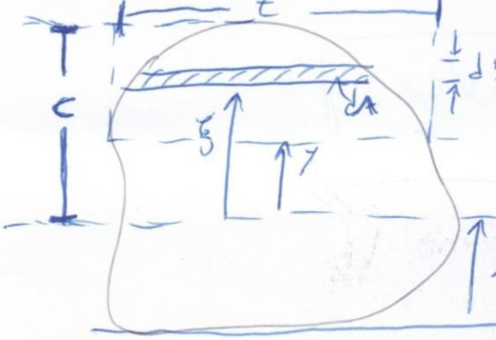
• Esfuerzo de corte en vigas




$$\boxed{\tau_{xy} = \frac{V(x)}{I_z \cdot t}\int_y^c \xi \, dA}$$

$$\boxed{\tau_{xy} = \frac{V(x)}{2 I_z}\left(\frac{h^2}{4} - y^2\right)}$$ (Sec. rectangular)

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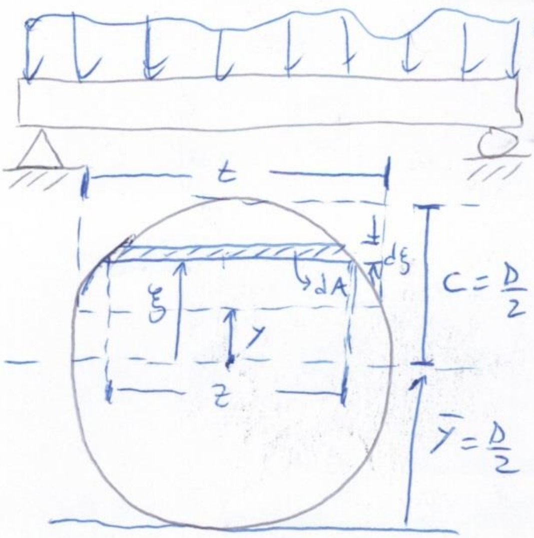
P1





• Por la ec. del círculo

$$\frac{D^2}{4} = \left(\frac{t}{2}\right)^2 + y^2$$

$$\boxed{\therefore \; t = 2\sqrt{\frac{D^2}{4} - y^2}}$$

• El diferencial de área es $dA = z \cdot d\xi$

↳ Por ec. del círculo: $\frac{D^2}{4} = \left(\frac{z}{2}\right)^2 + \xi^2 \Rightarrow z = 2\sqrt{\frac{D^2}{4} - \xi^2}$

$$\boxed{\therefore \; dA = 2\sqrt{\frac{D^2}{4} - \xi^2}\, d\xi}$$

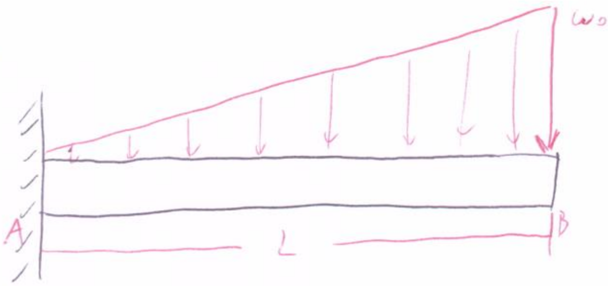
$$\Rightarrow \int_y^c \xi \, dA = \int_y^{D/2} 2\sqrt{\frac{D^2}{4} - \xi^2}\, d\xi = -\int_y^{D/2} \sqrt{v}\, dv \quad \left(\begin{array}{l} v = \frac{D^2}{4} - \xi^2 \\ dv = -2\xi \, d\xi \end{array}\right)$$

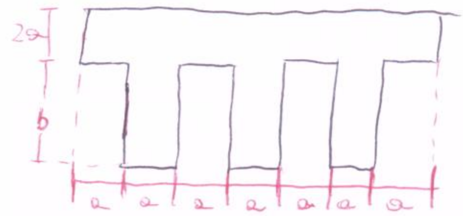
$$= -\frac{2}{3} v^{3/2} = -\frac{2}{3}\left(\frac{D^2}{4} - \xi^2\right)^{3/2} \Bigg|_y^{D/2}$$

$$\boxed{\therefore \int_y^c \xi \, dA = \frac{2}{3}\left(\frac{D^2}{4} - y^2\right)^{3/2}}$$

