Pauta Auxiliar 8

P1) Usando que $\vec{F} = -\nabla V$, se obtiene

$$\vec{F} = -\frac{\partial}{\partial x}\hat{\imath} - \frac{\partial}{\partial z}\hat{k} = -2C\hat{\imath} - C\hat{k} = -C(2\hat{\imath} + 1\hat{k})$$

Por equilibrio en $\hat{k}$: $\vec{N} + \vec{F}\cdot\hat{k} + \vec{p} = 0$

$$\vec{N} - C\hat{k} - mg\hat{k} = 0$$

$$\vec{N} = (mg + C)\hat{k}$$

Además, $\vec{F_r} = -\mu N \hat{\imath} = -\mu(mg+C)\hat{\imath}$.

Calculamos la energía inicial:

$$E_0 = \frac{1}{2} m v_0^2$$

y la final es nula, dado que se detiene:

$$E_f = 0.$$

Además, por integral de trabajo:

$$W = \int_\Gamma \vec{F}(\vec{r})\cdot d\vec{r} \quad , \quad \text{pero } d\vec{r} = dr\cdot\hat{t}.$$

En este caso: $dr = dx$, $\hat{t} = \hat{\imath}$

$$\therefore \quad W = \int_{x_0}^{x_f} \vec{F}((x),(y))\cdot\hat{\imath}\, dx$$

$$= \int_{x_0}^{x_f} [-\mu(mg+C) - 2C]\, dx$$

$$= -(x_f - x_0)(\mu(mg+C) + 2C)$$

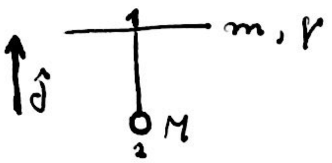
Este trabajo es negativo porque le quita energía al cuerpo hasta detenerlo.

Por último, $W = \Delta E$

$$\therefore \quad -d\left(\mu(mg+c)+2C\right) = -\frac{1}{2} m v_0^2$$

$$\longrightarrow \quad d = \frac{m v_0^2}{2\mu(mg+c)+4C}$$

P2



$$\Sigma F_1 = \gamma v^2 - T - mg = m \cdot \dot{v} \qquad (1)$$

$$\Sigma F_2 = T - Mg = M\dot{v} \qquad (2)$$

$$(2) \longrightarrow T = M(\dot{v} + g)$$

Reemplazando en (1): $\gamma v^2 - M\dot{v} - Mg - mg = m\dot{v}$

$$\longrightarrow \gamma v^2 - (M+m)g = (M+m)\dot{v}$$

$$\Longrightarrow \frac{\gamma v^2}{(M+m)} - g = \dot{v} \rightarrow \frac{\dot{v}}{\frac{\gamma v^2}{M+m} - g} = 1 \qquad \text{pero} \quad \dot{v} = \frac{dv}{dt}$$

$$\Longrightarrow \frac{dv}{\frac{\gamma v^2}{M+m} - g} = dt$$

buscamos la estructura $\frac{du}{1-u^2}$

$$\longrightarrow \frac{-dv}{g - \frac{\gamma}{M+m} v^2} = dt$$

$$\longrightarrow -\frac{dv}{g\left(1 - \frac{\gamma}{(M+m)g} v^2\right)} = dt$$

$$\longrightarrow \frac{dv}{1 - \frac{\gamma}{(M+m)g} v^2} = -g\, dt$$

Haciendo cambio de variable: $u^2 = \frac{\gamma}{(M+m)g} v^2$

$$\cancel{2} u\, du = \frac{\gamma}{(M+m)g} \cancel{2} v\, dv = \frac{\gamma}{(M+m)g} \cdot \sqrt{\frac{(M+m)g}{\gamma}}\, u\, dv$$

$$\rightarrow du = \sqrt{\frac{\gamma}{(M+m)g}}\, dv$$

$$\therefore dv = \sqrt{\frac{(M+m)g}{\gamma}}\, du$$

$$\longrightarrow \sqrt{\frac{(M+m)g}{\gamma}} \frac{du}{1-u^2} = -g\, dt \quad \int_0^u \int_0^t$$

$$\sqrt{\frac{(M+m)g}{\gamma}} \cdot \frac{1}{2} \ln\left(\frac{1+u}{1-u}\right) = -gt$$

$$\ln\left(\frac{1+u}{1-u}\right) = -2\sqrt{\frac{\gamma g}{M+m}}\, t \;\Big/ \exp$$

$$1+u = (1-u)\, e^{-2\sqrt{\frac{\gamma g}{M+m}}\, t}$$

$$u\left(1 + e^{-2\sqrt{\frac{\gamma g}{M+m}}\, t}\right) = e^{-2\sqrt{\frac{\gamma g}{M+m}}\, t} - 1$$

$$u = \frac{e^{-2\sqrt{\frac{\gamma g}{M+m}}\, t} - 1}{e^{-2\sqrt{\frac{\gamma g}{M+m}}\, t} + 1} \longrightarrow v = \sqrt{\frac{(M+m)g}{\gamma}} \frac{e^{-2\sqrt{\frac{\gamma g}{M+m}}\, t} - 1}{e^{-2\sqrt{\frac{\gamma g}{M+m}}\, t} + 1}$$

