**Solutions to problem set 5**

Part 1:

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1-

a)- True. b)- False. Basically the Phillips curve vanished after 1970. It is not stable, because it is a function of expected price level. c)- True. While aggregate supply relation always reflects the reality.
d)- True. e)- False. Before 1970, when $\pi^e = 0$, then if $\pi = 0$ ➔ $u = u_n$. f)- True.

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3-

a)- $\pi = 0$ ➔ $u = u_n$ ➔ $u_n = 0.1/2 = 5\%$
b)- $\pi_t = 0.1 - 2\, u_t$ ➔ $\pi_t = 0.1 - 2\, u_t = 0.1 - 2 * 3\% = 4\%$, $\pi_{t+1} = \pi_{t+2} = \pi_{t+10} = \pi_{t+15} = 4\%$

c)- No. Because, $\pi_t^e = 0$, and $\pi_t = 4\%$ forever, which means people are continuously wrong for ever. This does not seem to be reasonable.
d)- Because, finally, people realize that inflation is positive. But there will not change in natural rate of unemployment.

e)- $\pi_{t+1} = \pi_{t+2} = \ldots = \pi_{t+4} = 4\%$
Year 5, and on: $\pi_t = \pi_t^e + 0.1 - 2\, u_t$ ➔ $\pi_t = \pi_{t-1} + 0.1 - 2\, u_t$, $\pi_{t+5} = 4\% + 0.1 - (2 * 3\%)$ ➔ $\pi_{t+5} = 8\%$
By continuous substitution: $\pi_{t+10} = \pi_{t+9} + (0.1 - 2 * 3\%) = \pi_{t+8} + 2*(0.1 - 2 * 3\%) = \ldots$ ➔
$\pi_{t+10} = \pi_{t+5} + 5*(0.1 - 2 * 3\%) = 8\% + 5*(0.1 - 2 * 3\%) = 28\%$.
$\pi_{t+15} = \pi_{t+10} + 5*(0.1 - 2 * 3\%) = 28\% + 5*(0.1 - 2 * 3\%) = 48\%$.

f)- There is still and always a 4% difference between expected and actual inflation, which is not reasonable.

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5-

a)- $\pi_t = \pi_t^e + 0.1 - 2\, u_t$ ➔ $\pi_t = \pi_{t-1} + 0.1 - 2\, u_t$, $\pi_t = 0\% + 0.1 - (2 * 4\%)$ ➔ $\pi_t = 2\%$
$\pi_{t+1} = 2\% + 0.1 - (2 * 4\%)$ ➔ $\pi_{t+1} = 4\%$ $\pi_{t+2} = 4\% + 0.1 - (2 * 4\%)$ ➔ $\pi_{t+2} = 6\%$
$\pi_{t+3} = 6\% + 0.1 - (2 * 4\%)$ ➔ $\pi_{t+3} = 8\%$

b)- $\pi_t^e = 0.5 * \pi_t + 0.5 * \pi_{t-1}$, and $\pi_t = \pi_t^e + 0.1 - 2\, u_t$ ➔ By substitution: $\pi_t = (0.5 * \pi_t + 0.5 * \pi_{t-1}) + 0.1 - 2\, u_t$,
After simplification, we get the new Phillips curve: $\pi_t = \pi_{t-1} + 0.2 - 4\, u_t$,

c)- $\pi_t = \pi_{t-1} + 0.2 - 4\, u_t$, $\pi_t = 0\% + 0.2 - (4 * 4\%)$ ➔ $\pi_t = 4\%$
$\pi_{t+1} = 4\% + 0.2 - (4 * 4\%)$ ➔ $\pi_{t+1} = 8\%$ $\pi_{t+2} = 8\% + 0.2 - (4 * 4\%)$ ➔ $\pi_{t+2} = 12\%$
$\pi_{t+3} = 12\% + 0.2 - (4 * 4\%)$ ➔ $\pi_{t+3} = 16\%$

d)- Indexation leads to higher inflation rate at any level of unemployment.

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4-

a)- Because higher price of oil means higher production cost and higher output price at any given level of wage rate. We can characterize this using higher markup.
b)- $\pi^e = \pi$ ➔ $u = u_n$. Therefore: $u_n = 0.04 + 0.05\, \mu$
➔ Before the oil shock: $u_n = 0.04 + 0.05*0.2 = 0.05$. After the oil shock: $u_n = 0.04 + 0.05*0.4 = 0.06$
So, oil shock increases the natural rate of unemployment.

Part 2:

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1-

a)- False. Using Okun's law for Canada: $u_t - u_{t-1} = -0.33(g_{yt} - 3.7\%)$, output growth rate must be 3.7% to have constant unemployment rate.

b)- True. c)- True. d)- False. The growth rate of the money supply has a crucial role too, $\pi_t = g_{mt} - g_{yt}$.

e)- False. $\pi_t = g_{mt} - g_{yt}$, so inflation rate is equal to money growth rate minus output growth rate.

f)- True. g)- True. h)- True.

