• Cálculo de deflexión por integración:

$$\frac{d^4\hat{y}}{dx^4} = -\frac{w(x)}{EI} \quad \text{Fza distribuida}$$

* Modelación de Fzas mediante función escalón $(r(x-x_0))$ para Fzas distribuidas, y delta de Dirac $(\delta(x-x_0))$ para Fzas puntuales

* Lo que ocurre en los extremos de la viga se reemplaza por condiciones de borde (Son necesarias 4 cond. de borde)

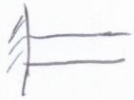
* Las cond. de borde son condiciones en la deflexión en los extremos, el ángulo de deflexión $\left(\frac{d\hat{y}}{dx}\right)$, el momento $\left(\frac{d^2\hat{y}}{dx^2}\right)$ ó la Fza. de corte $\left(\frac{d^3\hat{y}}{dx^3}\right)$

$$\hookrightarrow \boxed{M = EI\frac{d^2\hat{y}}{dx^2}} \ \wedge \ \boxed{V = -EI\frac{d^3\hat{y}}{dx^3}}$$

* Algunos bordes:

$\Rightarrow$ No hay deflexión $\rightarrow \hat{y}(0) = 0$

No hay momento $\rightarrow \frac{d^2\hat{y}}{dx^2} = 0$



$\Rightarrow$ No hay deflexión $\rightarrow \hat{y}(0) = 0$

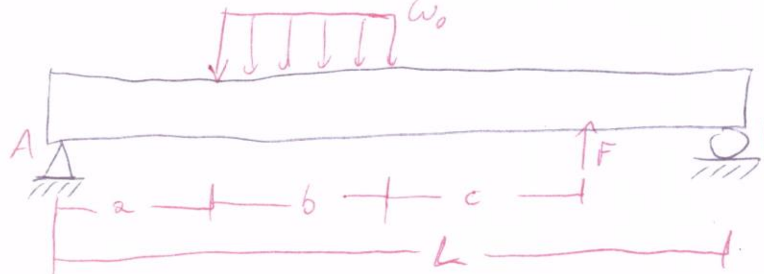
No hay ángulo $\rightarrow \frac{d\hat{y}}{dx}(0) = 0$

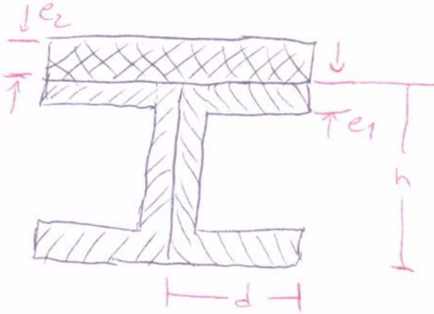
$\Rightarrow$ No hay momento $\rightarrow \frac{d^2\hat{y}}{dx^2}(0) = 0$

Balance de fzas $\rightarrow V(0) = -F = -k\hat{y}(0)$

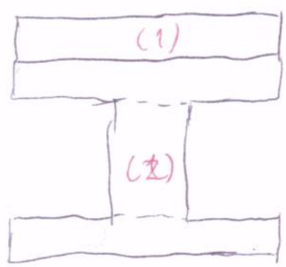
$$\therefore -EI\frac{d^3\hat{y}}{dx^3}(0) = -k\hat{y}(0)$$

P1





a)



$$\overline{Y_T} = \frac{\sum \overline{Y_i} \cdot A_i}{\sum A_i} \quad / \quad \overline{Y_1} = h + \frac{e_2}{2} \ ; \ A_1 = 2de_2$$

$$\overline{Y_2} = \frac{h}{2} \ , \ A_2 = 4de_1 + 2e_1(h-2e_1)$$

$$\Rightarrow \overline{Y_T} = \frac{(h+\frac{e_2}{2}) \cdot 2de_2 + \frac{h}{2}\left(4de_1 + 2e_1(h-2e_1)\right)}{2de_2 + 4de_1 + 2e_1(h-2e_1)}$$

$$\boxed{\overline{Y_T} = 0{,}177 \, m}$$

$$\leadsto \overline{I_z} = \overline{I_1} + \overline{I_2}$$

$$\hookrightarrow \overline{I_1} = I_1 + (\overline{Y_1} - \overline{Y_T})^2 \cdot A_1 = \frac{2de_2^3}{12} + (\overline{Y_1} - \overline{Y_T})^2 \cdot 2de_2$$

$$\boxed{\overline{I_1} = 5{,}538 \cdot 10^{-5} \, [m^4]}$$

$$\hookrightarrow \cancel{\overline{I_2} = 2 \frac{2de_1^3}{12}} +$$

$$* \overline{I_2} = \left(\frac{2de_1^3}{12} + \left(\frac{e_1}{2} - \frac{h}{2}\right)^2 \cdot 2de_1\right) + \left(\frac{2de_1^3}{12} + \left(h + \frac{e_1}{2} - \frac{h}{2}\right)^2 \cdot 2de_1\right) + (\overline{Y_T} - \overline{Y_2})^2 \cdot A_2$$

