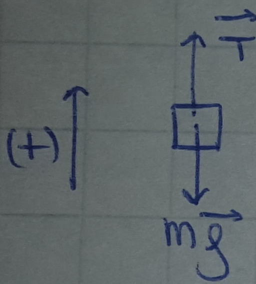
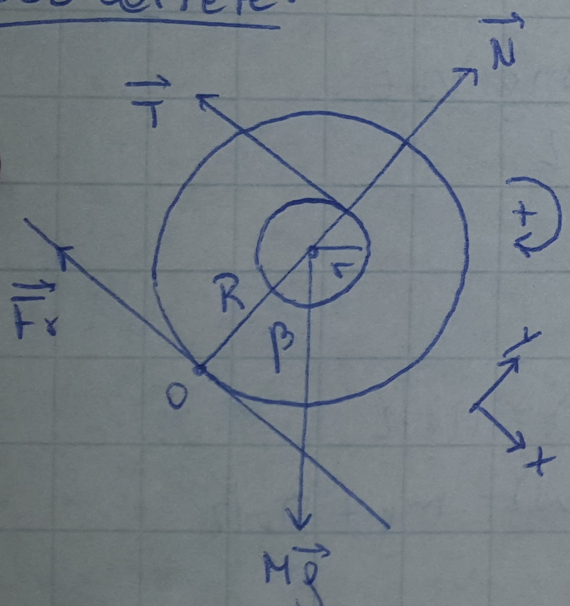
P1:

DCL masa:



$$\sum \vec{F} = m\vec{a} \begin{cases} F_x: 0 = 0 & (1) \\ F_y: T - mg = ma & (2) \end{cases}$$

DCL correte:



$$\sum \vec{F} = M\cdot\vec{a_c} \begin{cases} F_x: -T - F_r + Mg\cdot\sin\beta = M a_c & (3) \\ F_y: N - Mg\cdot\cos\beta = 0 & (4) \end{cases}$$

$$\sum \vec{\tau_o} = I_o\alpha : MgR\cdot\sin\beta - T(R+r) = I_o\alpha = \overbrace{(I + MR^2)}^{\text{Steiner}}\alpha \quad (5)$$

a- aceleración de m:

de ②: $a = \frac{T - mg}{m}$; de ⑤: $T = \frac{MgR\sin\beta - (I + MR^2)\alpha}{R + r} \Rightarrow a = \frac{MgR\sin\beta - (I + MR^2)\alpha - (R+r)mg}{m(R+r)}$

para α: el carrete se desplaza $\Delta\theta \Rightarrow \begin{cases} \text{el CM avanza } \Delta x = R\Delta\theta \\ \text{el hilo se enrolla } \Delta l = r\Delta\theta \end{cases}$ cond. de no resbalamiento

$\Rightarrow \Delta y = R\Delta\theta + r\Delta\theta = (R+r)\Delta\theta \Rightarrow a = (R+r)\alpha$

sustituyendo: $a = \dfrac{MgR\sin\beta - \frac{(I + MR^2)\cdot a}{R+r} - (R+r)mg}{m(R+r)} \Rightarrow a = \dfrac{MgR\sin\beta - (R+r)mg}{m(R+r) + \frac{I + MR^2}{R+r}}$

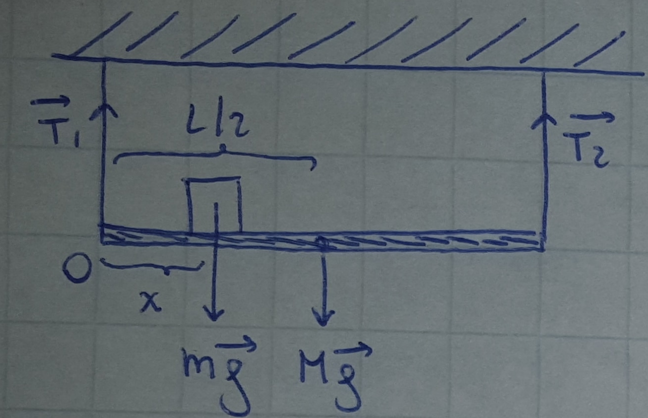
b- relación de rodadura deseable entre m y M:

