# Pauta Auxiliar 1

P1) a) En efecto, sea

$u = ax \wedge v = -ay$

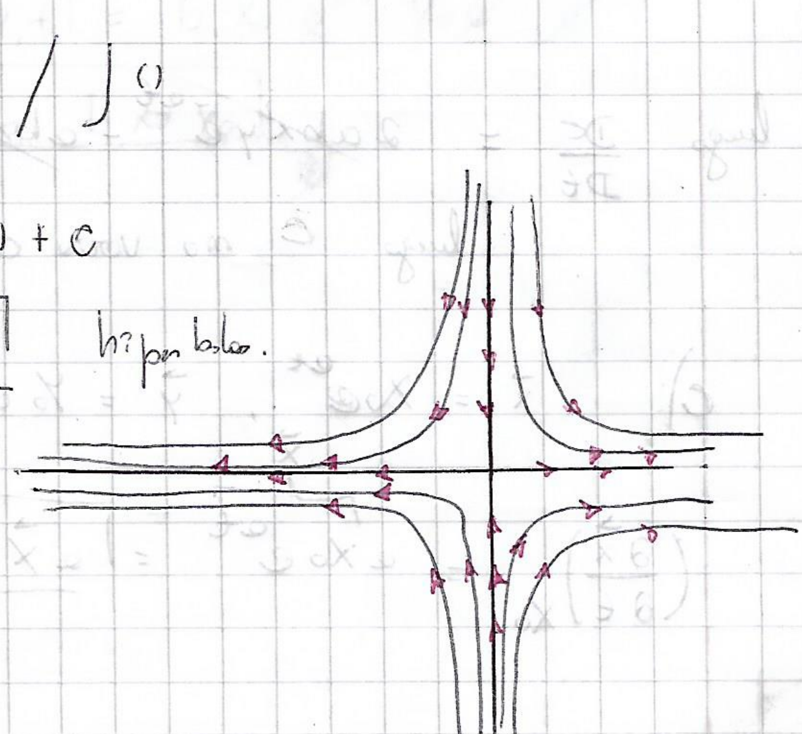
Líneas de Corriente: $\dfrac{dy}{dx} = \dfrac{v}{u}$

$\Rightarrow \quad \dfrac{dy}{dx} = -\dfrac{\cancel{a}y}{\cancel{a}x} = -\dfrac{y}{x}$

$\dfrac{dy}{y} = -\dfrac{dx}{x} \quad / \int ()$

$\ln(y) = -\ln(x) + C$

$\Rightarrow \quad \boxed{x \cdot y = c}$ hipérbola.



b) Sea

$$c(x,y,t) = b x^2 y e^{-at}$$

P. d. q. , $\frac{Dc}{Dt} \equiv 0$

en efecto. $$\frac{Dc}{Dt} = \underbrace{\left(\frac{\partial c}{\partial x}\right)}_{(1)} \overbrace{\left(\frac{\partial x}{\partial t}\right)}^{u} + \underbrace{\left(\frac{\partial c}{\partial y}\right)}_{(2)} \overbrace{\left(\frac{\partial y}{\partial t}\right)}^{v} + \underbrace{\left(\frac{\partial c}{\partial t}\right)}_{(3)}$$

(1) : $2bxye^{-at}$

(2) : $bx^2 e^{-at}$

(3) : $-abx^2ye^{-at}$

luego $$\frac{Dc}{Dt} = \cancel{2abx^2ye^{-at}} - \cancel{abx^2ye^{-at}} - \cancel{abx^2ye^{-at}} = 0$$

luego $c$ no varia con respecto al Tiempo.

c) $\vec{x} = x_0 e^{at}$ , $\vec{y} = y_0 e^{-at}$ .

