AUX 6 QFT

Consideremos una teoría con el siguiente lagrangiano

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi^i \partial_\mu \phi_i - V(\phi^i)$$

$i = 1, \dots, N$

$$\text{Con } V(\phi^i) = \frac{m^2}{2} \phi_i \phi^i + \frac{1}{4} \lambda (\phi_i \phi^i)^2 \; ;$$

linear sigma-model
posee simetría global $O(N)$

Analicemos el mínimo del potencial

$$\frac{\partial V(\phi^i)}{\partial \phi^j} = \frac{m^2}{2} 2 \phi_j + \frac{1}{2} \lambda (\phi_i \phi^i) \frac{\partial}{\partial \phi^j} (\phi_i \phi^i)$$

$$= m^2 \phi_j + \frac{\lambda}{2} (\phi_i \phi^i) \left(\delta_{ij} \phi^i + \delta_{ij} \phi^i \right)$$

$$= m^2 \phi_j + \lambda (\phi_i \phi^i) \phi_j \stackrel{!}{=} 0$$

$$\Rightarrow \phi_j = 0 \quad \text{ó} \quad m^2 + \lambda (\phi_i \phi^i) = 0$$

$$\Rightarrow (\phi^i)^2 = -\frac{m^2}{\lambda} \Rightarrow \phi_{1,2} = \pm \sqrt{-\frac{m^2}{\lambda}}$$

. Si $m^2 > 0 \longrightarrow$ sólo tengo la solución $\phi_j = 0$

. Si $m^2 = -\mu^2 < 0 \Rightarrow$ que el mínimo se encuentra en $\pm \sqrt{\frac{\mu^2}{\lambda}}$

(ϕ_j para a ser un mínimo local
y $\pm \sqrt{\frac{\mu^2}{\lambda}}$ corresponde al mínimo de la teoría

Si $m^2 = -\mu^2$ entonces la correlación del vacío es tal que

$\langle 0|\phi|0\rangle \neq 0$ es más $\langle 0|\phi|0\rangle = \pm\sqrt{\frac{\mu^2}{\lambda}}$

Luego $\langle \phi \rangle_0^2 = \frac{\mu^2}{\lambda} \equiv v^2$

~~Así~~ Para restablecer el vacío en 0 podemos hacer un shift sobre uno de los campos, de la siguiente forma

$$\phi_j = \pi_j(x) \quad (j<N) \quad y \quad \underbrace{\phi^N = v + \sigma(x)}_{\text{Este término rompe la simetría}} \qquad \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{pmatrix} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ v+\sigma \end{pmatrix}$$

Esta transformación restablece el vacío de la teoría en 0 pagando el precio de que ϕ^N ya no es invariante bajo $O(N)$. Luego la simetría se ha roto para $O(N-1)$

Para simplificar el calculo haremos $N=2$ de manera que

$$\phi_1 = \pi(x)$$
$$\phi_2 = v + \sigma(x)$$

Esto implica la siguiente forma del lagrangiano

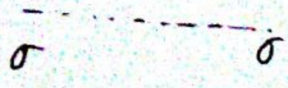
$$\mathcal{L} = \frac{1}{2}\Big(\partial_\mu \pi \partial^\mu \pi + \partial_\mu(v+\sigma(x))\partial^\mu(v+\sigma(x))\Big)$$
$$+ \frac{\mu^2}{2}\Big(\pi(x) + (v+\sigma(x))\Big)^2$$
$$- \frac{\lambda}{4}\Big(\pi^2 + (v+\sigma(x))^2\Big)^2$$

Scanned by CamScanner

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \pi\, \partial^\mu \pi + \frac{1}{2}\partial_\mu \sigma\, \partial^\mu \sigma - \frac{1}{2}(2\mu^2)\sigma^2$$

$$\mathcal{L}_1 = -\frac{\lambda}{4}\pi^4 - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{2}\pi^2\sigma^2 - \lambda v \sigma^3 - \lambda v \pi^2 \sigma$$

