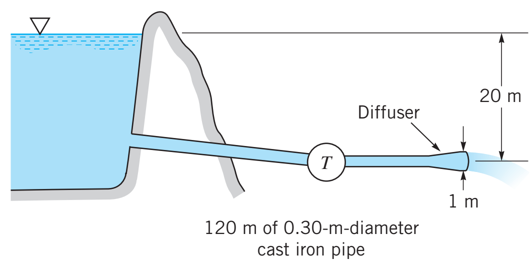
8.89 The pump shown in Fig. P8.89 adds 25 kW to the water and causes a flowrate of 0.04 m^3/s. Determine the flowrate expected if the pump is removed from the system. Assume $f = 0.016$ for either case and neglect minor losses.



■ **FIGURE P8.89**

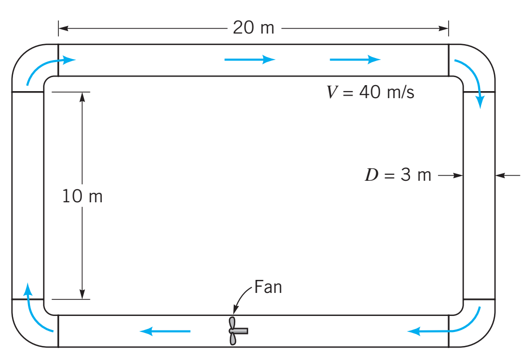
8.90 A certain process requires 2.3 cfs of water to be delivered at a pressure of 30 psi. This water comes from a large diameter supply main in which the pressure remains at 60 psi. If the galvanized iron pipe connecting the two locations is 200 ft long and contains six threaded 90° elbows, determine the pipe diameter. Elevation differences are negligible.

8.91 The turbine shown in Fig. P8.91 develops 400 kW. Determine the flowrate if **(a)** head losses are negligible or **(b)** head loss due to friction in the pipe is considered. Assume $f = 0.02$. *Note*: There may be more than one solution or there may be no solution to this problem.



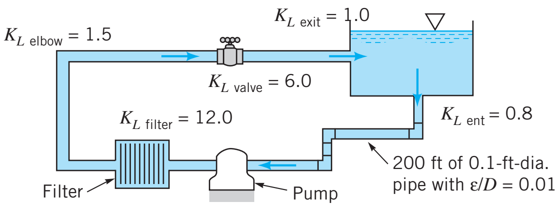
■ **FIGURE P8.91**

8.92 A fan is to produce a constant air speed of 40 m/s throughout the pipe loop shown in Fig. P8.92. The 3-m-diameter pipes are smooth, and each of the four 90-degree elbows has a loss coefficient of 0.30. Determine the power that the fan adds to the air.



■ **FIGURE P8.92**

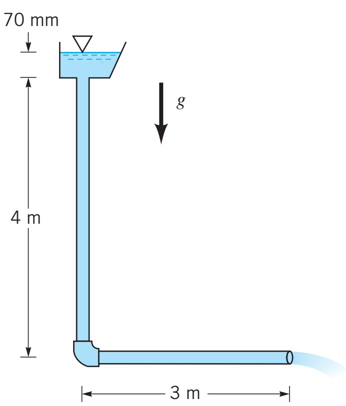
8.93 Water is circulated from a large tank, through a filter, and back to the tank as shown in Fig. P8.93. The power added to the water by the pump is 200 ft · lb/s. Determine the flowrate through the filter.



■ **FIGURE P8.93**

8.94 Water is to be moved from a large, closed tank in which the air pressure is 20 psi into a large, open tank through 2000 ft of smooth pipe at the rate of 3 ft^3/s. The fluid level in the open tank is 150 ft below that in the closed tank. Determine the required diameter of the pipe. Neglect minor losses.

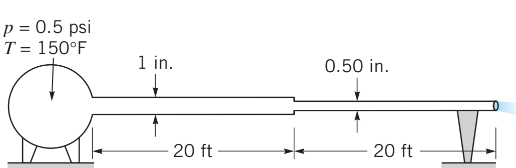
8.95 Rainwater flows through the galvanized iron downspout shown in Fig. P8.95 at a rate of 0.006 m^3/s. Determine the size of the downspout cross section if it is a rectangle with an aspect ratio of 1.7 to 1 and it is completely filled with water. Neglect the velocity of the water in the gutter at the free surface and the head loss associated with the elbow.