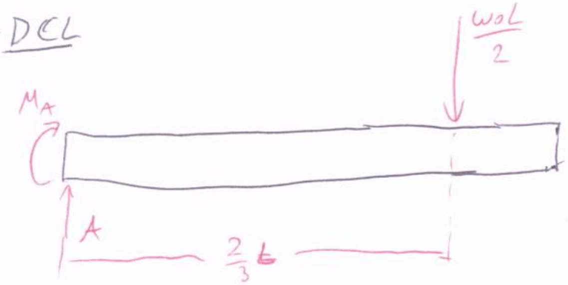
$$\Rightarrow \tau_{xy} = \frac{V(x)}{I_z \cdot t}\int_c^y \xi \, dA = \frac{V(x)}{\frac{\pi \cdot D^4}{64} \cdot 2\sqrt{\frac{D^2}{4} - y^2}} \cdot \frac{2}{3}\left(\frac{D^2}{4} - y^2\right)^{3/2}$$

$$\boxed{\therefore \tau_{xy} = \frac{64}{3\pi} \, \frac{D^2/4 - y^2}{D^4} \cdot V(x)}$$




* DCL

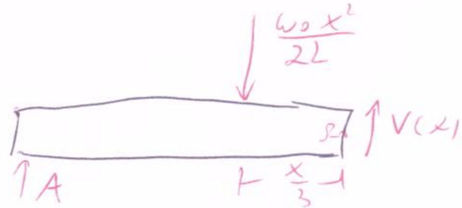



$$\sum_A M_z = 0 \quad -M_A + \frac{w_0 L^2}{3} = 0$$

$$\therefore M_A = -\frac{w_0 L^2}{3}$$

$$\sum F_y = 0 \rightsquigarrow A = \frac{w_0 L}{2}$$

* Corte para calcular V(x):

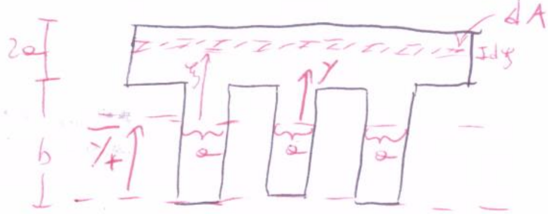

$$\sum F_y = 0 \rightsquigarrow V(x) + A - \frac{w_0 x^2}{2L} = 0$$

$$\therefore V(x) = \frac{w_0 x^2}{2L} - A = \frac{w_0}{2L}(x^2 - L^2)$$

* En $x = L/2 \rightsquigarrow \boxed{V(L/2) = -3750\ N}$

* Cálculo Esf. de corte $\tau_{xy}$:

$$\int_{y}^{c} \xi \, dA$$




* En el eje neutro $\rightarrow$

$$\left.\begin{aligned} y &= 0 \\ c &= 2a+b-\overline{Y}_T \\ t &= 3a \\ &\cancel{dA} \end{aligned}\right\} \quad dA = \begin{cases} 3a\,d\xi & 0<\xi<b-\overline{Y}_T \\ 7a\,d\xi & b-\overline{Y}_T<\xi<2a+b-\overline{Y}_T \end{cases}$$

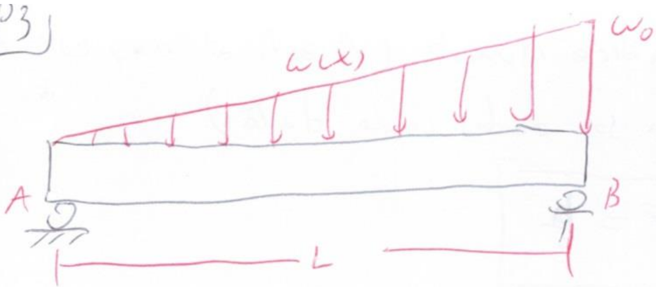
$$\Rightarrow \tau_{xy} = \frac{V(L/2)}{I_T \cdot t}\left( \int_{0}^{b-\overline{Y}_T} 3a\,\xi\,d\xi + \int_{b-\overline{Y}_T}^{2a+b-\overline{Y}_T} 7a\,\xi\,d\xi \right)$$

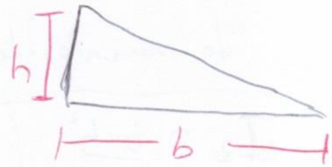
$$= \frac{V(L/2)}{I_T \cdot t}\left( \frac{3a(b-\overline{Y}_T)^2}{2} + \frac{7a}{2}\left( (2a+b-\overline{Y}_T)^2 - (b-\overline{Y}_T)^2 \right) \right)$$

* Calculando $I_T$ y $\overline{Y}_T$ de las fórmulas y reemplazando

$$\boxed{\tau_{xy} = -1,093 \text{ MPa}}$$

P3)