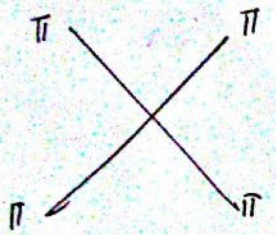
Propagadores de la teoría



$$= \frac{+i}{k^2 - 2\mu^2}$$

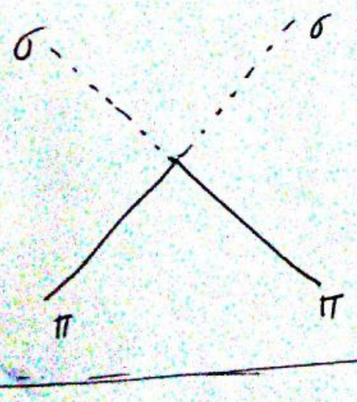
$$= \frac{+i}{k^2}$$

Calculemos los vértices

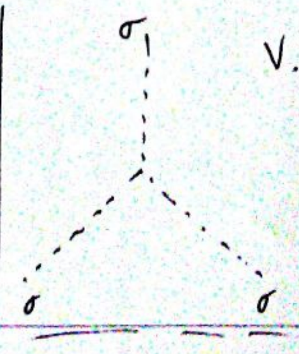


$$V.F_{\pi^4} = -\frac{\lambda}{4}\cdot 4!\, i\lambda = -6i\lambda$$

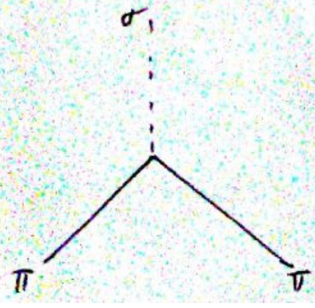
$$V.F_{\sigma^4} = \frac{-\lambda}{4}\cdot 4!\, i\lambda = -6i\lambda$$



$$VF_{\sigma^2\pi^2} = -\frac{\lambda}{2} i\, 2!\,2! = -2i\lambda$$

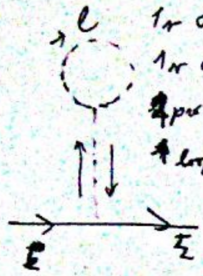


$$V.F_{\sigma^3} = -\lambda v i\, 3! = -6i\lambda v$$



$$V.F_{\pi^2\sigma} = -\lambda v i\, 2! = -2i\lambda v$$

Calculemos la autoenergía para π, queremos comprobar que π se mantiene sin masa incluso con las [redacted] correcciones de la autoenergía

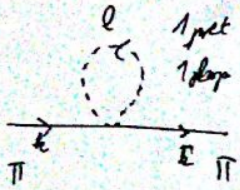


$1v\ \sigma^3$
$1v\ \sigma\pi^2$
$1\ pr\ \sigma$
$1\ loop\ \sigma$

$$i\Sigma_1(k^2) = \frac{(-2i\lambda v)\cdot(-6i\lambda v)}{2}\left(\frac{+i}{\cancel{k^2}-2\mu^2}\right)\int\frac{d^dl}{(2\pi)^4}\left(\frac{+i}{l^2-2\mu^2}\right)$$

factor del loop

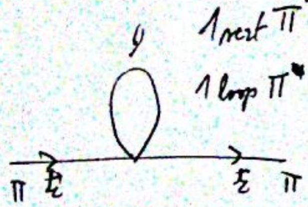
[redacted]



$1\ vert\ \pi^2\sigma^2$
$1\ loop\ \sigma$

$$i\Sigma_2'(k^2) = \frac{(-2i\lambda)}{2}\int\frac{d^dl}{(2\pi)^4}\left(\frac{+i}{l^2-2\mu^2}\right)$$

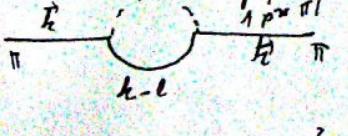
loop



$1\ vert\ \pi^4$
$1\ loop\ \pi^4$

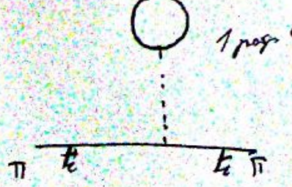
$$i\Sigma_3'(k^2) = \frac{(-6i\lambda)}{2}\int\frac{d^dl}{(2\pi)^4}\left(\frac{+i}{l^2}\right)$$

loop



$2\ vert\ \pi^2\sigma$
$1\ prop\ \sigma$ del loop
$1\ prop\ \pi$

$$i\Sigma_4'(k^2) = (-2i\lambda v)^2\int\frac{d^dl}{(2\pi)^4}\left(\frac{+i}{l^2-2\mu^2}\right)\left(\frac{+i}{(k-l)^2}\right)$$