■ **FIGURE P8.95**

***8.96** Repeat Problem 8.95 if the downspout is circular.

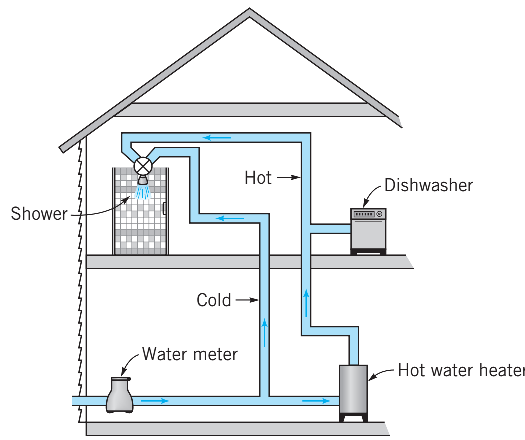
8.97 Air, assumed incompressible, flows through the two pipes shown in Fig. P8.97. Determine the flowrate if minor losses are neglected and the friction factor in each pipe is 0.015. Determine the flowrate if the 0.5-in.-diameter pipe were replaced by a 1-in.-diameter pipe. Comment on the assumption of incompressibility.



■ **FIGURE P8.97**

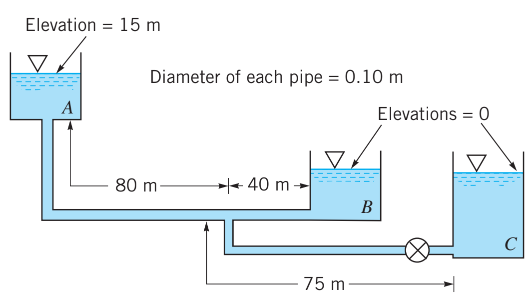
***8.98** Repeat Problem 8.97 if the pipes are galvanized iron and the friction factors are not known a priori.

† **8.99** As shown in Fig. P8.99, cold water (T = 50 °F) flows from the water meter to either the shower or the hot water heater. In the hot water heater it is heated to a temperature of 150 °F. Thus, with equal amounts of hot and cold water, the shower is at a comfortable 100 °F. However, when the dishwasher is turned on, the shower water becomes too cold. Indicate how you would predict this new shower temperature (assume the shower faucet is not adjusted). State any assumptions needed in your analysis.



■ **FIGURE P8.99**

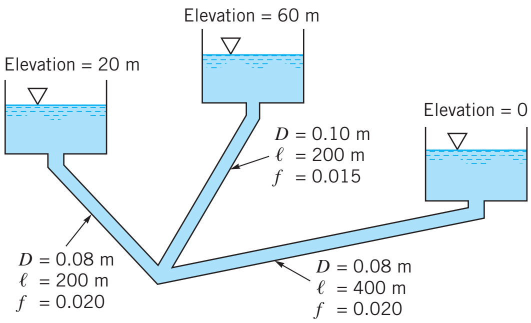
8.100 With the valve closed, water flows from tank A to tank B as shown in Fig. P8.100. What is the flowrate into tank B when the valve is opened to allow water to flow into tank C also? Neglect all minor losses and assume that the friction factor is 0.02 for all pipes.



■ **FIGURE P8.100**

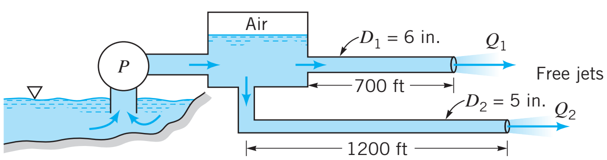
***8.101** Repeat Problem 8.100 if the friction factors are not known, but the pipes are steel pipes.

8.102 The three water-filled tanks shown in Fig. P8.102 are connected by pipes as indicated. If minor losses are neglected, determine the flowrate in each pipe.



■ **FIGURE P8.102**

8.103 Water is pumped from a lake, into a large pressurized tank, and out through two pipes as shown in Fig. P8.103. The pump head is $h_p = 45 + 27.5Q - 54Q^2$, where h_p is in feet and Q (the total flowrate through the pump) is in ft^3/s. Minor losses and gravity are negligible, and the friction factor in each pipe is 0.02. Determine the flowrates through each of the pipes, Q_1, and Q_2.



■ **FIGURE P8.103**

8.104 A 2-in.-diameter orifice plate is inserted in a 3-in.-diameter pipe. If the water flowrate through the pipe is 0.90 cfs, determine the pressure difference indicated by a manometer attached to the flow meter.