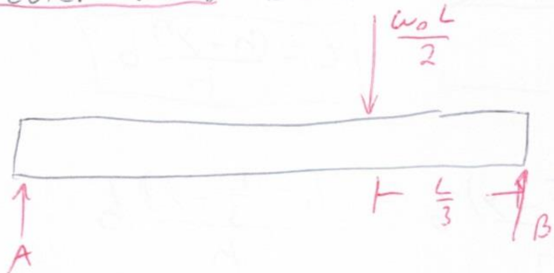



$$\tau_{xy} = \frac{V(x)}{I_z \cdot t} \int_y^c \xi \, dA$$

1-) Calcular $V(x)$
2-) Calcular $I_z$
3-) Calcular $\int_y^c \xi \, dA$

1-) Calcular $V(x)$: - DCL




$\cdot \sum_B M_z = 0$

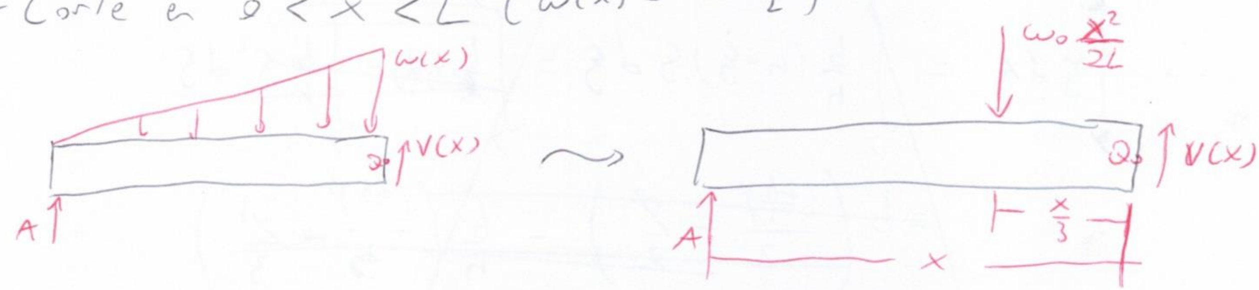
$\rightarrow \frac{L}{3} \cdot \frac{w_0 L}{2} - AL = 0$

$$\boxed{\therefore A = \frac{w_0 L}{6}}$$

$\cdot \sum F_y = 0 \Rightarrow B = \frac{w_0 L}{2} - A = \frac{w_0 L}{2} - \frac{w_0 L}{6} = \frac{w_0 L}{3}$

$$\boxed{B = \frac{w_0 L}{3}}$$

- Corte en $0 < x < L$ $\left( w(x) = w_0 \frac{x}{L} \right)$




$\Rightarrow A - w_0 \frac{x^2}{2L} + V(x) = 0$

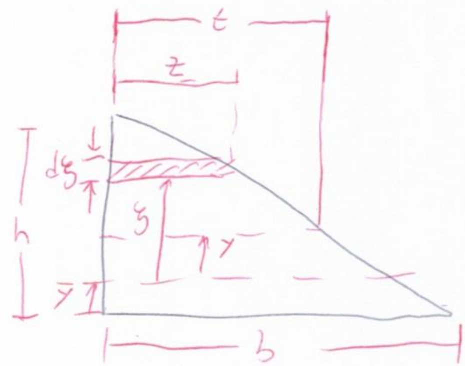
$$\boxed{\therefore V(x) = \frac{w_0}{2L} x^2 - \frac{w_0 L}{6}}$$

2-) Calcular $I_z$ ⇝ Resuelto en Apéndice Aux 3

$$\boxed{I_z = \frac{bh^3}{36} \; ; \; \bar{y} = \frac{h}{3}}$$

3-) Calcular $\int_y^c \xi \, dA$

↳ a) Calcular $t$



$$\rightsquigarrow t = \frac{(h-y')}{h} b \;, \text{ pero } y' = \bar{y} + y \Rightarrow t = \frac{(h - \frac{h}{3} - y)}{h} b$$

$$\boxed{t = \frac{2b}{3} - \frac{b}{h} y}$$

↳ b) Calcular $dA$: $\quad dA = z \cdot d\xi = \left(\frac{2b}{3} - \frac{b}{h}\xi\right) d\xi$

$$\therefore \int_y^c \xi \, dA = \int_y^{2h/3} \left(\frac{2b}{3} - \frac{b}{h}\xi\right) \xi \, d\xi = \frac{2b}{3} \int_y^{2h/3} \xi \, d\xi - \frac{b}{h} \int_y^{2h/3} \xi^2 \, d\xi$$

$$= \frac{b}{3}\left(\left(\frac{2h}{3}\right)^2 - y^2\right) - \frac{b}{3h}\left(\left(\frac{2h}{3}\right)^3 - y^3\right)$$

$$\boxed{\therefore \int_y^c \xi \, dA = \frac{b}{3}\left(\frac{4}{27}h^2 + \frac{y^3}{h} - y^2\right)}$$

$$\boxed{\therefore \tau_{xy} = \frac{w_0\left(\frac{x^2}{2L} - \frac{L}{6}\right)}{\left(\frac{bh^3}{36}\right)\left(\frac{2b}{3} - \frac{b}{h}y\right)} \cdot \frac{b}{3}\left(\frac{4}{27}h^2 + \frac{y^3}{h} - y^2\right)}$$