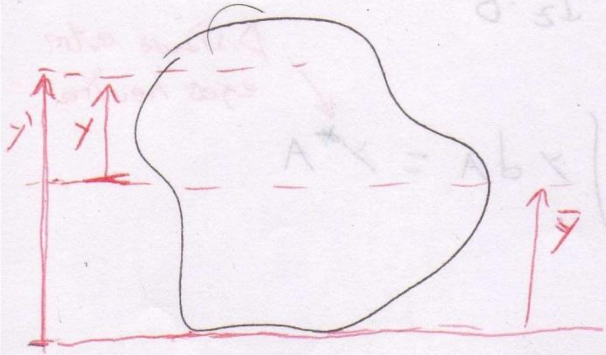
* Cálculo de propiedades de Área (I_z, Q)

↳ 2º momento de área ($\bar{I}_z$)



• $\bar{y}$ (Eje neutro) $= \dfrac{\int y' \, dA}{A}$

• $I_z = \int y^2 \, dA = \int (y' - \bar{y}) \, dA$

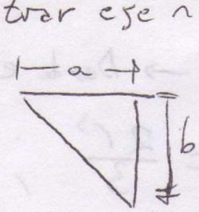
• Si hay varias fig. geométricas

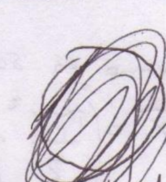
$$\bar{Y}_T = \frac{\sum \bar{Y}_i \cdot A_i}{\sum A_i}$$

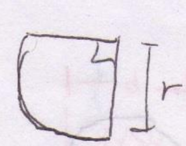
• Traslado de eje de rotación (Teorema de los ejes paralelos)

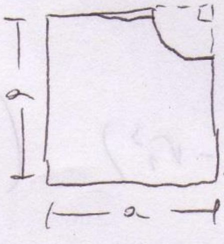
$$\bar{I}_z = I_z + \delta^2 A$$

* Encontrar eje neutro y ~~...~~ I_z de:

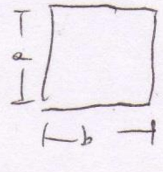






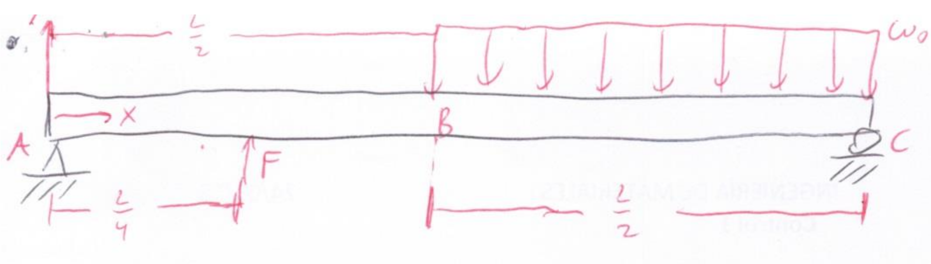


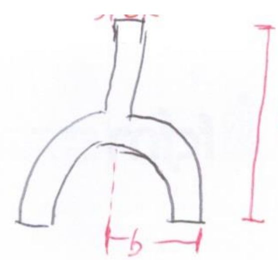
* Secciones conocidas:



$I_z = \dfrac{ba^3}{12}$

$I_z = \dfrac{\pi d^4}{64}$





a)

$$\overline{Y}_T = \frac{\sum A_i Y_i}{\sum A_i} = \frac{(Y_1 \cdot A_1 - Y_2 \cdot A_2) + Y_3 \cdot A_3}{A_1 - A_2 + A_3}$$

(1): Semicírculo de radio b completo

(2): Semicírculo de radio (b−e)

$$* Y_1 = \frac{4b}{3\pi} \quad \Big\| \quad * Y_2 = \frac{4(b-e)}{3\pi} \quad \Big\| \quad * Y_3 = \frac{(h-b)}{2} + b = \frac{b+h}{2}$$

Desde la base!

