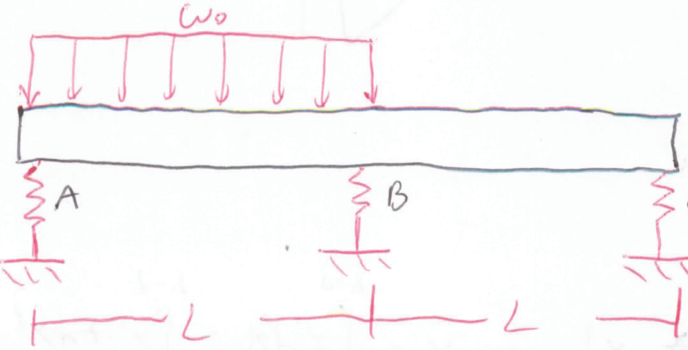
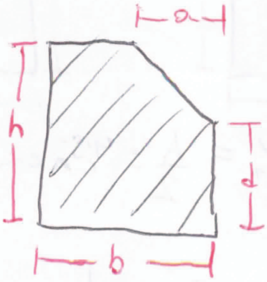
Pauta Aux. 5 ME3202

2013

P1)





Esquema general del problema:

$\boxed{\text{Determinar DCL}}$ $\Rightarrow$ $\boxed{\text{No pueden encontrarse todas las reacciones (Hiperestático)}}$

$\Downarrow$

$\boxed{\text{Para geometrías complejas, hace falta calcular } I_z}$ $\Leftarrow$ $\boxed{\text{Ecuación de la elástica} \quad \dfrac{d^4 \hat{y}}{dx^4} = -\dfrac{w(x)}{E I_z}}$

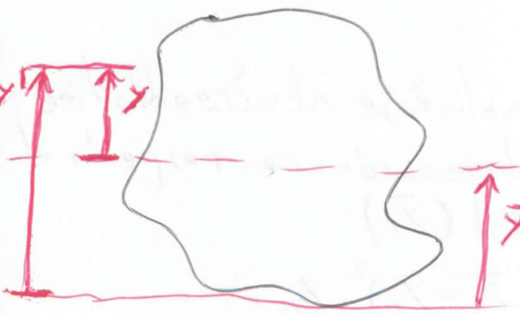
$\Downarrow$

$\boxed{\text{Calcular eje neutro de la geometría}}$ $\Rightarrow$ $\boxed{\text{Calcular } I_z}$ $\Rightarrow$ $\boxed{\text{Encontrar } \hat{y}(x) \text{ con sus cond. de borde}}$

$\Downarrow$

$\boxed{\text{Encontrar reacciones}}$

(I) Cálculo de propiedades de área (I_z)



- $\bar{y} = \dfrac{\int y' \, dA}{A}$

- $I = \int y^2 \, dA = \int (y' - \bar{y}) \, dA$

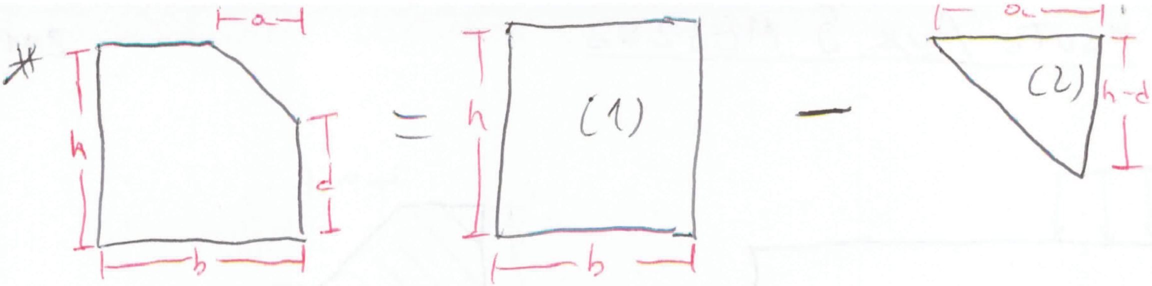
- Si hay varias fig. geométricas:

$$\cancel{Y}_T = \frac{\sum \bar{Y}_i \cdot A_i}{\sum A_i}$$

- Traslado de eje de rotación

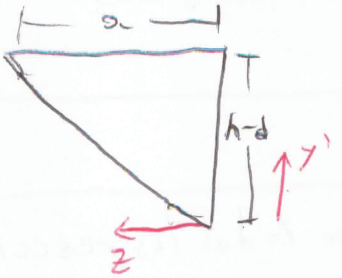
$$\bar{I}_z = I_z + \delta^2 \cdot A$$

*



• Para (1): $\bar{Y}_1 = \frac{h}{2} = 4{,}5\ cm$ | $I_{z1} = \frac{bh^3}{12}$

• Para (2):



$$z = \frac{a}{(h-d)} y' \Rightarrow \bar{Y}_2 = \frac{\int_0^{h-d} y'\, dA}{A} = \frac{\int_0^{h-d} y' \cdot z\, dy'}{\frac{a(h-d)}{2}}$$

$$= \frac{\int_0^{h-d} y'^2 \frac{a}{h-d}\, dy' \cdot 2}{a(h-d)}$$

$$\boxed{\therefore \bar{Y}_2 = \frac{2}{3}(h-d) \;\; 4\ cm}$$

$$\Rightarrow I_{z2} = \int y^2\, dA = \int (y'-\bar{Y}_2)^2\, dA = \int (y'^2 - 2y\bar{Y}_2 + \bar{Y}_2^2)\frac{a}{b-h} y'\, dy'$$

$$= \frac{a}{h-d}\left(\frac{(h-d)^4}{4} - 2\cdot\frac{2}{3}(h-d)\cdot\frac{(h-d)^3}{3} + \left(\frac{2}{3}(h-d)\right)^2 \frac{(h-d)^2}{2}\right)$$

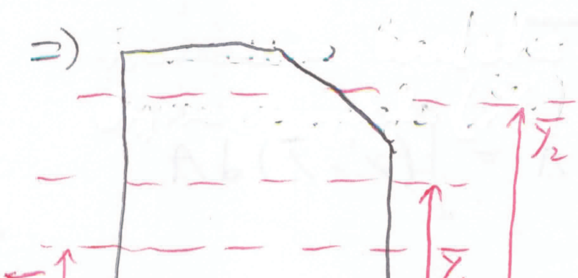
$$\boxed{I_{z2} = \frac{a}{36}(h-d)^3}$$

Desde la base!

$$\Rightarrow \bar{Y}_T = \frac{\bar{Y}_1 \cdot A_1 - \bar{Y}_2 \cdot A_2}{A_1 - A_2} = \frac{\frac{h}{2}\cdot bh - \left(\frac{2}{3}(h-d) + d\right)\cdot\frac{a(h-d)}{2}}{b\cdot h - \frac{a(h-d)}{2}}$$

$$\boxed{\bar{Y}_T = 3{,}8421\ cm}$$

⇒



• Es necesario encontrar el momento de inercia de cada sección con respecto al eje neutro total ($\bar{Y}_T$)

$$\Rightarrow \bar{I}_{z1} = I_{z1} + (\bar{Y}_1 - \bar{Y}_T)^2 \cdot A_1$$

$$= \frac{bh^3}{12} + \left(\frac{h}{2} - \bar{Y}_T\right)^2 \cdot b \cdot h$$

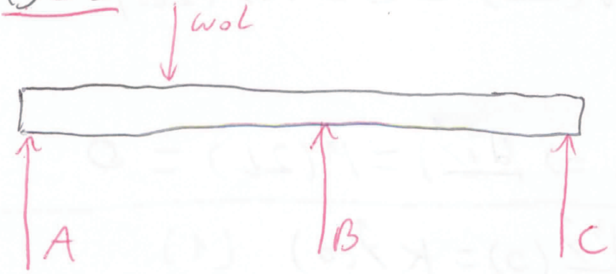
$$\boxed{\bar{I}_{z1} = 517{,}164\ cm^4}$$