$2\ vert\ \pi^2\sigma$
$1\ prop\ \sigma$

$$i\Sigma_5(k^2) = \frac{(-2i\lambda v)^2}{2}\left(\frac{+i}{\cancel{k^2}-2\mu^2}\right)\int\frac{d^dl}{(2\pi)^4}\left(\frac{+i}{l^2}\right)$$

Luego

$$i\Sigma_1'(0) = -3\frac{\lambda^2 v^2}{\mu^2}\int\frac{d^d l}{(2\pi)^4}\left(\frac{1}{l^2-2\mu^2}\right) = -3\lambda\int\frac{d^d l}{(2\pi)^4}\left(\frac{1}{l^2-2\mu^2}\right)$$

$$i\Sigma_2'(0) = \lambda\int\frac{d^d l}{(2\pi)^4}\left(\frac{1}{l^2-2\mu^2}\right)$$

$$i\Sigma_3(0) = 3\lambda\int\frac{d^d l}{(2\pi)^4}\left(\frac{1}{l^2}\right)$$

$$i\Sigma_4'(0) = 4\frac{\lambda^2 v^2}{2\mu^2}\int\frac{d^d l}{(2\pi)^4}\left(\left(\frac{1}{l^2-2\mu^2}\right)-\left(\frac{1}{l^2}\right)\right)$$

$$= 2\lambda\int\frac{d^d l}{(2\pi)^4}\left(\frac{1}{l^2-2\mu^2}\right) - 2\lambda\int\frac{d^d l}{(2\pi)^4}\left(\frac{1}{l^2}\right)$$

$$i\Sigma_5(0) = -\frac{\lambda^2 v^2}{\mu^2}\int\frac{d^d l}{(2\pi)^4}\left(\frac{1}{l^2}\right) = -\lambda\int\frac{d^d l}{(2\pi)^4}\left(\frac{1}{l^2}\right)$$

$$\Rightarrow \Sigma(0) = \Sigma_1(0) + \Sigma_2(0) + \Sigma_3(0) + \Sigma_4(0) + \Sigma_5(0)$$

$$= 0$$

$$\Rightarrow m_R = \Sigma(0) = 0$$

Problema 2

$$i\Pi(k^2) = \frac{1}{2}(ig)^2\left(\frac{1}{i}\right)^2 \int \frac{d^d\ell}{(2\pi)^d}\, \tilde{\Delta}((\ell+k)^2)\,\tilde{\Delta}(\ell^2)$$
$$- i\,(Ak^2 + Bm^2) + O(g^4)$$

La autoenergía ha de cumplir con

$$\left.\begin{array}{l} \Pi(-m^2) = 0 \\ \Pi'(-m^2) = 0 \end{array}\right\}$$ Podemos usar esto para fijar A y B

Empezamos con

$$\Pi(k^2) = \frac{1}{2} g^2 I(k^2) - Ak^2 - Bm^2 + O(g^3)$$

Con $I(k^2) = \int_0^1 dx \int \frac{d^dq}{(2\pi)^d} \frac{1}{(q^2+D)^2}$; $D \equiv x(1-x)k^2 + m^2$

$$\Rightarrow \Pi(k^2) = \frac{1}{2}\alpha \int_0^1 dx\, D \ln\left(\frac{D}{m^2}\right)$$
$$- \left\{\frac{1}{6}\alpha\left[\frac{1}{\varepsilon} + \ln\left(\frac{\mu}{m}\right) + \frac{1}{2}\right] + A\right\} k^2$$
$$- \left\{\alpha\left[\frac{1}{\varepsilon} + \ln\left(\frac{\mu}{m}\right) + \frac{1}{2}\right] + B\right\} m^2 + O(\alpha^2)$$

def: $\alpha = O(g^2)$

$$\boxed{\alpha = \frac{g^2}{(4\pi)^3}}$$

Torniamo

$$A = -\frac{1}{6}\alpha\left[\frac{1}{3} + \ln\left(\frac{\mu}{m}\right) + \frac{1}{2} + K_A\right] + \mathcal{O}(\alpha^2)$$