8.105 Air to ventilate an underground mine flows through a large 2-m-diameter pipe. A crude flowrate meter is constructed by placing a sheet metal "washer" between two sections of the pipe. Estimate the flowrate if the hole in the sheet metal has a diameter of 1.6 m and the pressure difference across the sheet metal is 8.0 mm of water.

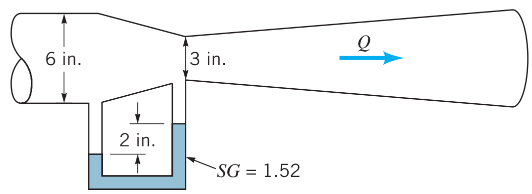
8.106 Gasoline flows through a 35-mm-diameter pipe at a rate of $0.0032\ \text{m}^3/\text{s}$. Determine the pressure drop across a flow nozzle placed in the line if the nozzle diameter is 20 mm.

8.107 Air at 200 °F and 60 psia flows in a 4-in.-diameter pipe at a rate of 0.52 lb/s. Determine the pressure at the 2-in-diameter throat of a Venturi meter placed in the pipe.

8.108 A 50-mm-diameter nozzle meter is installed at the end of a 80-mm-diameter pipe through which air flows. A manometer attached to the static pressure tap just upstream from the nozzle indicates a pressure of 7.3 mm of water. Determine the flowrate.

8.109 A 2.5-in.-diameter nozzle meter is installed in a 3.8-in.-diameter pipe that carries water at 160 °F. If the inverted air-water U-tube manometer used to measure the pressure difference across the meter indicates a reading of 3.1 ft, determine the flowrate.

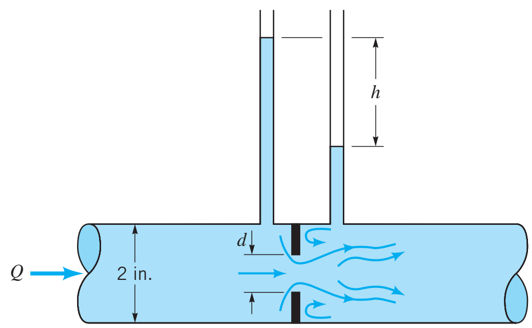
8.110 Water flows through the Venturi meter shown in Fig. P8.110. The specific gravity of the manometer fluid is 1.52. Determine the flowrate.



■ **FIGURE P8.110**

8.111 If the fluid flowing in Problem 8.110 were air, what would be the flowrate? Would compressibility effects be important? Explain.

8.112 Water flows through the orifice meter shown in Fig. P8.112 at a rate of 0.10 cfs. If $d = 0.1$ ft, determine the value of h.

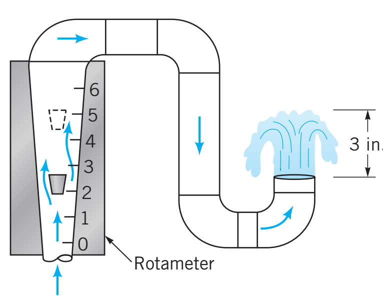


■ **FIGURE P8.112**

8.113 Water flows through the orifice meter shown in Fig. P8.112 at a rate of 0.10 cfs. If $h = 3.8$ ft, determine the value of d.

8.114 Water flows through the orifice meter shown in Fig. P8.112 such that $h = 1.6$ ft with $d = 1.5$ in. Determine the flowrate.

8.115 The scale reading on the rotameter shown in Fig. P8.115 and **Video V8.6** (also see Fig. 8.46) is directly proportional to the volumetric flowrate. With a scale reading of 2.6 the water bubbles up approximately 3 in. How far will it bubble up if the scale reading is 5.0?



■ **FIGURE P8.115**

8.116 This problem involves the determination of the friction factor in a pipe for laminar and transitional flow conditions. To proceed with this problem, ***click here*** in the E-book.

8.117 This problem involves the calibration of an orifice meter and a Venturi meter. To proceed with this problem, ***click here*** in the E-book.

8.118 This problem involves the flow of water from a tank and through a pipe system. To proceed with this problem, ***click here*** in the E-book.

8.119 This problem involves the flow of water pumped from a tank and through a pipe system. To proceed with this problem, ***click here*** in the E-book.

8.120 This problem involves the pressure distribution in the entrance region of a pipe. To proceed with this problem, ***click here*** in the E-book.

8.121 This problem involves the power loss due to friction in a coiled pipe. To proceed with this problem, ***click here*** in the E-book.