$a > 0 \Rightarrow \dfrac{MgR\sin\beta - (R+r)mg}{m(R+r) + \frac{I+MR^2}{R+r}} > 0 \Rightarrow M\cancel{g}R\sin\beta - (R+r)m\cancel{g} > 0$

$\Leftrightarrow \frac{M}{m} > \frac{R+r}{R\sin\beta}$

P2:

a-



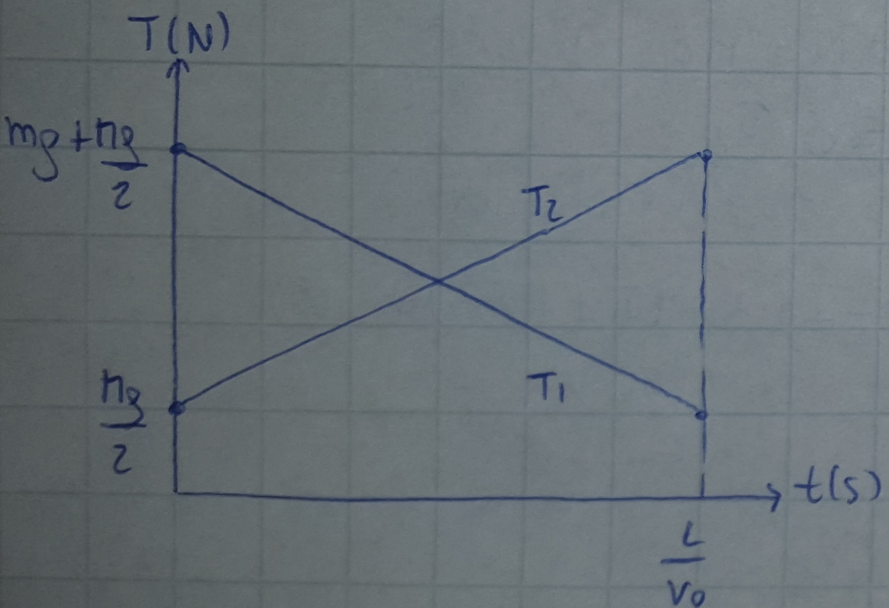
$$\sum \vec{F} = \vec{0} \begin{cases} F_x : 0 = 0 & ① \\ F_y : T_1 + T_2 - mg - Mg = 0 & ② \end{cases}$$

$$\sum \vec{\tau}_o = \vec{0} : -mgx - Mg\frac{L}{2} + T_2 L = 0 \quad ③$$

de ③ : $T_2 = \frac{Mg}{2} + \frac{mgx}{L}$; $x = V_0 t \Rightarrow T_2 = \frac{Mg}{2} + \frac{mg}{L} V_0 t$

de ② $T_1 = mg + Mg - T_2 = mg + Mg - \frac{Mg}{2} - \frac{mg}{2} V_0 t$

$\Rightarrow T_1 = mg + \frac{Mg}{2} - \frac{mg}{L} V_0 t.$



$T_1(t=0) = mg + Mg/2$; $T_2(t=0) = Mg/2$

$T_1(t = L/V_0) = Mg/2$; $T_2(t = L/V_0) = mg + Mg/2.$

b- $T_2 = \frac{M}{2}g + mg\frac{x}{l} \Rightarrow x = \left(\frac{T_{máx}}{mg} - \frac{M}{2m}\right)l$ 1.

$$\frac{T_{máx}}{mg} = \frac{3,2\pm0,1}{0,1\pm0,005}\cdot\frac{1}{9,8} = \frac{1}{9,8}\left(\frac{3,2}{0,1} \pm \frac{3,2}{0,1}\sqrt{\left(\frac{0,1}{3,2}\right)^2+\left(\frac{0,005}{0,1}\right)^2}\right) = \frac{1}{9,8}(32\pm1,9) = 3,3\pm0,2$$