$$A_1 = \frac{\pi b^2}{2} \quad \Big\| \quad A_2 = \frac{\pi (b-e)^2}{2} \quad \Big\| \quad A_3 = e(h-b)$$

$$\boxed{\therefore \overline{Y}_T = 0{,}07468 \, \cancel{m} = 7{,}468 \text{ cm}}$$

b) $\overline{I}_T = \overline{I}_1 - \overline{I}_2 + \overline{I}_3$ ← C/r a eje neutro de fig. completa $\overline{Y}_T$

$$\rightsquigarrow I_1 = 0{,}1098\, b^4 \Rightarrow \overline{I}_1 = I_1 + (Y_1 - \overline{Y}_T)^2 \cdot A_1$$

$$= 0{,}1098\, b^2 + \left(\frac{4b}{3\pi} - \overline{Y}_T\right)^2 \cdot \frac{\pi b^2}{2} = 1643{,}56 \text{ cm}^4$$

$$\rightsquigarrow I_2 = 0{,}1098 (b-e)^4 \Rightarrow \overline{I}_2 = I_2 + (Y_2 - \overline{Y}_T)^2 A_2$$

$$= 0{,}1098 (b-e)^4 + \left(\frac{4(b-e)}{3\pi} - \overline{Y}_T\right)^2 \cdot \frac{\pi (b-e)^2}{2}$$

$$= 2411{,}88 \text{ cm}^4$$

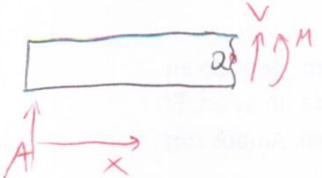
$$\rightsquigarrow I_g = \frac{e \cdot (h-b)^3}{12} \Rightarrow \overline{I}_3 = I_3 + (Y_3 - \overline{Y}_T)^2 A_3$$

$$= \frac{e(h-b)^3}{12} + \left(\frac{(h+b)}{2} - \overline{Y}_T\right)^2 \cdot e(h-b) = 283{,}28 \text{ cm}^4$$

$$\boxed{\therefore I_T = 4340{,}72 \text{ cm}^4 = 4{,}341 \cdot 10^{-5} \text{ m}^4}$$

c) Momento interno $M(x)$:

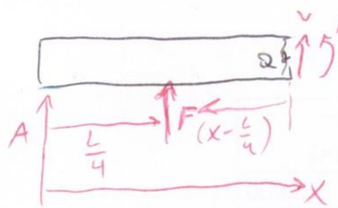
i) $0 < x < \frac{L}{4}$



$$\sum F_y = 0 \Rightarrow V(x) = A$$

$$\sum_Q M_z = 0 \Rightarrow -Ax + M = 0 \Rightarrow \boxed{M(x) = Ax}$$

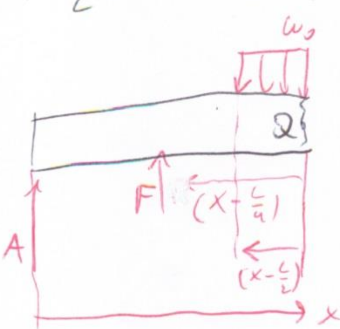
ii) $\frac{L}{4} < x < \frac{L}{2}$



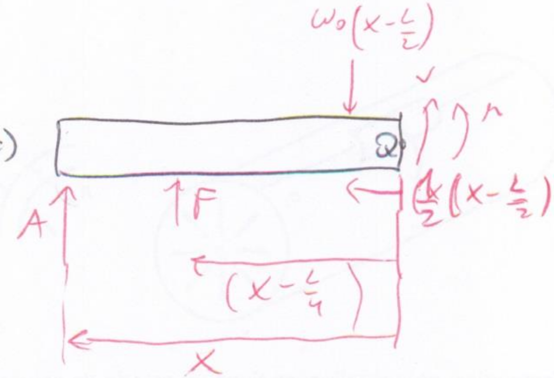
$$\sum_Q M_z = 0 \Rightarrow -Ax - F\left(x-\frac{L}{4}\right) + M = 0$$