$$\Rightarrow I_{z2} = I_{z2} + (Y_2 - Y_T)^2 \cdot A_2 = \frac{a}{36}(h-d)^3 + \left(\frac{2}{3}(h-d) + d - Y_T\right)^2 \cdot \frac{a(h-d)}{2}$$

$$\boxed{\bar{I}_{z2} = 179{,}585\ cm^4}$$

$$\boxed{\therefore I_T = \bar{I}_{z1} - \bar{I}_{z2} = 337{,}579\ cm^4 = 3{,}3758 \cdot 10^{-6}\ m^4}$$

(II) DCL



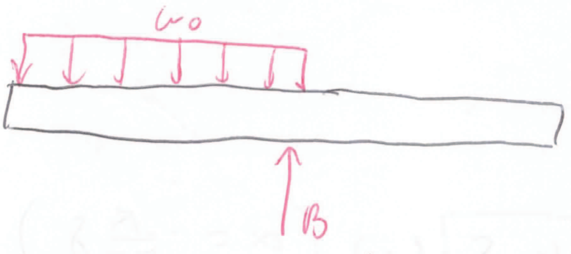
$$\sum F_y = 0 \Rightarrow A + B + C = \omega_0 L$$

$$\sum_A M_z = 0 \Rightarrow -\frac{\omega_0 L^2}{2} + BL + 2CL = 0$$

$$\boxed{\therefore C = \frac{1}{2}\left(\omega_0 \frac{L}{2} - B\right) \wedge A = \frac{1}{2}\left(\frac{3\omega_0 L}{2} - B\right)}$$

(III) Deflexión:

- Modelación de fuerzas mediante función escalón (Fzas. distribuidas) o delta de Dirac (Fzas puntuales)
- Lo que ocurre en los bordes se reemplaza por condiciones de Borde. Son necesarias 4.



$$\Rightarrow \omega(x) = \omega_0\left(r(x) - r(x-L)\right) - B \cdot \delta(x-L)$$

$$\rightsquigarrow \frac{d^4 y}{dx^4} = -\frac{\omega(x)}{EI_z} = -\frac{1}{EI_z}\left(\omega_0\left(r(x) - r(x-L)\right) - B \cdot \delta(x-L)\right)$$

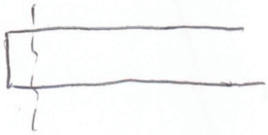
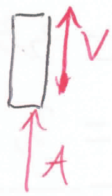
$$\rightsquigarrow \frac{d^3 y}{dx^3} = -\frac{1}{EI_z}\left(\omega_0\left(x\,r(x) - (x-L)\,r(x-L)\right) - B \cdot r(x-L)\right) + \alpha$$

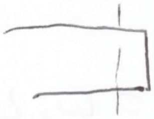
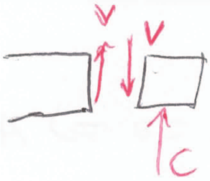
$$\rightsquigarrow \frac{d^2 y}{dx^2} = -\frac{1}{EI_z}\left(\omega_0\left(\frac{x^2}{2} r(x) - \frac{(x-L)^2}{2} r(x-L)\right) - (x-L) B\, r(x-L)\right) + \alpha x + \beta$$

$$\rightsquigarrow \frac{dy}{dx} = -\frac{1}{EI_z}\left(\omega_0\left(\frac{x^3}{6} r(x) - \frac{(x-L)^3}{6} r(x-L)\right) - \frac{(x-L)^2}{2} B\, r(x-L)\right) + \frac{\alpha x^2}{2} + \beta x + \gamma$$

$$\rightarrow \hat{y} = -\frac{1}{EI_z}\left(w_0\left(\frac{x^4}{24} r(x) - \frac{(x-L)^4}{24} r(x-L)\right) - \frac{(x-L)^3}{6} B\, r(x-L)\right) + \frac{\alpha x^3}{6} + \frac{\beta x^2}{2} + \gamma x + \delta$$