Esta expresión para $t \to \infty$ se hace $-\sqrt{\frac{(M+m)g}{r}}$

P3)

a) $\vec{F_g} = -\frac{GMm}{r^2}\hat{r}$

Usando que $\vec{F} = -\nabla V$, tenemos:

$$-\frac{GMm}{r^2}\hat{r} = -\frac{\partial V}{\partial r}\hat{r}$$

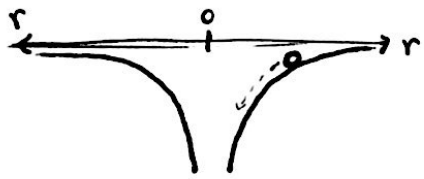
$$\longrightarrow \frac{-GMm}{r^2}dr = -dV \qquad \int_\infty^r \int_0^V$$

$$\longrightarrow -GMm\left(-\frac{1}{r}\right)\Big|_\infty^r = -V$$

$$\longrightarrow -GMm\left(\frac{-1}{r} + \cancel{\frac{1}{\infty}}^{0}\right) = -V$$

$$\longrightarrow -V = \frac{GMm}{r} \longrightarrow V = \frac{-GMm}{r}$$

Si lo dibujamos, queda:



b) Como $\vec{v} \perp \vec{r}$, $l = m \cdot r \cdot v$.

$\therefore$ $l_0 = m(R+h)\cdot v_0$ & $l_f = mRv_f$

pero como las fuerzas centrales son conservativas: $l_0 = l_f$

$$\longrightarrow v_f = \left(\frac{R+h}{R}\right)v_0$$

c) Por energía:

$$E_0 = \frac{1}{2} m v_0^2 + V(R+h) = \frac{1}{2} m v_0^2 + \left(\frac{-GMm}{R+h}\right)$$

$$E_f = \frac{1}{2} m v_f^2 + V(R) = \frac{1}{2} m \left(\frac{R+h}{R}\right)^2 v_0^2 + \left(\frac{-GMm}{R}\right)$$

Como el campo es conservativo:

$$\frac{1}{2} \cancel{m} v_0^2 - \frac{GM\cancel{m}}{R+h} = \frac{1}{2} \cancel{m} \left(\frac{R+h}{R}\right)^2 v_0^2 - \frac{GM\cancel{m}}{R}$$

$$\frac{v_0^2}{2}\left(1 - \left(\frac{R+h}{R}\right)^2\right) = -GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\frac{v_0^2}{2}\left(\frac{R^2 - R^2 - 2Rh - h^2}{R^2}\right) = GM\left(\frac{R-R-h}{R(R+h)}\right)$$

$$-\frac{v_0^2}{2}\left(\frac{2R\cancel{h} + h^{\cancel{2}}}{R^{\cancel{2}}}\right) = -GM\left(\frac{\cancel{h}}{\cancel{R}(R+h)}\right)$$

$$+ v_0^2\left(\frac{2R+h}{2R}\right) = \frac{+GM}{R+h}$$

$$\therefore \quad M = \frac{(R+h)v_0^2}{G}\left(1 + \frac{h}{2R}\right)$$

d) $$v_0^2 = \frac{GM}{(R+h)} \cdot \frac{1}{1 + \cancel{\frac{h}{2R}}^{\to 0}}$$ si $h \to 0$ tenemos

$$v_0^2 = \frac{GM}{R} \longrightarrow v_0 = \sqrt{\frac{GM}{R}}$$

e) $$\vec{F} = -\frac{GMm}{r^2}\hat{r} = m \cdot \vec{a} \longrightarrow \frac{-GMm}{r^2} = -\frac{mv^2}{r}$$ / Pero $r = R$

$$\longrightarrow \frac{+GM\cancel{m}}{R^{\cancel{2}}} = \frac{+\cancel{m}v^2}{\cancel{R}} \longrightarrow v = \sqrt{\frac{GM}{R}}$$

Donde se han usado coordenadas polares para r constante y $\dot{\theta}$ constante.

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