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2-

a)- $u_t - u_{t-1} = -0.33(g_{yt} - 3.7\%)$ ➔ $1\% = -0.33(g_{yt} - 3.7\%)$ ➔ $g_{yt} = 0.7\%$. Output growth rate should be large enough to cover the expansion of the labor force, and increase in the labor productivity.

b)- We need to decrease unemployment 0.25% a year. So: $-0.25 = -0.33(g_{yt} - 3.7\%)$ ➔ 4.45% a year.

c)- Okun's law changes to: $u_t - u_{t-1} = -0.33(g_{yt} - 5.7\%)$

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3)-

a)-

Phillips curve for the US: $\pi_t - \pi_{t-1} = -(u_t - 5\%)$, $\pi_t = \pi_{t-1}$ ➔ $u_n = 5\%$

Phillips curve for Canada: $\pi_t - \pi_{t-1} = -0.5\,(u_t - 8.6\%)$, $\pi_t = \pi_{t-1}$ ➔ $u_n = 8.6\%$

b)- $u_t - u_{t-1} = -0.4(g_{yt} - 3\%)$, $u_t = u_{t-1}$ ➔ $g_{yt} = 3\%$

$\pi_t = g_{mt} - g_{yt}$, $8\% = g_{mt} - 3\%$ ➔ $g_{mt} = 11\%$

c)-

$\pi_{t-1} = 8\%$, $u_{t-1} = 5\%$, $g_{yt-1} = 3\%$, $g_{mt-1} = 11\%$

$\pi_t = 4\%$

$\pi_t - \pi_{t-1} = -(u_t - 5\%)$ ➔ $4\% - 8\% = -(u_t - 5\%)$ ➔ $u_t = 9\%$

$u_t - u_{t-1} = -0.4\,(g_{yt} - 3\%)$ ➔ $9\% - 5\% = -0.4\,(g_{yt} - 3\%)$ ➔ $g_{yt} = -7\%$

$\pi_t = g_{mt} - g_{yt}$, $4\% = g_{mt} + 7\%$ ➔ $g_{mt} = -3\%$

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$\pi_{t+1} = 4\%$

$\pi_{t+1} - \pi_t = -(u_{t+1} - 5\%)$ ➔ $4\% - 4\% = -(u_{t+1} - 5\%)$ ➔ $u_{t+1} = 5\%$

$u_{t+1} - u_t = -0.4\,(g_{yt+1} - 3\%)$ ➔ $5\% - 9\% = -0.4\,(g_{yt+1} - 3\%)$ ➔ $g_{yt+1} = 13\%$

$\pi_{t+1} = g_{mt+1} - g_{yt+1}$, $4\% = g_{mt+1} - 13\%$ ➔ $g_{mt+1} = 17\%$

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$\pi_{t+2} = 4\%$

$\pi_{t+2} - \pi_{t+1} = -(u_{t+2} - 5\%)$ ➔ $4\% - 4\% = -(u_{t+2} - 5\%)$ ➔ $u_{t+2} = 5\%$

$u_{t+2} - u_{t+1} = -0.4\,(g_{yt+2} - 3\%)$ ➔ $5\% - 5\% = -0.4\,(g_{yt+2} - 3\%)$ ➔ $g_{yt+2} = 3\%$

$\pi_{t+2} = g_{mt+2} - g_{yt+2}$, $4\% = g_{mt+2} - 3\%$ ➔ $g_{mt+2} = 7\%$

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4)-

a)-

$\pi_t - \pi_{t-1} = -(u_t - 5\%)$, $u_t - u_{t-1} = -0.4\,(g_{yt} - 3\%)$, $\pi_t = g_{mt} - g_{yt}$

➔ $\pi_t - \pi_{t-1} = -(u_t - 5\%)$, $u_t - u_{t-1} = -0.4\,(g_{mt} - \pi_t - 3\%)$

$\pi_t = 10\%$, $u_{t-1} = u_t = 5\%$, $g_{yt-1} = 3\%$, $g_{mt} = 13\%$

b)-

$\pi_{t+1} - \pi_t = -(u_{t+1} - 5\%)$, $u_{t+1} - u_t = -0.4\,(g_{mt+1} - \pi_{t+1} - 3\%)$

$\pi_{t+1} - 10\% = -(u_{t+1} - 5\%)$ ➔ $\pi_{t+1} + u_{t+1} = 15\%$

$u_{t+1} - 5\% = -0.4\,(0 - \pi_{t+1} - 3\%)$ ➔ $u_{t+1} - 0.4\,\pi_{t+1} = 6.2\%$ ➔ $u_{t+1} = 8.71\%$, $\pi_{t+1} = 6.29\%$

$\pi_{t+2} - \pi_{t+1} = -(u_{t+2} - 5\%)$, $u_{t+2} - u_{t+1} = -0.4\,(g_{mt+2} - \pi_{t+2} - 3\%)$