$$\boxed{\therefore M(x) = Ax + F\left(x - \frac{L}{4}\right)}$$

iii) $\frac{L}{2} < x < L$



$\sum \Rightarrow$



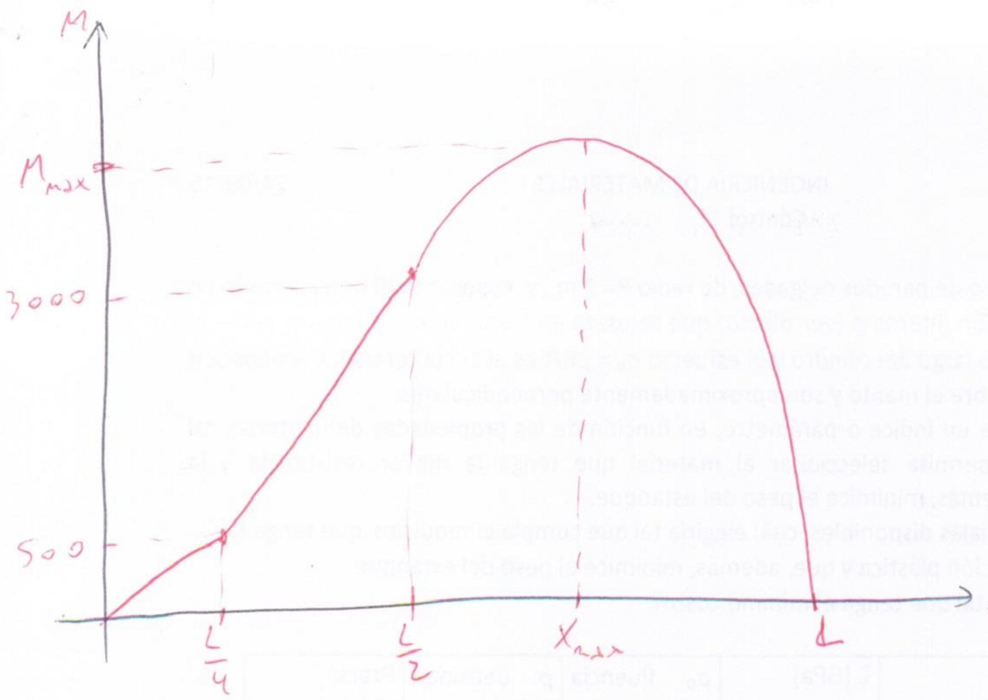
$$\sum_Q M_z = 0 \Rightarrow -Ax - F\left(x-\frac{L}{4}\right) + \frac{w_0}{2}\left(x-\frac{L}{2}\right)^2 + M = 0$$

$$\boxed{\therefore M(x) = Ax + F\left(x-\frac{L}{4}\right) - \frac{w_0}{2}\left(x-\frac{L}{2}\right)^2}$$

* DCL

$$\sum_B M_z = 0 \Rightarrow -A\cdot L - F\cdot\frac{3L}{4} + \frac{w_0 L^2}{8} = 0$$

$$\boxed{\therefore A = \frac{w_0 L}{8} - \frac{3}{4}F = 500\ N}$$



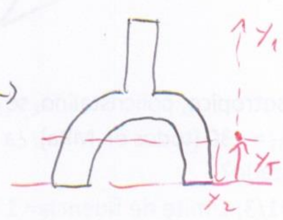
. Si el máximo se encuentra en el 3º tramo:

$$\left.\frac{dM(x)}{dx}\right|_{M_{max}} = 0 \Rightarrow A + F - \omega_0\left(x - \frac{L}{2}\right) = 0$$

$$x_{max} = \frac{A+F}{\omega_0} + \frac{L}{2} = 2,625 \text{ m}$$

$$\Rightarrow \boxed{M_{max} = 3781,25 \text{ N·m}}$$

. Esf. máximo $\Rightarrow \sigma = -\frac{M_{max}}{I_z} \cdot y_{max} \rightarrow$