ó

$$* \overline{I_2} = 2\left(\frac{dh^3}{12} - \frac{(d-e_1)(h-2e_1)^3}{12} + (\overline{Y_T} - \overline{Y_2})^2 \frac{A_2}{2}\right) = 1{,}701 \cdot 10^{-4} \, m^4$$

$$\boxed{\therefore\ \bar{I}_z = \cancel{7,239} = 2,255 \cdot 10^{-4}\ [m^4]}$$

b) $$\frac{d^4 y}{dx^4} = -\frac{w(x)}{E\bar{I}_z} \quad ; \quad \cancel{w(x) = w_0\left(r(x-(a+b))\right)}$$

$$w(x) = w_0\left(r(x-a) - r(x-a-b)\right) + F\delta(x-a-b-c)$$

$$\Rightarrow \frac{d^4 y}{dx^4} = -\frac{1}{EI_z}\left[w_0\left[r(x-a) - r(x-\overbrace{a-b}^{k_1})\right] + F\delta(x-\overbrace{a-b-c}^{k_2})\right]$$

$$\frac{d^3 y}{dx^4} = -\frac{1}{EI_z}\left[w_0\left((x-a)r(x-a) - (x-k_1)r(x-k_1)\right) - F\,r(x-k_2)\right] + \alpha$$

$$\frac{d^2 y}{dx^4} = -\frac{1}{EI_z}\left[w_0\left(\frac{(x-a)^2}{2}r(x-a) - \frac{(x-k_1)^2}{2}r(x-k_1)\right) - F(x-k_2)\cdot r(x-k_2)\right] + \alpha x + \beta$$

$$\frac{dy}{dx} = -\frac{1}{EI_z}\left[w_0\left(\frac{(x-a)^3}{6}r(x-a) - \frac{(x-k_1)^3}{6}r(x-k_1)\right) - F\frac{(x-k_2)^2}{2}r(x-k_2)\right] + \frac{\alpha x^2}{2} + \beta x + \gamma$$

$$\boxed{y(x) = -\frac{1}{EI_z}\left[w_0\left(\frac{(x-a)^4}{24}r(x-a) - \frac{(x-k_1)^4}{24}r(x-k_1)\right) - F\frac{(x-k_2)^3}{6}\cdot r(x-k_2)\right] + \frac{\alpha x^3}{6} + \frac{\beta x^2}{2} + \gamma x + \delta}$$

· Condiciones de Borde:

1-) $\frac{d^2 y}{dx^2}(0) = 0 \Rightarrow \beta = 0$; 2-) $y(0) = 0 \Rightarrow \delta = 0$

3-) $$\frac{d^2 y}{dx^2}(L) = 0 \Rightarrow \boxed{-\frac{1}{EI_z}\left[w_0\left(\frac{(L-a)^2}{2} - \frac{(L-k_1)^2}{2}\right) - F(L-k_2)\right] + \alpha L = 0} \quad (1)$$

4-) $$y(0) = 0 \Rightarrow \boxed{-\frac{1}{EI_z}\left[w_0\left(\frac{(L-a)^4}{24} - \frac{(L-k_1)^4}{24}\right) - \frac{F(L-k_2)^3}{6}\right] + \frac{\alpha L^3}{6} + \gamma L = 0} \quad (2)$$

$* \ B_c(1) \rightsquigarrow \alpha = \frac{1}{L E I_z}\left[w_0\left(\frac{(L-a)^2}{2} - \frac{(L-x_1)^2}{2}\right) - F\frac{(L-x_2)}{}\right]$

$$\boxed{\alpha = \cancel{3{,}167\cdot 10^{-4}}} \ \boxed{= -7{,}849\cdot 10^{-5}}$$