$\pi_{t+2} - 6.29\% = -(u_{t+2} - 5\%)$ ➔ $\pi_{t+2} + u_{t+2} = 11.29\%$

$u_{t+2} - 8.71\% = -0.4\,(0 - \pi_{t+2} - 3\%)$ ➔ $u_{t+2} - 0.4\,\pi_{t+2} = 9.91\%$ ➔ $u_{t+1} = 10.3\%$, $\pi_{t+2} = 1\%$

c)- $\pi_{t+1} - \pi_t = -(u_{t+1} - 5\%)$, $u_{t+1} - u_t = -0.4\,(g_{mt+1} - \pi_{t+1} - 3\%)$

In medium run $u_{t+1} = u_t$ ➔ $\pi_{t+1} = -3\%$ ➔ $u_t = 5\%$ ➔ $g_{yt} = 3\%$.

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6-

$\pi_t - \pi_{t-1} = -(u_t - 8.6\%) + 0.1\mu$ ➔ $\pi_t - \pi_{t-1} = -(u_t - u_n)$ and $u_n = 8.6\% + 0.1\mu$

a)- After the shock, natural rate of unemployment goes up. Therefore, current unemployment would fall below natural rate of unemployment; therefore, inflation starts increasing (see the Phillips Curve form).

b)- They should let the unemployment go up, to higher level of natural rate of unemployment.

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7-

a)- $\pi_t = \pi_t^e + K - 2\,u_t$ , $\pi_t = \pi_t^e$ ➔ Natural rate of unemployment: $u_n = u_t = K/2$

$\pi_t - \pi_t^e = -2(u_t - K/2)$ ➔ Sacrifice ratio = 1/2.
Sacrifice ration does not depend on natural rate of unemployment

b)- $\pi_t - \pi_t^e = -2(u_t - K/2)$, and $\pi^e_t = \pi_{t-1}$ ➔
$\pi_t - \pi_{t-1} = -2(u_t - K/2)$ ➔ $\pi_t - 12\% = -2*1\%$ ➔ $\pi_t = 10\%$
$\pi_{t+1} - \pi_t = -2(u_{t+1} - K/2)$ ➔ $\pi_{t+1} - 10\% = -2*1\%$ ➔ $\pi_{t+1} = 8\%$

c)-
$\pi_{t+2} - \pi_{t+1} = -2(u_{t+2} - K/2)$ ➔ $\pi_{t+2} - 8\% = -2*1\%$ ➔ $\pi_{t+1} = 6\%$
$\pi_{t+3} - \pi_{t+2} = -2(u_{t+3} - K/2)$ ➔ $\pi_{t+3} - 6\% = -2*1\%$ ➔ $\pi_{t+1} = 4\%$
$\pi_{t+4} - \pi_{t+3} = -2(u_{t+4} - K/2)$ ➔ $\pi_{t+4} - 4\% = -2*1\%$ ➔ $\pi_{t+1} = 2\%$

So, The authorities need to hold on the policy for 5 year, from t to t+4.
Sacrifice ratio= 5 point years of excess unemployment/10 percentage point reduction in inflation rate=0.5
Which is consistent with the sacrifice ratio calculated from Phillips curve in part (a).

d)- $\pi_t - \pi_t^e = -2(u_t - K/2)$, and $\pi_t = \pi_{t-1}$ ➔ $\pi_t^e = 0.25 * 2\% + 0.75 * \pi_{t-1}$ ➔ $\pi_t^e = 0.5\% + 0.75 * \pi_{t-1}$
$\pi_t - 0.5\% - 0.75 * \pi_{t-1} = -2(u_t - K/2)$ ➔ $\pi_t - 0.75 * \pi_{t-1} = -2(u_t - K/2) + 0.5\%$

$\pi_t - 0.75 * \pi_{t-1} = -2(u_t - K/2) + 0.5\%$ ➔ $\pi_t - 0.75 * 12\% = -2*1\% + 0.5\%$ ➔ $\pi_t = 7.5\%$
$\pi_{t+1} - 0.75 * \pi_t = -2(u_{t+1} - K/2) + 0.5\%$ ➔ $\pi_{t+1} - 0.75 * 7.5\% = -2*1\% + 0.5\%$ ➔ $\pi_{t+1} = 4.125\%$
$\pi_{t+2} - 0.75 * \pi_{t+1} = -2(u_{t+2} - K/2) + 0.5\%$ ➔ $\pi_{t+2} - 0.75 * 4.125\% = -2*1\% + 0.5\%$ ➔ $\pi_{t+2} = 1.594\%$
So, The authorities need to hold on the policy for 3 year, from t to t+2.
Sacrifice ratio= 3 point years of excess unemployment/10 percentage point reduction in inflation rate=0.3
Which is less than the sacrifice ratio calculated from Phillips curve in part (a).

e)- At the same time t+1, authorities can return to natural rate of unemployment.
$\pi_{t+1} = \pi_{t+1}^e = 2\%$, $u_n = u_{t+1} = K/2$

f)- They need to make sure to their policy has the highest credibility.

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