$y_1 = h - \overline{y_T}$

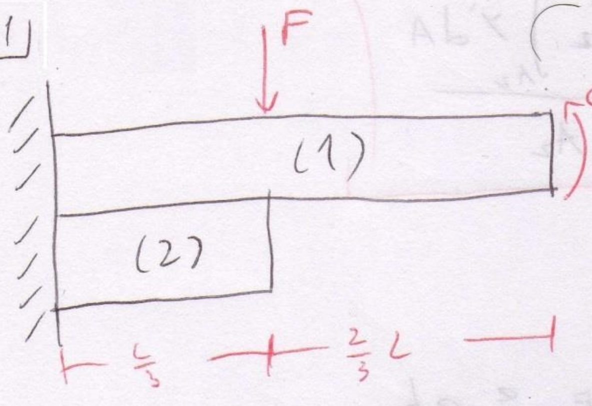
$y_2 = -\overline{y_T}$

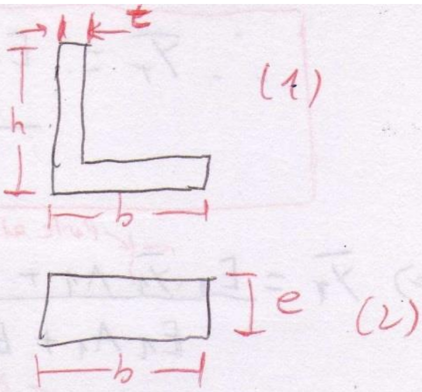
$$\Rightarrow \boxed{y_1 = 9,532 \text{ cm}} \quad ; \quad y_2 = -7,468 \text{ cm}$$

$$\therefore \sigma = -\frac{3781,25 \text{ N·m}}{4,341 \cdot 10^{-5} \text{ m}^4} \cdot 9,532 \cdot 10^{-2} \text{ m} \Rightarrow \boxed{\therefore \sigma_{max} = -8,303 \text{ MPa}}$$

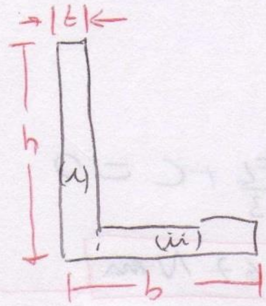
(Compresión)

P1)





a) Viga (1)



$$\bar{y}_1 = \frac{\sum_i y_i \cdot A_i}{\sum_i A_i} = \frac{\bar{y}_{(i)} A_{(i)} + \bar{y}_{(ii)} \cdot A_{(ii)}}{A_{(i)} + A_{(ii)}}$$

$$= \frac{\frac{h}{2} \cdot h \cdot t + \frac{t}{2} \cdot (t(b-t))}{ht + t(b-t)}$$

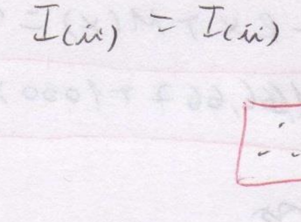
$$\Rightarrow \boxed{\bar{y}_1 = 3{,}038 \text{ cm}}$$

$$\rightarrow \bar{I}_{(i)} = I_{(i)} + \delta^2 A_{(i)} = \frac{th^3}{12} + \left(\bar{y} - \frac{h}{2}\right) \cdot t \cdot h = 249{,}249 \text{ cm}^4$$

$$I_{(ii)} = I_{(ii)} + \delta^2 A_{(ii)} = \frac{(b-t) \cdot t^3}{12} + \left(\bar{y} - \frac{t}{2}\right)^2 (b-t) \cdot t = 91{,}38 \text{ cm}^4$$

$$\boxed{\therefore I_1 = 3{,}406 \cdot 10^{-6} \text{ m}^4}$$

b)



• Para calcular el eje neutro de una viga compuesta se usa:

$$E_1 \int_{A_1} y \, dA + E_2 \int_{A_2} y \, dA = 0$$

• Haciendo $y = y' - \bar{y}_T$

$$\Rightarrow E_1 \int_{A_1} (y' - \bar{y}_T) dA + E_2 \int_{A_2} (y' - \bar{y}_T) dA = 0$$

$$\Rightarrow \left[E_1 \int_{A_1} y' dA + E_2 \int_{A_1} y' dA\right] + \left[E_2 \bar{y}_T A_1 + E_2 \bar{y}_T A_c\right] = 0$$

$$\boxed{\therefore \bar{Y}_T = \frac{E_1 \int_{A_1} y' dA + E_2 \int_{A_2} y' dA}{E_1 \cdot A_1 + E_2 A_2}}$$

Parte el / $\frac{e}{2}$

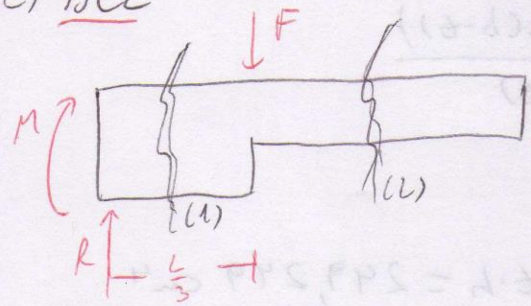
$$\Rightarrow \bar{Y}_T = \frac{E_1 \bar{y}_1 A_1 + E_2 \bar{y}_2 A_2}{E_1 A_1 + E_2 A_2}$$