(IV) Condiciones de Borde

(A)  $\Rightarrow$  $\Rightarrow V(0) = -A = -K\hat{y}(0)$

(C)  $\Rightarrow$  $\Rightarrow V(2L) = C = K\hat{y}(2L)$

* No hay empotramientos en los bordes $\Rightarrow M(0) = M(2L) = 0$

$\therefore$ Condiciones de Borde: $-EI\frac{d^3\hat{y}}{dx^3}(0) = K\hat{y}(0)$ (1)

$$-EI\frac{d^3\hat{y}}{dx^3}(2L) = -K\hat{y}(2L) \quad (2)$$

$$\frac{d^2\hat{y}}{dx^2}(0) = 0 \; (3) \;\wedge\; \frac{d^2\hat{y}}{dx^2}(2L) = 0 \; (4)$$

* Ahora el problema puede resolverse:

(3): $\frac{d^2\hat{y}}{dx^2}(0) = 0 \Rightarrow \boxed{\beta = 0}$

(1): $-EI\frac{d^3\hat{y}}{dx^3}(0) = K\hat{y}(0) \Rightarrow \boxed{-EI\alpha = K\cdot\delta}$ (i)

(4): $\frac{d^2\hat{y}}{dx^2}(2L) = 0 \Rightarrow -\frac{1}{EI}\left(w_0\cdot 2L^2 - w_0\frac{(2L-L)^2}{2} - LB\right) + 2L\alpha = 0$

$$\alpha = \frac{1}{2EIL}\left(w_0\cdot\frac{3}{2}L^2 - LB\right)$$

$$\boxed{\alpha = 1{,}17\cdot 10^{-3} - 7{,}8\cdot 10^{-7}B}$$

$$\boxed{\delta = -\frac{EI}{K}\alpha = -7{,}5\cdot 10^{-4} + 5\cdot 10^{-7}B}$$

(2) $EI\dfrac{d^3y}{dx^3}(2L) = Ky(2L)$

$$\Rightarrow w_0(2L-L)+B+\alpha = -\frac{K}{EI}\left(\frac{w_0}{24}\left((2L)^4-L^4\right)-\frac{L^3}{6}B\right)+\left(\frac{\alpha(2L)^3}{6}+\gamma\cdot 2L+\delta\right)K$$

$$w_0L+B+\alpha = -\frac{K}{EI}\left(15L^4-\frac{L^3}{6}B\right)+\frac{4}{3}\alpha L^3+2L\gamma+\delta$$

$$\Rightarrow \boxed{\gamma = -4{,}23284\cdot 10^{-5}+3{,}87771\cdot 10^{-7}B}$$

*Finalmente, con las constantes calculadas, podemos encontrar la fuerza B, sabiendo que: $B = -Ky(L)$