$$B = -\alpha\left[\frac{1}{\epsilon} + \ln\left(\frac{\mu}{m}\right) + \frac{1}{2} + K_B\right] + \mathcal{O}(\alpha^2)$$

$$\Rightarrow \Pi(k^2) = \frac{\alpha}{2}\int_0^1 dx\, D \ln\left(\frac{D}{m^2}\right) + \alpha\left(\frac{1}{6}K_A k^2 + K_B m^2\right) + \mathcal{O}(\alpha^2)$$

Impongono $\Pi(-m^2) = 0$

$$\boxed{\Pi' = \frac{\partial \Pi}{\partial k^2}}$$

$$\Rightarrow \Pi(-m^2) = \frac{\alpha}{2}\int_0^1 dx\,(1-x(1-x))m^2 \ln\{1-x(1-x)\} + \alpha m^2\left(-\frac{1}{6}K_A + K_B\right) + \mathcal{O}(\alpha^2)$$

$$= 0$$

$$\Rightarrow K_B = \frac{K_A}{6} - \frac{1}{2}\int_0^1 dx\,(1-x(1-x))\ln\{1-x(1-x)\}$$

$$\boxed{\int_0^1 dx \ln(1-x(1-x)) = \frac{\pi}{\sqrt{3}} - 2}$$

$$\boxed{\int_0^1 dx\,(x^2-x)\ln(1-x+x^2) = \frac{51}{54} - \frac{\pi\sqrt{3}}{6}}$$

$$\frac{\partial}{\partial k^2}\Pi(k^2) = \frac{\partial}{\partial k^2}\left[\frac{\alpha}{2}\int_0^1 dx\, D\ln\left(\frac{D}{m^2}\right) + \alpha\left(\frac{1}{6}K_A k^2 + K_B m^2\right) + O(\alpha^2)\right]$$

$$= \frac{\alpha}{2}\int_0^1 dx\left(\frac{\partial D}{\partial k^2}\cdot\ln\left(\frac{D}{m^2}\right) + D\frac{\partial}{\partial D}\left(\ln\left(\frac{D}{m^2}\right)\right)\cdot\frac{\partial D}{\partial k^2}\right)$$

$$+ \alpha\frac{1}{6}K_A + O(\alpha^2)$$

$$= \frac{\alpha}{2}\int_0^1 dx\left((x(1-x))\cdot\ln\left(\frac{x(1-x)k^2+m^2}{m^2}\right) + \frac{\cancel{D}}{\cancel{D}}\cdot\frac{\partial D}{\partial k^2}\right)$$

$$+ \frac{\alpha}{6}K_A$$

$$= \frac{\alpha}{2}\int_0^1 dx\left[(x(1-x))\ln\left(\frac{x(1-x)k^2+m^2}{m^2}\right) + x(1-x)\right] + \frac{\alpha}{6}K_A$$

Luego

$$\Pi'(m^2) = \frac{\alpha}{2}\int_0^1 dx\left[(x(1-x))\ln\left(1-x(1-x)\right) + \overbrace{x(1-x)}^{1/6}\right] + \frac{\alpha}{6}K_A$$

$$= \alpha\left(\frac{1}{12} + \frac{K_A}{6} + \frac{1}{2}\int_0^1 x(1-x)\ln(1-x(1-x))\right) = 0$$

$$K_A = 6\left(-\frac{1}{12} - \frac{1}{2}\int_0^1 x(1-x)\ln\left(1 - x(1-x)\right)\right)$$

$$= -\frac{1}{2} - 3\int_0^1 x(1-x)\ln\left(1-x(1-x)\right)$$

$$= -\frac{1}{2} - 3\left(\frac{51}{54} - \frac{\pi\sqrt{3}}{6}\right) \simeq -0.3874$$

$$K_B \approx -\frac{0.3874}{6} - \frac{1}{2}\left(\left(\frac{\pi}{\sqrt{3}} - 2\right) - \left(\frac{51}{54} - \frac{\pi\sqrt{3}}{6}\right)\right)$$

$$= \boxed{0.009767}$$