$* \ B_c(2)$

$$\gamma = \frac{1}{L E I_z}\left[w_0\left(\frac{(L-a)^4}{24} - \frac{(L-x_1)^4}{24}\right) - \frac{F(L-x_2)^3}{6}\right] - \frac{\alpha L^2}{6}$$

$$\boxed{\gamma = 1{,}402\cdot 10^{-3}}$$

$*$ Una vez calculadas las ctes., calculamos la deflexión en $x = L/2$

$$\hat{y}(L/2) = -\frac{1}{E I_z}\left[w_0 \cdot \frac{(\frac{L}{2}-a)^4}{24}\right] + \frac{\alpha (\frac{L}{2})^3}{6} + \gamma \cdot \frac{L}{2}$$

$$\boxed{\hat{y}(L/2) = 5{,}363\cdot 10^{-3}\ [m]}$$

$*$ El esf. normal ~~máximo~~ se calcula como:

$$\cancel{\sigma_{max} = -M(x)\cdot y}$$

$$\sigma(x) = \frac{-M(x)\cdot y}{I_z} \quad ; \quad M(x) = EI\frac{d^2\hat{y}}{dx^2}$$

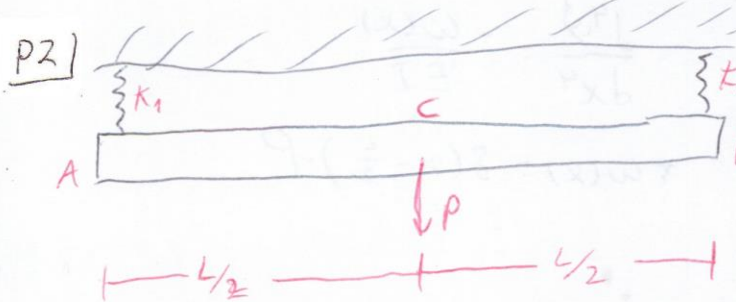
$$\boxed{\therefore \sigma(x) = \frac{-d^2\hat{y}}{dx^2} E y}$$

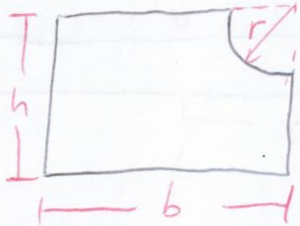
$\hookrightarrow \frac{d^2\hat{y}}{dx^2}\left(\frac{L}{2}\right) = \frac{-w_0}{2 E I_z}\left(\frac{L}{2}-a\right)^2 + \alpha\cdot\frac{L}{2} = -4{,}279\cdot 10^{-4}$

$\hookrightarrow y_{max} = -\bar{y}_T = -0{,}177$

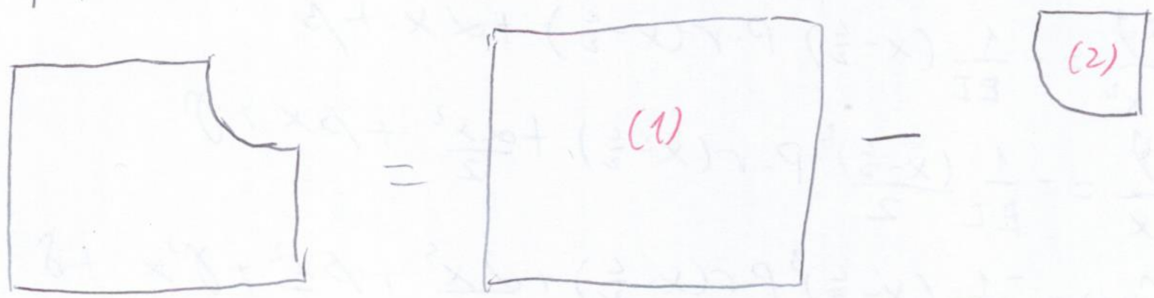
$$\boxed{\therefore \sigma(L/2) = \cancel{-151174{,}66}\ [Pa]} \ \boxed{1{,}515\cdot 10^{7}\ [Pa]}$$

P2)





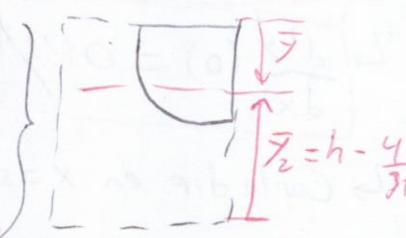
a) Propiedades de área:



(1): $\bar{y}_1 = \frac{h}{2} \;\wedge\; I_1 = \frac{b \cdot h^3}{12}$

(2):

$\cancel{\bar{y}_2 = \frac{4r}{3\pi}} \;\wedge\; I_2 = \frac{\pi r^4}{16}$



Desde la base

$$\Rightarrow \bar{y}_T = \frac{\bar{y}_1 \cdot A_1 - \bar{y}_2 A_2}{A_1 - A_2} = \frac{\frac{h}{2} \cdot hb - \left(h - \frac{4r}{3\pi}\right) \cdot \frac{1}{4}\pi r^2}{hb - \frac{1}{4}\pi r^2} \cancel{= 4{,}567\text{ cm}}$$

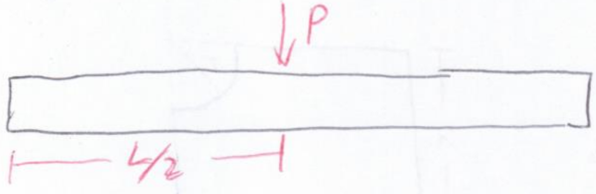
$$\boxed{\bar{y}_T = 4{,}567\text{ cm}}$$

$$\Rightarrow \bar{I}_1 = I_1 + \delta^2 A = \frac{bh^3}{12} + \left(\frac{h}{2} - \bar{y}_T\right)^2 bh \Rightarrow \boxed{\bar{I}_1 = 1{,}278 \cdot 10^{-5}\text{ m}^4}$$

$$\bar{I}_2 = I_2 + \delta^2 A = \frac{\pi r^4}{16} + \left(\bar{y}_T - \left(h + \frac{4r}{3\pi}\right)\right)^2 \cdot \frac{1}{4}\pi r^2 \Rightarrow \boxed{\bar{I}_2 = 3{,}38 \cdot 10^{-6}\text{ m}^4}$$

$$\therefore \bar{I}_T = \bar{I}_1 - \bar{I}_2 \Longrightarrow \boxed{\bar{I}_T = 9{,}401 \cdot 10^{-6}\text{ m}^4}$$

b) Aplicando la ec. de la curva elástica



$$\frac{d^4 y}{dx^4} = -\frac{\omega(x)}{EI}$$

$$* \ \omega(x) = \delta\left(x - \frac{L}{2}\right) \cdot P$$

· Integrando:

$$\rightsquigarrow \frac{d^3 y}{dx^3} = -\frac{1}{EI} \cdot P \cdot r\left(x - \frac{L}{2}\right) + \alpha$$

$$\rightsquigarrow \frac{d^2 y}{dx^2} = -\frac{1}{EI}\left(x - \frac{L}{2}\right) \cdot P \cdot r\left(x - \frac{L}{2}\right) + \alpha x + \beta$$

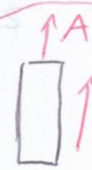
$$\rightsquigarrow \frac{dy}{dx} = -\frac{1}{EI} \frac{\left(x - \frac{L}{2}\right)^2}{2} P \cdot r\left(x - \frac{L}{2}\right) + \frac{\alpha x^2}{2} + \beta x + \gamma$$

$$\Rightarrow \hat{y}(x) = \frac{-1}{6EI}\left(x - \frac{L}{2}\right)^3 P \cdot r\left(x - \frac{L}{2}\right) + \frac{\alpha x^3}{6} + \beta \frac{x^2}{2} + \gamma x + \delta$$

· Condiciones de Borde:

↳ $\boxed{\frac{d^2 y}{dx^2}(0) = 0 \ (1)} \ \boxed{\frac{d^2 y}{dx^2}(L) = 0 \ (2)}$

↳ Corte dif. en $x = 0 \rightsquigarrow$

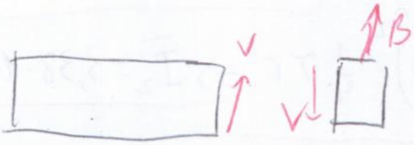


$$\left.\begin{array}{l} \end{array}\right\} \begin{array}{l} V(0) + A = 0 \\ \Rightarrow V(0) = -A \end{array}$$

$$* \ V(x) = -EI \frac{d^3 y}{dx^3}(x) \quad y \quad A = -k_1 \cdot \hat{y}(0)$$

$$\boxed{\therefore -EI \frac{d^3 y}{dx^3}(0) = k_1 \hat{y}(0) \ (3)}$$

↳ Corte dif. en $x = L$



$$\left.\begin{array}{l} \end{array}\right\} \begin{array}{l} V(L) - B = 0 \Rightarrow V(L) = B \\ * \ B = -k_2 \hat{y}(L) \end{array}$$