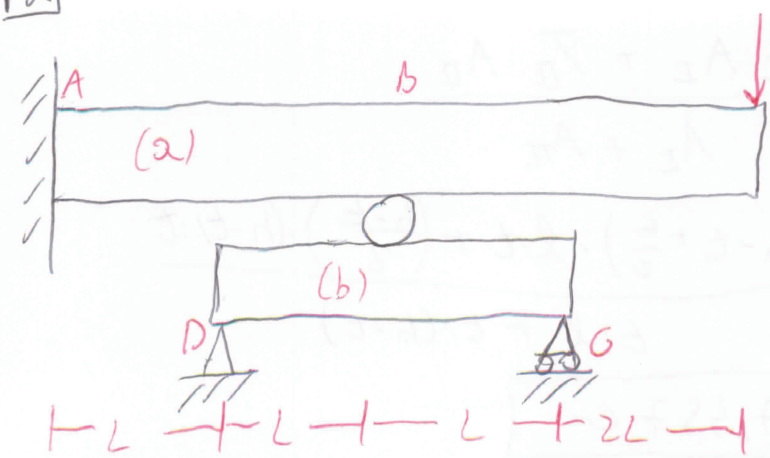
$$\Rightarrow B = \frac{K}{EI}w_0\frac{L^4}{24}+\left(\frac{\alpha L^3}{6}+\gamma L+\delta\right)K$$

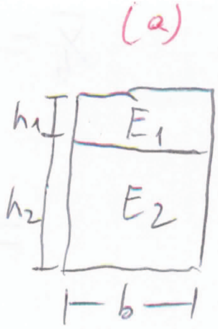
$$\boxed{\therefore B = 376{,}4\ N}$$

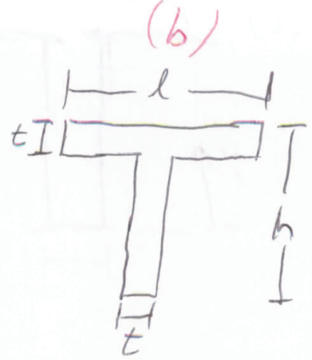
*Reemplazando en el DCL, se tiene finalmente que

$$\boxed{C = 61{,}8\ [N] \quad \wedge \quad A = 561{,}801\ [N]}$$

P2

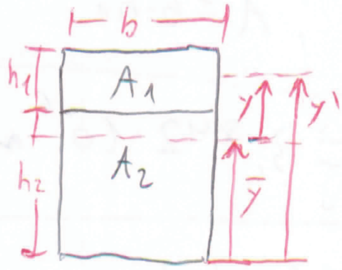






a) Cálculo de ejes neutros:

Viga (a):



$$E_1\int_{A_1} y\,dA + E_2\int_{A_2} y\,dA = 0$$

$$\Rightarrow E_1\int_{A_1} y'\,dA + E_2\int_{A_2} y'\,dA = E_1\int_{A_1}(\bar{y}+y)\,dA + E_2\int_{A_2}(\bar{y}+y)\,dA$$

$$= E_1\int_{A_1}\bar{y}\,dA + E_2\int_{A_2}\bar{y}\,dA + \cancel{\left(E_1\int_{A_1} y\,dA + E_2\int_{A_2} y\,dA\right)}^{0}$$

$$= E_1\bar{y}A_1 + E_2\bar{y}A_2$$

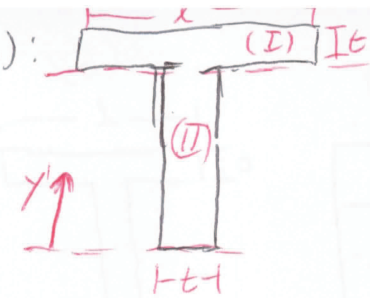
$$\boxed{\therefore\ \bar{Y}_a = \frac{E_1\int_{A_1} y'\,dA + E_2\int_{A_2} y'\,dA}{E_1A_1 + E_2A_2}}$$

$$\Rightarrow \int_{A_1} y'\,dA = \int_{h_2}^{h_2+h_1} y'\cdot b\,dy' = \frac{b}{2}\left((h_2+h_1)^2 - h_2^2\right) = \frac{bh_1}{2}(2h_2+h_1) = 384\ cm^3$$

$$\int_{A_2} y'\,dA = \int_0^{h_2} y'\,b\,dy' = \frac{bh_2^2}{2} = 400\ cm^3$$

$$\boxed{\therefore\ \bar{Y}_a = 8{,}733\ cm}$$

• Viga (b):