Desde la base!

$$= \frac{E_1 (\bar{y}_1 + e)(ht + t(b-t)) + E_2 \cdot \frac{e}{2} \cdot eb}{E_1 (ht + t(b-t)) + E_2 eb}$$

$$\boxed{\therefore \bar{Y}_T = 4,6967 \text{ cm}}$$

c) DCL



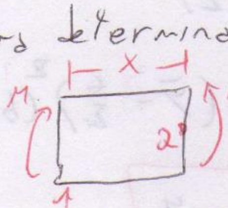
$\cdot \sum M_z = 0 \rightsquigarrow M - \frac{FL}{3} + C = 0$

$\boxed{\therefore M = 166,667 \text{ N·m}}$

$\cdot \sum F_y = 0 \rightsquigarrow \boxed{R = F = 1000 \text{ N}}$

• Hacemos cortes para determinar la distribución de momento interno

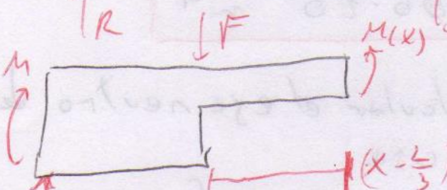
(1) $0 < x < \frac{L}{3}$ $\Rightarrow$



$\cdot \sum M_z = 0 \rightsquigarrow -M - Rx + M(x) = 0$

$\boxed{\therefore M(x) = 166,667 + 1000x}$

(2) $\frac{L}{3} < x < L$ $\Rightarrow$



$\cdot \sum M_z = 0 \rightsquigarrow$

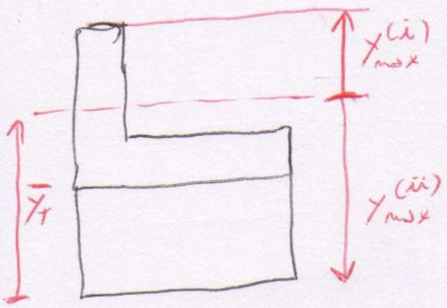
$\Rightarrow M - Rx + (x - \frac{L}{3})F + M(x) = 0$

$\boxed{\therefore M(x) = \frac{FL}{3} + M = 500 \text{ N·m}}$

• El momento máximo se da para cada intervalo (1): $M_{max} = M(\frac{L}{2}) = 166,667 + 1000 \cdot \frac{L}{3}$

$\boxed{M_{max} = 500 \text{ N·m}}$

• Para determinar Y_{max}, se hace para cada material



$$\Rightarrow Y_{max}^{(i)} = h + e - \overline{Y}_T = 0{,}143 \text{ cm (superior)}$$

$$Y_{max}^{(ii)} = -\overline{Y}_T = 4{,}6967 \text{ cm (inferior)}$$

• Usando las fórmulas para Esf. máximo en vigas compuestas:

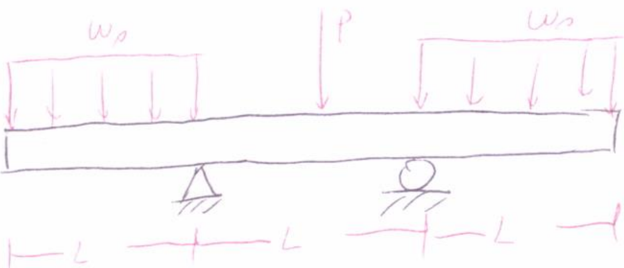
$$\boxed{\sigma_{(i),max} = \frac{-E_1}{E_1 I_1 + E_2 I_2} \cdot M_{max} \cdot Y_{max}^{(i)} = -4{,}152 [MPa] \text{ (Compresión)}}$$

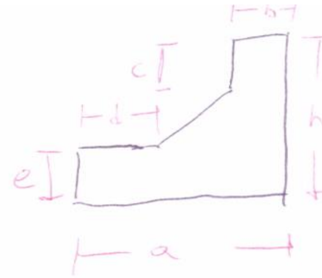
$$\boxed{\sigma_{(ii),max} = \frac{-E_2}{E_1 I_1 + E_2 I_2} M_{max} \cdot Y_{max}^{(ii)} = 1{,}507 [MPa] \text{ (Tracción)}}$$