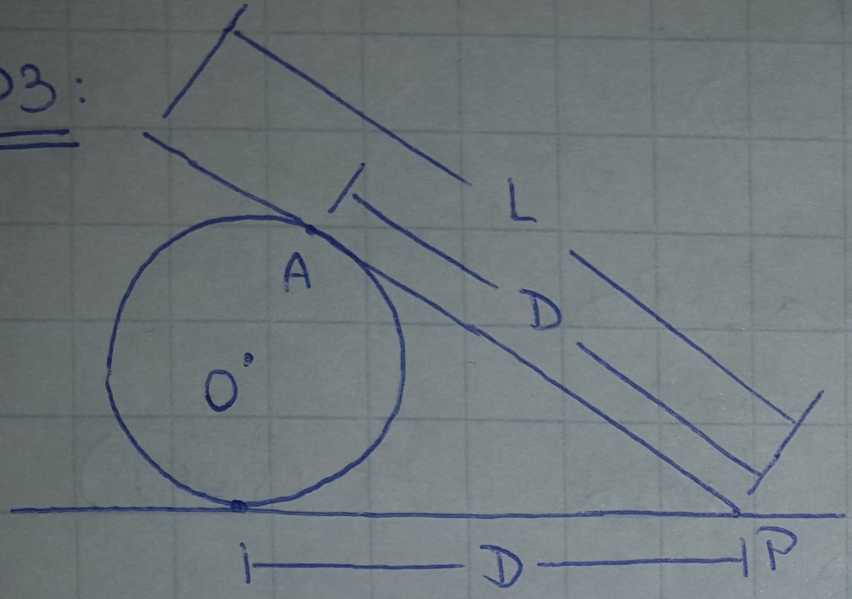
$$\frac{M}{2m} = \frac{0,5}{2}\left(\frac{1}{0,1\pm0,005}\right) = \frac{0,5}{2}\left(\frac{1}{0,1}\pm\frac{1}{0,1}\sqrt{\left(\frac{0}{1}\right)^2+\left(\frac{0,005}{0,1}\right)^2}\right) = 0,25(10\pm0,5) = 2,5\pm0,125$$

$$\Rightarrow x = (3,3\pm0,2)-(2,5\pm0,1) = (3,3-2,5)\pm\sqrt{0,2^2+0,1^2} = 0,8\pm0,2.$$

Luego, por distribución gaussiana, el 95% de los datos está en $[\{x\}-2\sigma, \{x\}+2\sigma]$.
Luego, $x_{max} \leq \{x\}-2\sigma$ evita que la cuerda se corte con un 95% de probabilidad.

así, $x_{máx} = 0,8 - 2\cdot0,2 = 0,4$.

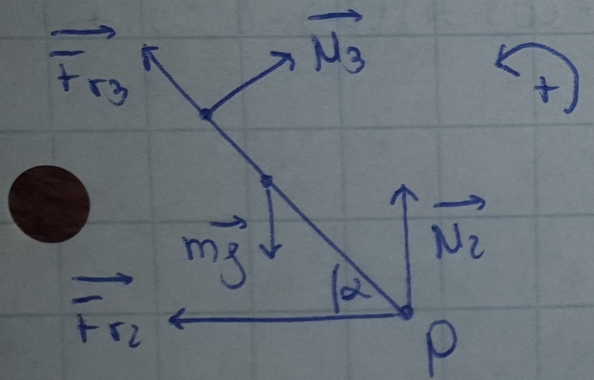
P3:



por el teorema de los tangentes exteriores,

$\overline{AP} = D$.