$$\bar{y}_b = \frac{\bar{y}_I \cdot A_I + \bar{y}_{II} \cdot A_{II}}{A_I + A_{II}}$$

$$= \frac{(h - t + \frac{t}{2}) \cdot l \cdot t + \left(\frac{h-t}{2}\right) \cdot (h-t) \cdot t}{t \cdot l + t \cdot (h-t)}$$

$$\boxed{\bar{y}_b = 7{,}357 \text{ cm}}$$

b) Cálculo de momentos de área: Usando teorema de ejes paralelos

• Viga (a): • $I_1 = \frac{b \cdot h_1^3}{12}$; $\delta^2 = \left(h_2 + \frac{h_1}{2} - \bar{y}_a\right)^2$; $A = b \cdot h_1$

$$\boxed{\therefore \bar{I}_1 = \frac{b \cdot h_1^3}{12} + \left(h_2 + \frac{h_1}{2} - \bar{y}_a\right)^2 \cdot A = 3{,}842 \cdot 10^{-6} \text{ m}^4}$$

• $I_2 = \frac{b \cdot h_2^3}{12}$; $\delta^2 = \left(\frac{h_2}{2} - \bar{y}_a\right)^2$; $A = b \cdot h_2$

$$\boxed{\therefore \bar{I}_2 = \frac{b \cdot h_2^3}{12} + \left(\frac{h_2}{2} - \bar{y}_a\right)^2 \cdot b \cdot h_2 = 1{,}781 \cdot 10^{-5} \text{ m}^4}$$

• Viga (b): $\bar{I}_b = \bar{I}_{(I)} + \bar{I}_{(II)}$

$\rightsquigarrow I_{(I)} = \frac{l t^3}{12}$; $\delta^2 = \left(h - t + \frac{t}{2} - \bar{y}_b\right)^2$; $A = l t$

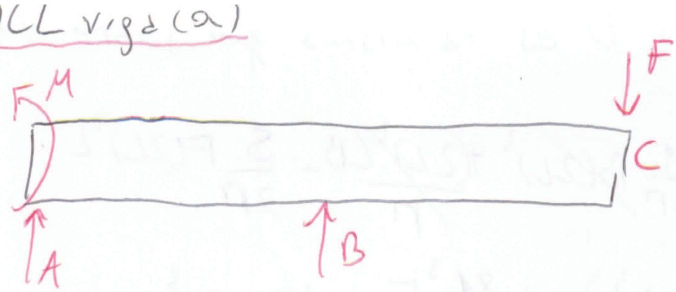
$$\Rightarrow \bar{I}_{(I)} = \frac{l t^3}{12} + \left(h - t + \frac{t}{2} - \bar{y}_b\right)^2 \cdot l t = 56{,}109 \text{ cm}^4$$

$\rightsquigarrow I_{(II)} = \frac{t \cdot (h-t)^3}{12}$; $\delta^2 = \left(\frac{h-t}{2} - \bar{y}_b\right)^2$; $A = t(h-t)$

$$\Rightarrow \bar{I}_{(II)} = \frac{t(h-t)^3}{12} + \left(\frac{h-t}{2} - \bar{y}_b\right)^2 \cdot t(h-t) = 134{,}212 \text{ cm}^4$$

$$\boxed{\therefore \bar{I}_b = 190{,}321 \text{ cm}^4 = 1{,}903 \cdot 10^{-6} \text{ m}^4}$$

c) DCL viga (a)



$$\sum F_y = 0 \Rightarrow A + B = F$$

$$\sum_A M_z = 0 \Rightarrow M + 2LB - 5LF = 0$$

• Deflexión: $\omega(x) = B\,\delta(x-2L) \;\wedge\; \Gamma = E_1 \cancel{A_1} I_1 + E_2 I_2$

$$\rightsquigarrow \frac{d^4\hat{y}}{dx^4} = \frac{B}{\Gamma}\,\delta(x-2L)$$

$$\rightsquigarrow \frac{d^3\hat{y}}{dx^3} = \frac{B}{\Gamma}\,r(x-2L) + \alpha$$

$$\rightsquigarrow \frac{d^2\hat{y}}{dx^2} = \frac{B}{\Gamma}(x-2L)\,r(x-2L) + \alpha x + \beta$$

