Integral Ej. #3.

Método del cambio de variable

$$\ddot{y} = -g - \frac{(\gamma_1 + \gamma_2)}{m_1+m_2}\dot{y}$$

- utilizamos $v = \dot{y} \rightarrow \dot{v} = \ddot{y}$

$$\dot{v} = \left(-g - \frac{(\gamma_1+\gamma_2)}{m_1+m_2} v\right)$$

$$\frac{dv}{dt} = -g - \frac{\gamma_1+\gamma_2}{m_1+m_2} v$$

$$\frac{-dv}{g + \frac{\gamma_1+\gamma_2}{m_1+m_2}\cdot v} = dt$$

Cambio de variable:

$$u = g + \frac{\gamma_1+\gamma_2}{m_1+m_2} v$$

$$du = \frac{\gamma_1+\gamma_2}{m_1+m_2} dv$$

$$\hookrightarrow dv = \frac{m_1+m_2}{\gamma_1+\gamma_2} du$$

$$-\left(\frac{m_1+m_2}{\gamma_1+\gamma_2}\right)\frac{du}{u} = dt \int_g^u \int_0^t$$

- integramos CUIDANDO los límites

$$\ln(u) - \ln(g) = -t\left(\frac{\gamma_1+\gamma_2}{m_1+m_2}\right)$$

$$\ln\left(\frac{u}{g}\right) = -t\left(\frac{\gamma_1+\gamma_2}{m_1+m_2}\right) \quad \left| \frac{u}{g} = 1 + \frac{\gamma_1+\gamma_2}{g(m_1+m_2)} v \right.$$

$$\ln\left(1 + \frac{\gamma_1+\gamma_2}{g(m_1+m_2)} v\right) = -t\left(\frac{\gamma_1+\gamma_2}{m_1+m_2}\right) \quad / \exp(\ )$$

$$1 + v\left(\frac{\gamma_1+\gamma_2}{g(m_1+m_2)}\right) = e^{-t\left(\frac{\gamma_1+\gamma_2}{m_1+m_2}\right)}$$

$$v = \frac{g(m_1+m_2)}{\gamma_1+\gamma_2}\left(e^{-t\left(\frac{\gamma_1+\gamma_2}{m_1+m_2}\right)} - 1\right)$$