$$\boxed{\therefore EI \frac{d^3 y}{dx^3}(L) = k_2 \hat{y}(L) \ (4)}$$

• Aplicando las cond. de Borde

$$(1)\ \frac{d^2y}{dx^2}(0)=0 \Rightarrow \boxed{\beta=0}$$

$$(2)\ \frac{d^2y}{dx^2}(L)=0 \rightsquigarrow -\frac{1}{EI}\left(L-\frac{L}{2}\right)\cdot P+\alpha L=0$$

$$\Rightarrow \boxed{\alpha=\frac{P}{2EI}}$$

$$(3)\ -EI\frac{d^3y}{dx^3}(0)=k_1\hat{y}(0)$$

$$-\alpha EI=k_1\delta \rightsquigarrow \delta=-\frac{\alpha EI}{k_1}=-\frac{P}{2\cancel{EI}}\cdot\frac{\cancel{EI}}{k_1}\Rightarrow \boxed{\delta=\frac{-P}{2k_1}}$$

$$(4)\ EI\frac{d^3y}{dx^3}(L)=k_2\hat{y}(L)$$

$$-P+\alpha EI=k_2\left(-\frac{1}{6EI}\left(L-\frac{L}{2}\right)^3P+\frac{\alpha L^3}{6}+\gamma L+\delta\right)$$

~~Despejando~~

$$-P+\frac{P}{2}=k_2\left(-\frac{L^3P}{48EI}+\frac{\overset{\frac{P}{2EI}}{\alpha}L^3}{6}-\frac{P}{2k_1}\right)+k_2L\gamma$$

$$\boxed{\therefore \gamma=\frac{1}{k_2L}\left(-\frac{P}{2}-k_2P\left(-\frac{L^3}{48EI}+\frac{L^3}{12EI}-\frac{1}{2k_1}\right)\right)}$$

• Una vez calculadas todas las constantes, evaluemos $\hat{y}\left(\frac{L}{2}\right)$ para encontrar el desplazamiento en el punto C.

$$\hat{y}\left(\frac{L}{2}\right)=\cancel{\frac{\alpha L^3}{6}+\frac{\beta L^2}{2}+\gamma L+\delta}=\frac{\alpha\left(\frac{L}{2}\right)^3}{6}+\gamma\frac{L}{2}+\delta$$

• Calculando las ctes: $\alpha=1{,}4\cdot10^{-3}$

$$\gamma=-7{,}4\cdot10^{-3}$$

$$\delta=-8{,}3\cdot10^{-3}$$

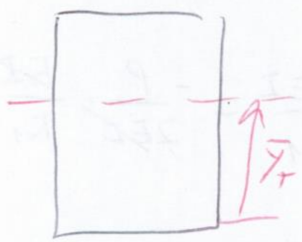
$$\boxed{\therefore \hat{y}\left(\frac{L}{2}\right)=-0{,}0241\ m=-2{,}41\ cm}$$

a) ~~$M = EI \frac{d^2y}{dx^2} \Rightarrow M_{max} \sim$ derivamos $M(x)$, obtenemos x para luego evaluar en $M(x)$~~

~~$\Rightarrow \frac{d^3y}{dx^3} = 0 \sim -\frac{1}{EI} P \cdot r(x - \frac{L}{2}) + \frac{P}{2EI}$~~

c) $M_{max} \sim \left(\frac{d^2y}{dx^2}\right)_{max} \sim x = \frac{L}{2}$

$Y_{max} \sim$



$* \; y = -\overline{Y}_T = -4{,}867 \text{ cm}$

$* \; y = h - \overline{Y}_T = 5{,}433 \text{ cm} \rightarrow$ Máximo
(Extr. sup)

$\therefore \sigma_{max} = -\frac{M}{I} y \quad \wedge \quad M = EI \frac{d^2y}{dx^2}$

$$\left.\Rightarrow \sigma_{max} = -E\, y_{max} \left(\frac{d^2y}{dx^2}\right)_{max} \right\} \quad \frac{d^2y}{dx^2}(L/2) = \frac{PL}{4EI}$$

$$\boxed{\therefore \sigma_{max} = -43{,}342 \text{ MPa}}$$