$$\left(\frac{\partial \vec{x}}{\partial t}\right)_{x_0} = a\overbrace{x_0 e^{at}}^{\vec{x}} = \boxed{a\vec{x} = \vec{u}}$$

$$\left(\frac{\partial \vec{y}}{\partial t}\right)_{y_0} = -a \underbrace{y_0 e^{-at}}_{\vec{y}} = -a\vec{y} = \vec{v}\,.$$

$$\left(\frac{\partial \vec{u}}{\partial t}\right) = a^2 x_0 e^{at} = \boxed{a^2 \vec{x}\,.}$$

$$\left(\frac{D\vec{u}}{Dt}\right) = \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial x}{\partial t}\right) + \cancel{\left(\frac{\partial u}{\partial y}\right)}^{\,0}\left(\frac{\partial y}{\partial t}\right) + \cancel{\left(\frac{\partial u}{\partial t}\right)}^{\,0}$$

$$\frac{D\vec{u}}{Dt} = a \cdot (\vec{u}) = \boxed{a^2 \vec{x}}$$

$$\Rightarrow \left(\frac{\partial \vec{u}}{\partial t}\right) = \frac{D\vec{u}}{Dt}\,.$$

En efecto. $c(x,y,t) = b\, x_0^2 \cancel{e^{2at}}\, y_0 \cancel{e^{-at}}\, \cancel{e^{-at}}$

$$\boxed{c(x_0, y_0) = b\, x_0^2\, y_0}$$

P2) En efecto.

a) $$\frac{dy}{dx} = -\frac{y+t}{x}$$

$$\frac{dy}{-y+t} = \frac{dx}{x} \Big/ \int ()$$

$$-\ln(y+t) = \ln(x) + C$$

$$\Rightarrow \quad C = x(y+t)$$

Condición de borde, $C = 1(1+0)$

$$\Rightarrow C = 1$$

Luego la Stream line $\boxed{1 = x(y+t)}$

b) $$\frac{dx}{dt} = x$$

$$\frac{dx}{x} = dt \Big/ \int \Rightarrow \ln(x) = t + c$$

Luego $\boxed{x(t) = A_1 e^{t}}$ Condición de borde. $\Rightarrow A_1 = 1$

$$\boxed{x(t) = e^{t}}$$

$$\frac{dy}{dt} = -y - t.$$

$$\dot{y} + y = -t$$

$$\mu(t) = e^t.$$

$$\dot{y}e^t + e^t y = -te^t.$$

$$(\dot{ye^t}) = -te^t \Big/ \int dt.$$

$$ye^t = e^t(1-t) + C$$

$$y(t) = (1-t) + \frac{C}{e^t}$$

$$1 = 1 + Ce^{-0} \Rightarrow C = 0$$

$$\boxed{y(t) = (1-t)}$$

$$\boxed{y(x) = 1 - \ln(x)}$$

c) ahora calculamos la shock line.

$$x = c_1 e^{\tau} = 1 \quad \text{me} \quad \tau \geq t_0 \qquad \text{con } t_0 = 0.$$

$$y = (1-\tau) + c_2 e^{-\tau} = 1$$

$$\Rightarrow \boxed{c_1 = e^{-\tau}}, \quad \boxed{c_2 = \tau e^{\tau}}$$

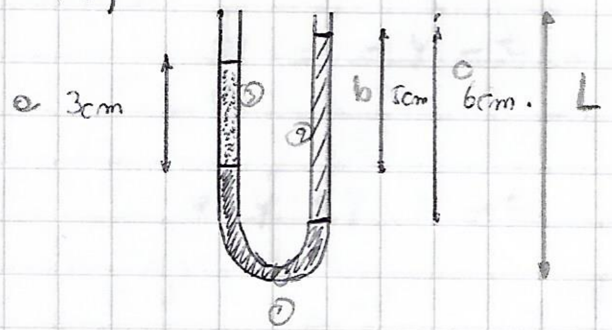
$$x = e^{-\tau} e^{t}, \qquad y = \tau e^{\tau} e^{-t} + (1-t).$$

evaluamos en $t = 0$

$$x = e^{-\tau}, \qquad y = \tau e^{\tau} + 1$$

$$\boxed{y = 1 - \frac{\ln(x)}{x}}$$

P3)



$\rho_1 = 2000 \ kgr/m^3$.

$\rho_2 = 500 \ kgr/m^3$.

En efecto. Sea $L$ el largo Total entonces, si hacemos un balance:

$$c\rho_2 + (L-c)\rho_1 = \rho_3 a + (L-b)\rho_1$$

$$\frac{c\rho_2 - c\rho_1 + b\rho_1}{a} = \rho_3$$

$$\frac{c(\rho_2 - \rho_1) + b\rho_1}{a} = \rho_3$$

luego $\boxed{\rho_3 = 333.\overline{3} \ kgr/m^3}$