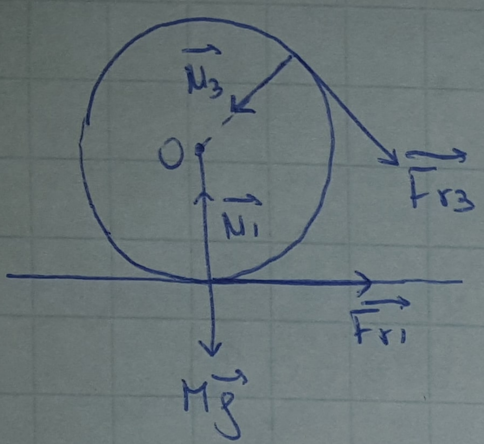
DCL barra:



$$\sum \vec{F} = \vec{0} \begin{cases} \overline{F}_x : -\overline{F}_{r2} + N_3 \cdot \sin\alpha - \overline{F}_{r3} \cdot \cos\alpha = 0 & (1) \\ \overline{F}_y : N_2 - mg + N_3 \cdot \cos\alpha + F_{r3} \cdot \sin\alpha = 0 & (2) \end{cases}$$

$$\sum \vec{\tau}_P = \vec{0} : mg \cdot \frac{L}{2} \cos\alpha - N_3 \cdot D = 0 \quad (3)$$

DCL cilindro:



$$\sum \vec{F} = \vec{0} : \begin{cases} F_x : F_{r1} - N_3 \cdot \sin\alpha + F_{r3} \cdot \cos\alpha = 0 & (4) \\ F_y : N_1 - Mg - N_3 \cdot \cos\alpha - F_{r3} \cdot \sin\alpha = 0 & (5) \end{cases}$$

$$\sum \vec{\tau}_0 = \vec{0} : -F_{r3} R + F_{r1} R = 0 \quad (6)$$

de (6): $F_{r1} = F_{r3}$

de (4): $F_{r1}(1+\cos\alpha) = N_3 \cdot \sin\alpha$

de (3): $N_3 = \dfrac{mgL}{2D}\cos\alpha$

$$F_{r1} = F_{r3} = \frac{mgL\cos\alpha \cdot \sin\alpha}{2D(1+\cos\alpha)}$$

de (1): $F_{r2} = N_3 \cdot \sin\alpha - F_{r3} \cdot \cos\alpha = \dfrac{mgL}{2D}\cos\alpha\sin\alpha - \dfrac{mgL\cos^2\alpha\sin\alpha}{2D(1+\cos\alpha)}$

$$F_{r2} = \frac{mgL\cos\alpha\sin\alpha(1+\cos\alpha) - mgL\cos^2\alpha\sin\alpha}{2D(1+\cos\alpha)} = \frac{mgL\cos\alpha\sin\alpha}{2D(1+\cos\alpha)}(1+\cancel{\cos\alpha}-\cancel{\cos\alpha})$$

$F_{r1} \le \mu_1 N_1$:

de (5): $N_1 = Mg + N_3\cos\alpha + F_{r3}\sin\alpha = Mg + \dfrac{mgL\cos^2\alpha}{2D} + \dfrac{mgL\cos\alpha \cdot \sin^2\alpha}{2D(1+\cos\alpha)}$

$$N_1 = Mg + \frac{mgL\cos\alpha}{2D(1+\cos\alpha)}\left(\cos\alpha(1+\cos\alpha) + \sin^2\alpha\right) = Mg + \frac{mgL\cos\alpha}{2D\cancel{(1+\cos\alpha)}}\cancel{\left(\cos\alpha + \cos^2\alpha + \sin^2\alpha\right)}$$

$$N_1 = \frac{2MgD + mgL \cdot \cos\alpha}{2D} \Rightarrow \frac{mgL\cos\alpha\sin\alpha}{\cancel{2D}(1+\cos\alpha)} \le \mu_1 \cdot \frac{2MgD + mgL\cos\alpha}{\cancel{2D}}$$

$$\Leftrightarrow \mu_1 \geq \frac{mgL\cos\alpha\sin\alpha}{(1+\cos\alpha)(2MgD + mgL\cos\alpha)} = \frac{mL\cos\alpha\cdot\sin\alpha}{(1+\cos\alpha)(2MD + mL\cos\alpha)}.$$

$F_{r2} \leq \mu_2 N_2$:

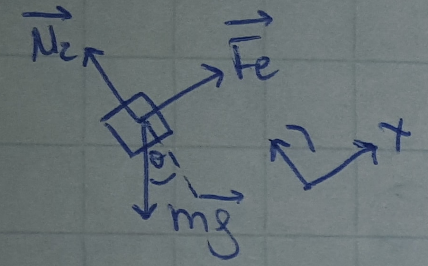
de ②: $$N_2 = mg - N_3\cdot\cos\alpha - F_{r3}\cdot\sin\alpha = mg - \frac{mgL}{2D}\cos^2\alpha - \frac{mgL\cos\alpha\cdot\sin^2\alpha}{2D(1+\cos\alpha)}$$

$$N_2 = mg - \frac{mgL\cos\alpha}{2D(1+\cos\alpha)}\left(\cos\alpha(1+\cos\alpha) + \sin^2\alpha\right) = mg - \frac{mgL\cos\alpha}{2D\cancel{(1+\cos\alpha)}}\left(\cancel{\cos\alpha + \cos^2\alpha + \sin^2\alpha}\right)$$

$$N_2 = \frac{mg}{2D}(2D - L\cos\alpha) \Rightarrow \frac{\cancel{mg}L\cos\alpha\sin\alpha}{\cancel{2D}(1+\cos\alpha)} \leq \mu_2\frac{\cancel{mg}}{\cancel{2D}}(2D - L\cos\alpha) \Rightarrow \mu_2 \geq \frac{L\cos\alpha\cdot\sin\alpha}{(1+\cos\alpha)(2D - L\cos\alpha)}$$

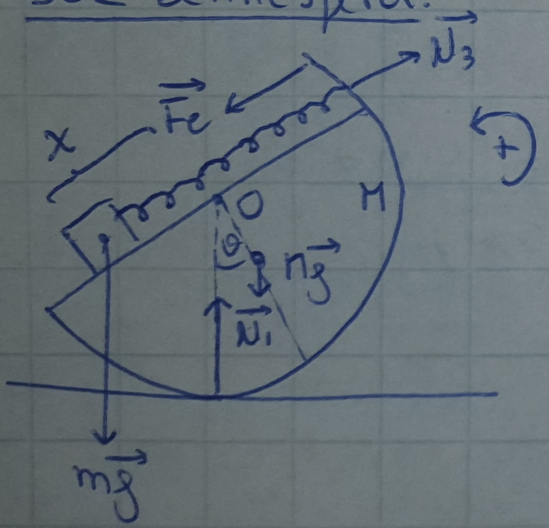
P4:

DCL masa:



$$\sum \vec{F} = \vec{0} \begin{cases} F_x: kx - mg \cdot \sin\theta = 0 & (1) \\ F_y: N_2 - mg \cdot \cos\theta = 0 & (2) \end{cases}$$

DCL semiesfera:



$$\sum \vec{F} = \vec{0} \begin{cases} F_x: -kx \cdot \sin\theta + N_3 \cdot \sin\theta = 0 & (3) \\ F_y: -mg + N_1 - Mg - kx \cdot \cos\theta + N_3 \cdot \cos\theta = 0 & (4) \end{cases}$$

$$\sum \vec{\tau_o} = \vec{0} : -dMg \cdot \sin\theta + x\, mg \cdot \cos\theta = 0 \quad (5)$$

a- ángulo de inclinación: de (1): $x = \frac{mg}{k} \sin\theta$

de ⑤: $-dMg\sin\theta + \frac{mg}{k}\sin\theta \cdot mg \cdot \cos\theta = 0 \Rightarrow \cos\theta = \frac{dMk}{m^2 g} \Rightarrow \theta = \arccos\left(\frac{dMk}{m^2g}\right)$

b- estiramiento del resorte: de ①: $x = \frac{mg}{k}\sin\theta$; $\sin\theta = \sqrt{1-\cos^2\theta} \Rightarrow x = \frac{mg}{k}\sqrt{1-\left(\frac{dMk}{m^2g}\right)^2}$

c- relaciones de existencia entre m y M:

$\text{rec}(\cos\theta) = [-1, 1] \Rightarrow \frac{dMk}{m^2 g} \leq 1 \quad \wedge \quad x < R \Rightarrow \frac{mg}{k}\sqrt{1-\left(\frac{dMk}{m^2g}\right)^2} < R.$