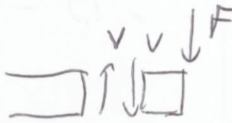
$$\rightsquigarrow \frac{d\hat{y}}{dx} = \frac{B}{2\Gamma}(x-2L)^2\,r(x-2L) + \frac{\alpha x^2}{2} + \beta x + \gamma$$

$$\rightsquigarrow \hat{y}(x) = \frac{B}{6\Gamma}(x-2L)^3\,r(x-2L) + \frac{\alpha x^3}{6} + \frac{\beta x^2}{2} + \gamma x + \delta$$

• Condiciones de borde

A: $\boxed{\hat{y}(0) = 0}$ (1) $\quad \boxed{\frac{d\hat{y}}{dx}(0) = 0}$ (2)

C: $\boxed{\frac{d^2\hat{y}}{dx^2}(5L) = 0}$ (3)



$\Rightarrow V(5L) = -F$

$\therefore \boxed{\frac{d^3\hat{y}}{dx^3}(5L) = \frac{F}{\Gamma}}$ (4)

$\Rightarrow$ (1): $\delta = 0$

(2): $\gamma = 0$

(3): $\boxed{\frac{3BL}{\Gamma} + 5\alpha L + \beta = 0}$ (*)

(4): $\frac{B}{\Gamma} + \alpha = \frac{F}{\Gamma} \Rightarrow \boxed{\alpha = \frac{1}{\Gamma}(F - B)}$

$\Rightarrow$ En (*): $\beta = -\left(\frac{3BL}{\Gamma} + \frac{5L}{\Gamma}F - \frac{5L}{\Gamma}B\right) \Rightarrow \boxed{\beta = \frac{2BL}{\Gamma} - \frac{5FL}{\Gamma}}$

* La deflexión que sufre la viga (a) en B es la misma que sufre (b) en el mismo punto. Luego:

- Viga (a): $\hat{y}_a(2L) = \frac{1}{6\Gamma}F(2L)^3 - \frac{1}{6\Gamma}\cancel{B(2L)^3} + \cancel{\frac{(2L)^2LB}{\Gamma}} - \frac{5}{2\Gamma}F(2L)^2L$

$$= B\left(\frac{4L^3}{\Gamma} - \frac{4L^3}{3\Gamma}\right) + \frac{8L^3}{6\Gamma}F - \frac{10}{\Gamma}FL^3$$

$$\boxed{\hat{y}_a(2L) = \frac{8L^3}{3\Gamma}B - \frac{22}{3\Gamma}L^3F}$$

- Viga (b): $\hat{y}_b(L) = \frac{-BL^3}{6EI_b}$

$$\Rightarrow \hat{y}_a(2L) = \hat{y}_b(L) \rightsquigarrow \frac{8\cancel{L^3}}{\cancel{3}\Gamma}B - \frac{22}{\cancel{3}\Gamma}\cancel{L^3}F = \frac{-B\cancel{L^3}}{\cancel{6}_2EI_b}$$

$$\frac{8B}{\Gamma} + \frac{B}{2EI_b} = \frac{22}{\Gamma}F$$

$$\boxed{\therefore B = \frac{22/\Gamma \cdot F}{\frac{8}{\Gamma} + \frac{1}{2EI_b}} = 2410{,}355\ N}$$

Volviendo al DCL:

$$\boxed{A = F - B = -1410{,}355\ N}$$

$$\boxed{M = 5LF - 2LB = 179{,}29\ N\cdot m}$$