• (2): $M_{max} = 500 \text{ Nm (cte)}$

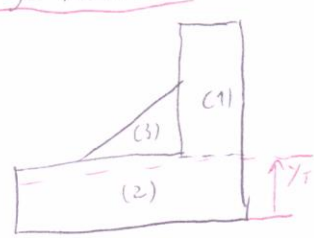
$$Y_{max} = 12 - \overline{Y}_1 = 8{,}962 [cm] \text{ (Parte superior)}$$

$$\Rightarrow \boxed{\sigma_{max} = -\frac{M_{max} \cdot Y_{max}}{I_1} = 13{,}\cancel{4}54 [MPa]}$$





i) Eje neutro



$$\overline{Y}_T = \frac{\sum Y_i \cdot A_i}{\sum A_i} = \frac{Y_1 \cdot A_1 + Y_2 \cdot A_2 + Y_3 \cdot A_3}{A_1 + A_2 + A_3}$$

$$* \; Y_1 = e + \frac{(h-e)}{2} = \frac{h+e}{2} \quad \wedge \quad A_1 = b \cdot (h-e)$$

$$Y_2 = \frac{e}{2} \quad \wedge \quad A_2 = e \cdot a$$

$$Y_3 = e + \frac{(h-c-e)}{3} = \frac{(h-c+2e)}{3} \quad \wedge \quad A_3 = \frac{(a-d-b)\cdot(h-e-c)}{2}$$

$$\boxed{\therefore \; \overline{Y}_T = 0{,}0331 \; m}$$

ii) Momento de Área

$$\overline{I}_z = \overline{I}_1 + \overline{I}_2 + \overline{I}_3$$

$$= (I_1 + \delta_1^2 A_1) + (I_2 + \delta_2^2 A_2) + (I_3 + \delta_3^2 A_3)$$

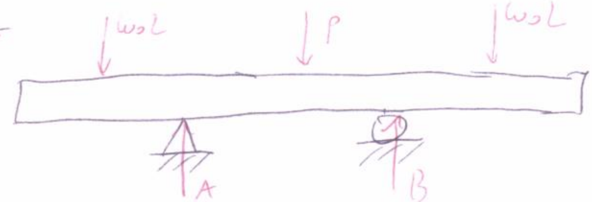
$$* \; I_1 = \frac{b(h-e)^3}{12} \qquad I_2 = \frac{a \cdot e^3}{12} \qquad I_3 = \frac{(a-d-b)\cdot(h-e-c)^3}{36}$$

$$\delta_1 = (Y_1 - \overline{Y}_T) \qquad \delta_2 = (Y_2 - \overline{Y}_T) \qquad \delta_3 = (Y_3 - \overline{Y}_T)$$

$$\boxed{\overline{I}_z = 1{,}174 \cdot 10^{-6} \; m^4}$$

iii) Cargas internas ($M(x)$):

→ DCL



- $\sum_A M_z = 0$

$$\frac{w_0 L^2}{2} - \frac{PL}{2} + BL - \frac{3w_0 L^2}{2} = 0$$

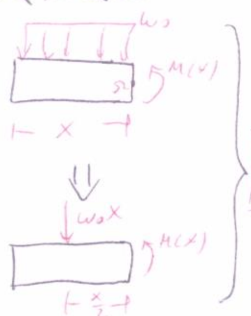
$$\boxed{B = \frac{P}{2} + w_0 L^2 = 1100 \text{ kN}}$$

- $\sum F_y = 0 \rightarrow -2w_0 L + A + B - P = 0$

$$\boxed{\therefore A = P + 2w_0 - B = 1100 \text{ KN}}$$

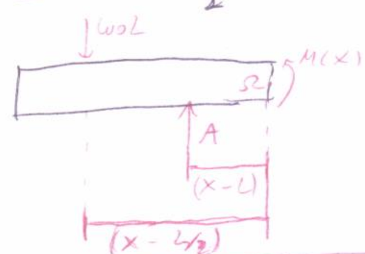
→ Cortes:

a) $0 < x < L$



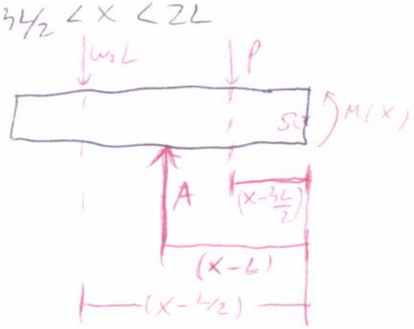
$$\boxed{M(x) = -\frac{w_0 x^2}{2}}$$

b) $L < x < \frac{3L}{2}$



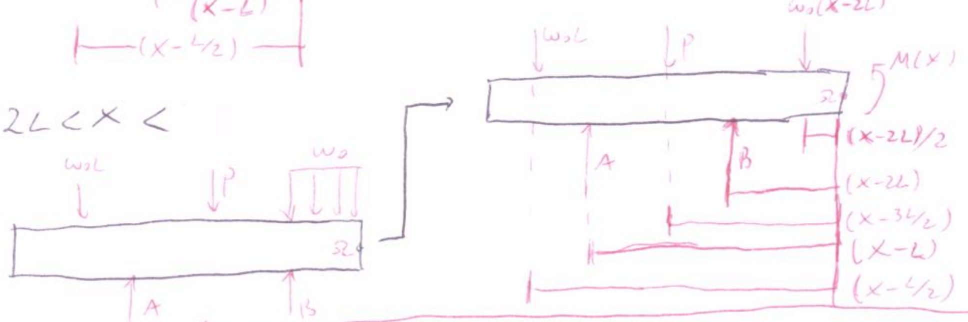
$$\boxed{M(x) = -w_0 L(x - L/2) + A(x - L)}$$

c) $3L/2 < x < 2L$



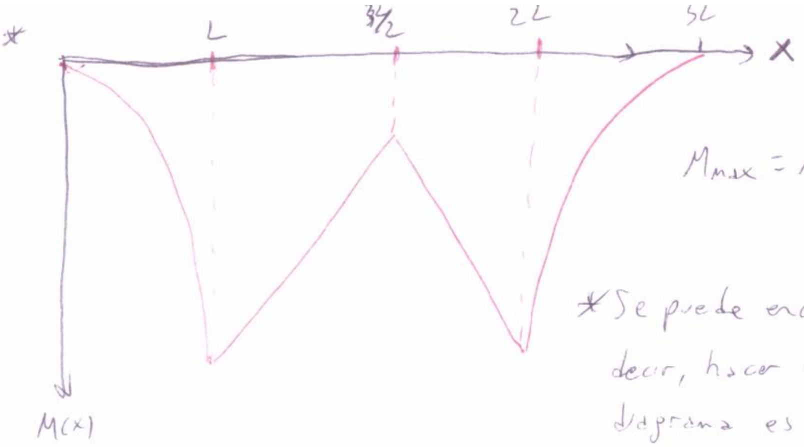
$$\boxed{M(x) = -w_0 L(x - L/2) + A(x - L) - P(x - 3L/2)}$$

d) $2L < x <$



$$\boxed{M(x) = w_0 L(x - L/2) - A(x - L) + P(x - 3L/2) - B(x - 2L) + \frac{w_0(x - 2L)^2}{2}}$$

*



$$M_{max} = M(x=L) = -\frac{\omega_0 L^2}{2} = -600\ N\cdot m$$

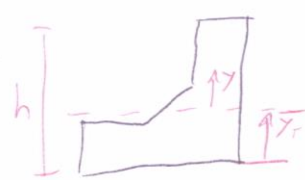
* Se puede encontrar el diagrama por simetría, es decir, hacer los cortes hasta 3L/2 y el resto del diagrama es simétrico

~ Esfuerzo normal por flexión

$$\sigma_x = -\frac{M(x)}{I_z} y \Leftrightarrow \sigma_{max} \text{ cuando } \begin{cases} M_{max} \\ y_{max} \end{cases}$$

* $M_{max} = -600\ N\cdot m$

* y_{max}



"y" en los extremos:

$$y_{max}^{(1)} = -\bar{Y}_T = -0{,}0331\ m$$

$$\boxed{y_{max}^{(2)} = h - \bar{Y}_T = 0{,}04688\ m}$$

$$y_{max} = y_{max}^{(2)}$$

$$\therefore \sigma_x = \frac{-(-600)\ N\cdot m}{1{,}174\cdot 10^{-6}\ m^4} \cdot 0{,}04688\ m \Rightarrow \boxed{\sigma_x = 23{,}96\ MPa}$$